Week 14: Cosmology

HW 11 due Dec. 3rd (Thursday next week)

HW 1-10 and Midterm 2 grades released

Last in-person class is today — entirely over Zoom next week

Project Presentations next week
Be prepared to present on Tuesday - students will be chosen at random

1. You take a spectrum of a star and determine its spectral type to be G2. You notice the widths of its Balmer absorption lines are narrower than the widths of the same lines in the Sun.

   (a) (5 pts) What can you conclude about the luminosity class of the star? Justify your answer.

Pressure broadening less for larger stars —> narrower lines —> must be a giant (IV or less)

   (b) (10 pts) If both the mass and the radius of the star are 10× larger than the mass and radius of the Sun, what is the star's surface gravity?

$$g = \frac{GM}{R^2} = \frac{G(10M_\odot)}{(10R_\odot)^2} = 0.1 g_\odot$$

   (c) (5 pts) If the star's composition is comparable to that of the Sun (i.e., $\mu \approx \mu_\odot$), how should its central temperature T_c compare to the Sun's?

$$T_c \approx \frac{2GM\mu m_p}{Rk} = \frac{2G(10M_\odot)\mu_\odot m_p}{10Rk} = T_{c,\odot}$$

2. In high mass stars, silicon-28 (^{28}Si, mass of 27.9769 m_u) is fused into ^{40}Ca (39.9626 m_u) through a chain of reactions involving 3 alpha particles (^{4}He, 4.0026 m_u) and intermediate products ^{32}S (31.9721 m_u) and ^{36}Ar (35.9675 m_u).

(a) (10 pts) How much energy is produced by each reaction?

$$\Delta E = \Delta m c^2 = (m_\mathrm{Si} + 3m_\mathrm{He} - m_\mathrm{Ca})c^2 = 3.3 \times 10^{-12}\ \mathrm{J}$$

(b) (10 pts) If 0.05 $M_\odot$ of silicon is completely burned into calcium over 1000 years, what should the star's luminosity be (in units of $L_\odot$) assuming all photons coming from the star are produced in these reactions?

$$L = \frac{E_\mathrm{tot}}{t} = \frac{N_\mathrm{reactions}\Delta E}{t} = \frac{0.05 M_\odot}{28 m_u}\frac{\Delta E}{t} = 5.9 \times 10^5\ L_\odot$$

3. Imagine you observe a star through a gas cloud, and you measure its V-band magnitude to be $m_V = 15.0$ and its color to be $B - V = 1.3$.

   (a) (10 pts) If the cloud's optical depth is $\tau_V \approx 5$, what would the star's apparent magnitude be in the absence of the cloud?

$$m_{\rm obs} = m_0 + 1.086\tau$$
$$15 = m_0 + 1.086 \cdot 5 \longrightarrow m_0 = 9.6$$

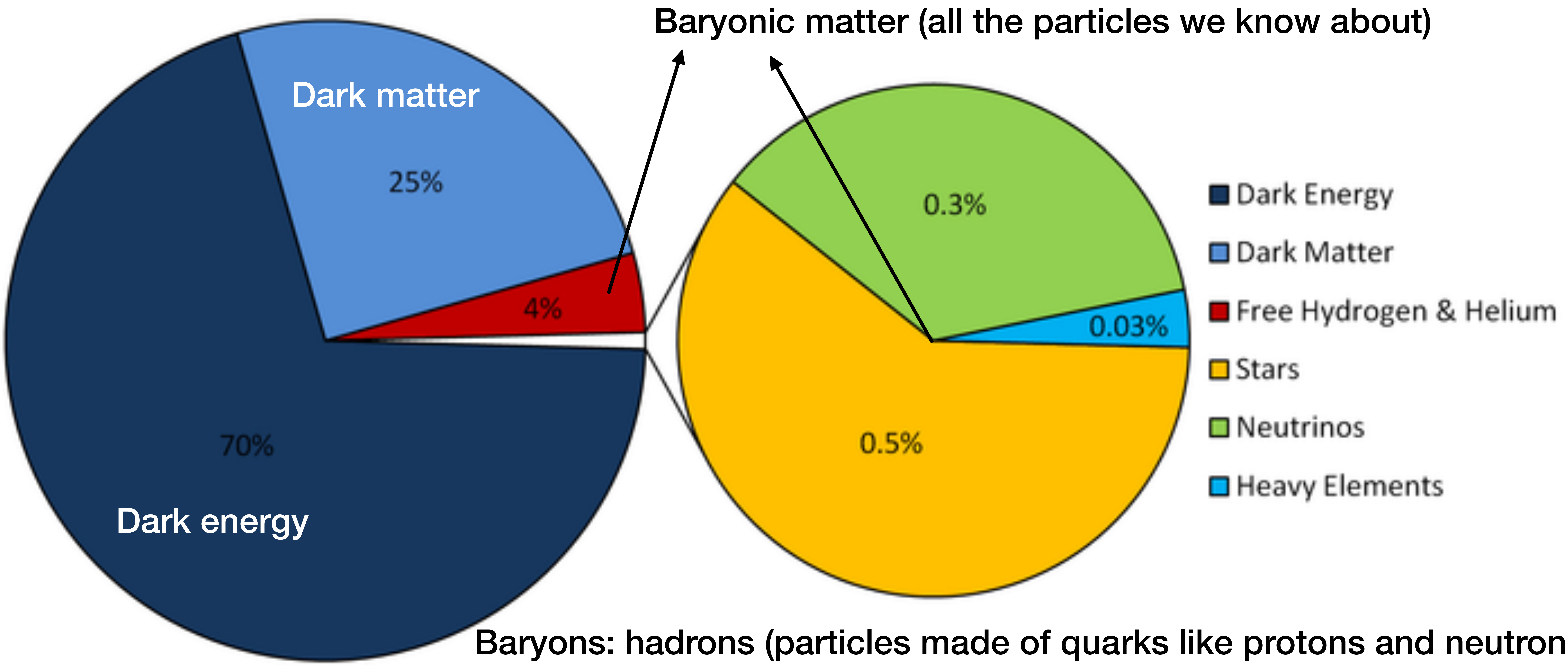
   (b) (10 pts) Assuming that the local reddening law holds in the cloud (i.e., $R_V \approx 3.1$), what would the star's apparent B magnitude be in the absence of the cloud?

$$R_V = \frac{A_V}{E(B-V)} = \frac{1.086 \cdot 5}{E(B-V)} = 3.1$$

$$(B-V)_{\rm obs} = (B-V)_0 + E(B-V)$$
$$B_0 = V_0 + (B-V)_{\rm obs} - E(B-V) = 9.6 + 1.3 - 1.75 = 9.15$$

Cosmology

Relative Contents of the Universe



Baryons: hadrons (particles made of quarks like protons and neutrons)
Leptons: lighter particles not made of quarks, including electrons

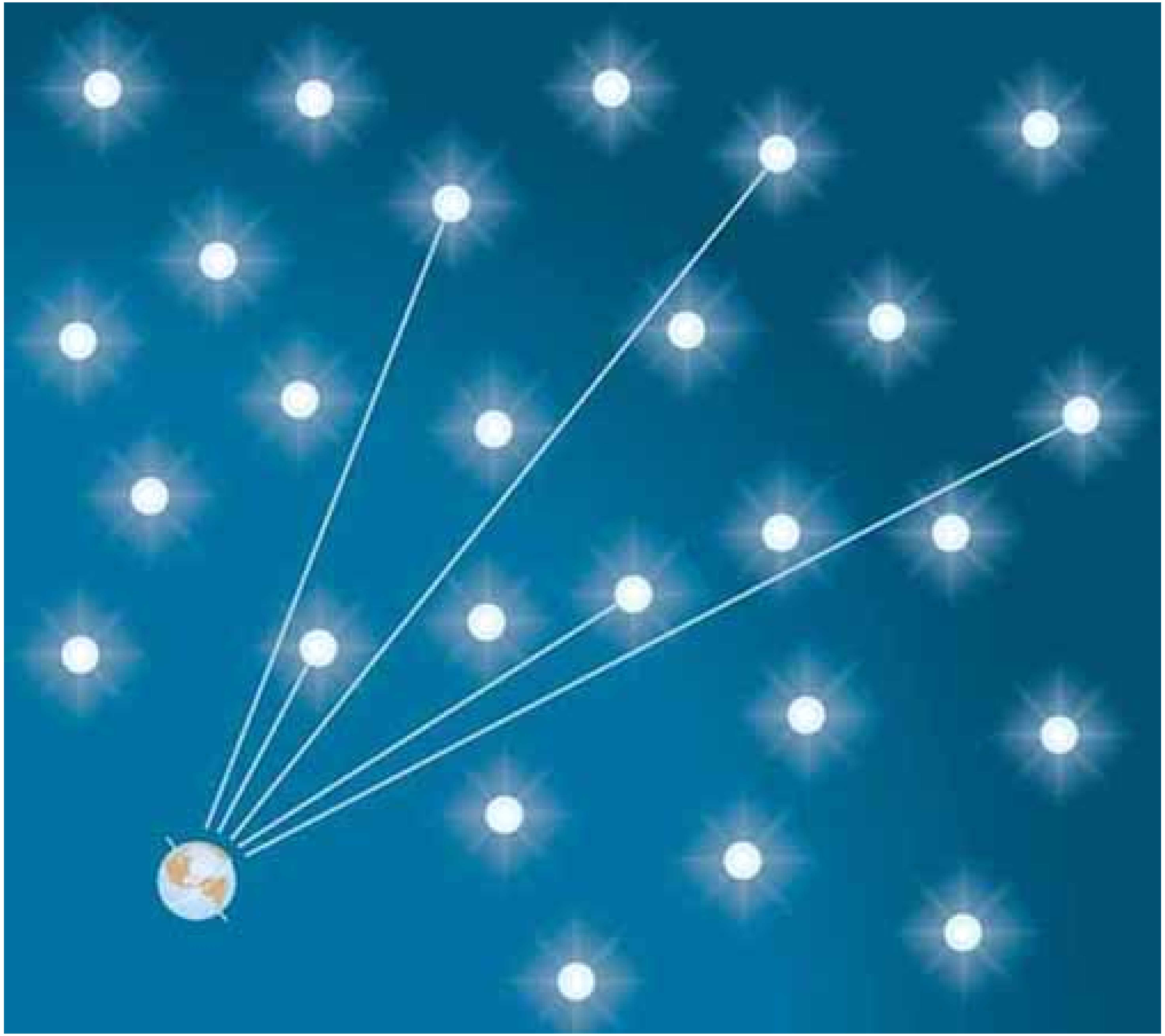
First observation:
The night sky is dark!

What does this imply?

Olber's paradox:

Infinitely old, infinitely large universe full of stars

Sky should be as bright as the disk of the Sun!



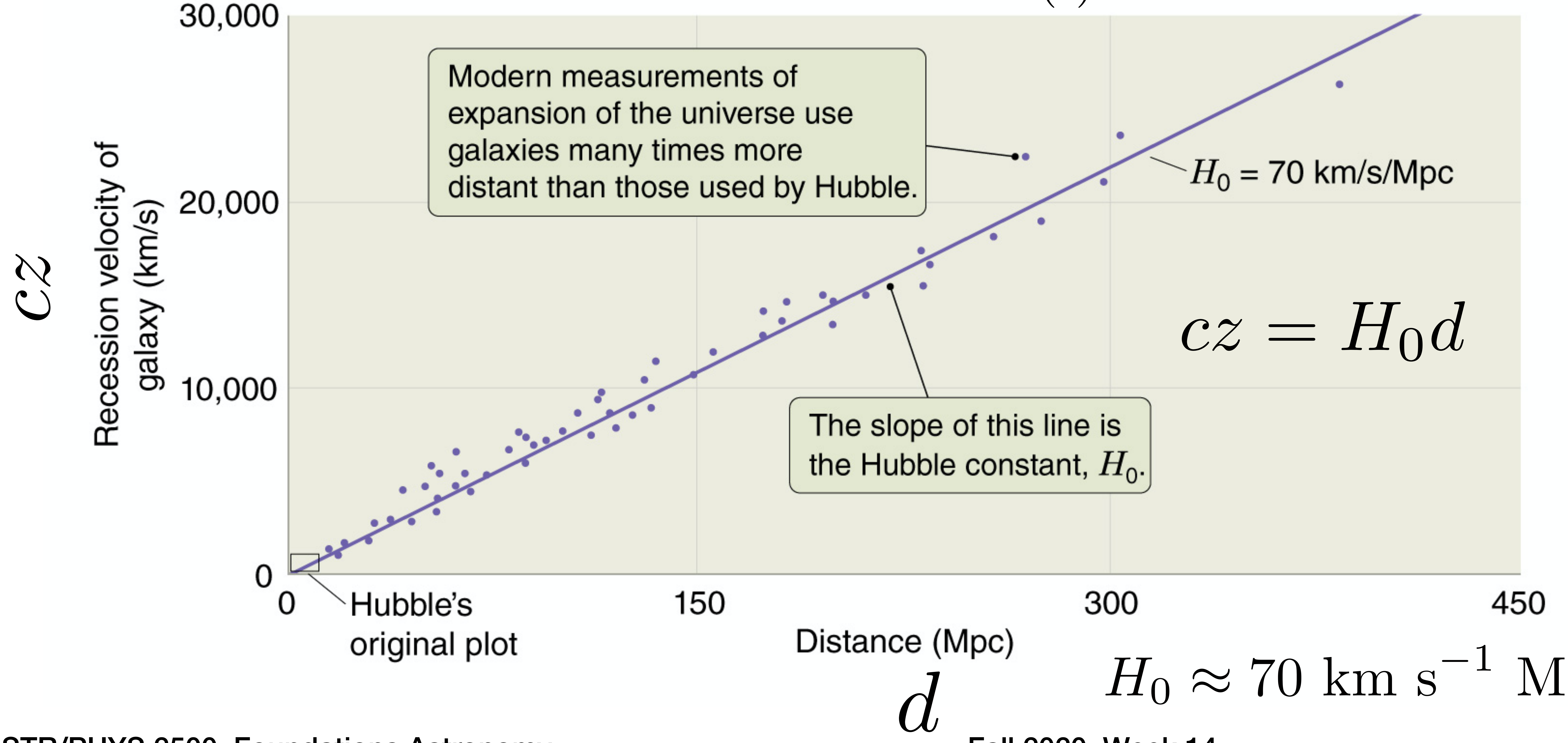
Hubble's Law

$$r(t) = a(t) r_0$$

$$v(r) = \dot{r} = \dot{a} r_0 = \frac{\dot{a}}{a(t)} r(t)$$

$$v(t) = H(t) r(t)$$

↓ Hubble parameter



$$cz = H_0 d$$

$$H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$r(t) = a(t) r_0$$
$$v(r) = \dot{r} = \dot{a} r_0 = \frac{\dot{a}}{a(t)} r(t)$$

$$v(t) = H(t) r(t)$$
↓ Hubble parameter

$$v(t_0) = H(t_0) r(t_0) = cz = H_0 d$$
↓

Current time = age of the universe $t_0 \approx 13.8\ \mathrm{Gyr}$

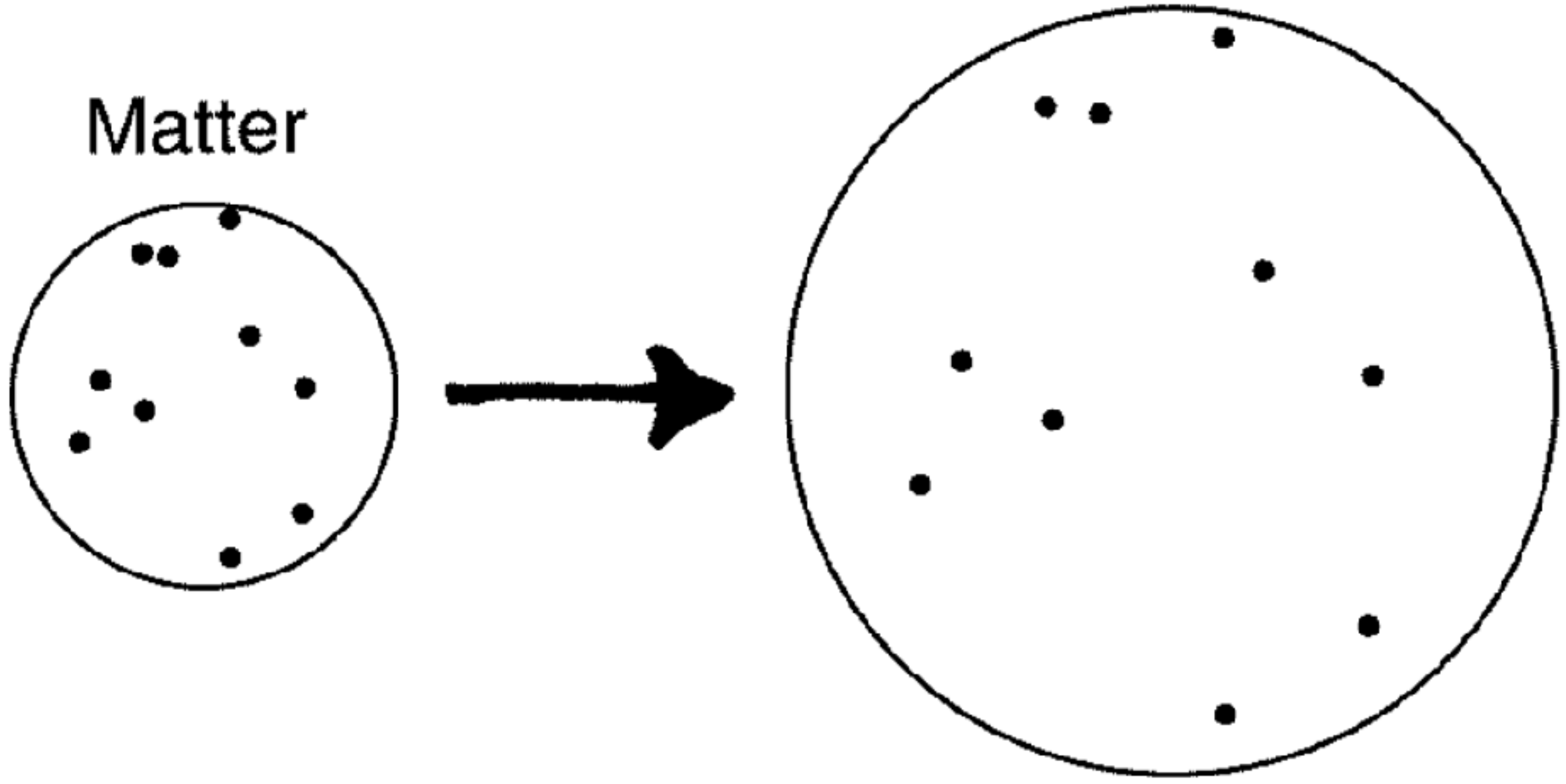
Horizon distance
(Farthest away we can see)

$$r(t_0) \sim c t_0 \sim c/H_0$$

Expanding universe implies a hot, dense state in the past

Call this the Big Bang
(Should include radiation, which would still be around today)

How does the expansion speed change with time?



When dense, the gravitational force is stronger
(But also the expansion speed was higher)

Like shooting a cannonball straight up:

1) $v < v_{esc}$: go up and come back down
2) $v = v_{esc}$: stop when infinitely far away
3) $v > v_{esc}$: reach coasting velocity

Same is true if all particles exploding away from each other

Friedmann Equation

Master equation of the universe

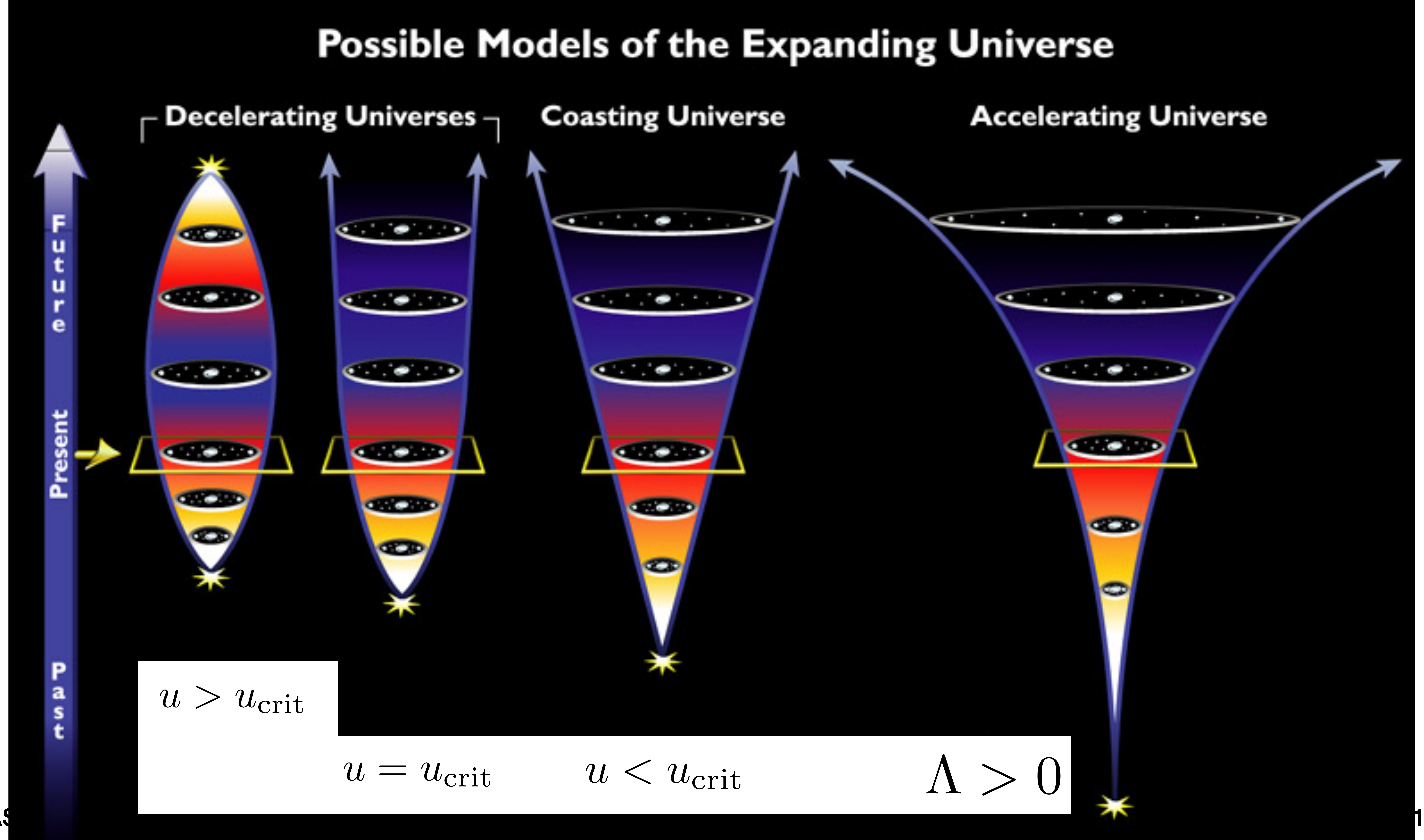
Curvature of space
(Flat =0, spherical <0, or hyperbolic >0 geometry?)

Cosmological constant
(Dark energy)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}u(t) + \frac{\kappa c^2}{r_{c,0}^2}\frac{1}{a(t)^2} + \frac{\Lambda}{3}$$

Hubble parameter
(Speed of expansion)

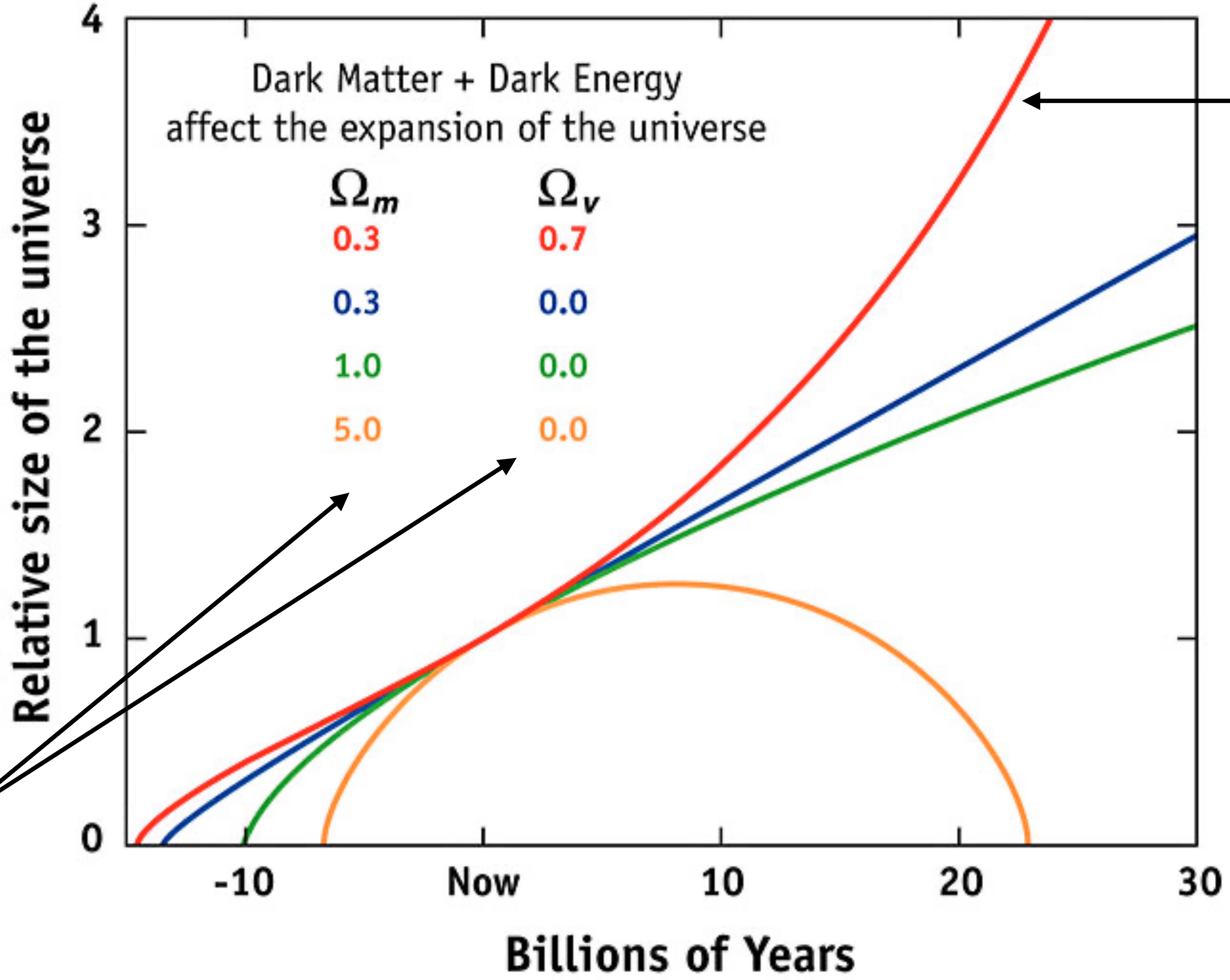
"Energy density"
(Sum of rest mass & KE of all particles + radiation)



$u > u_{\rm crit}$

$u = u_{\rm crit}$ $u < u_{\rm crit}$ $\Lambda > 0$

Expansion of the Universe

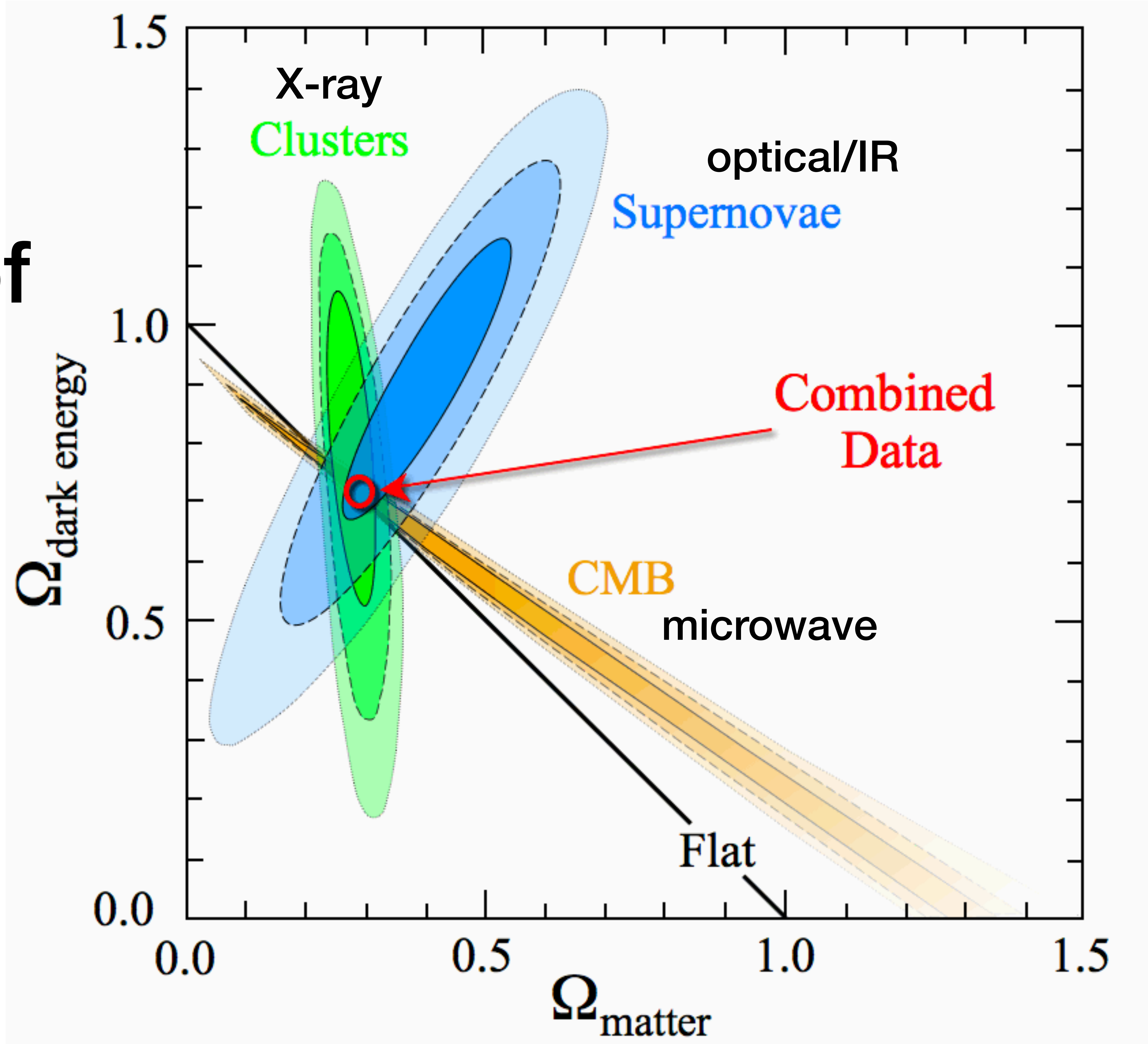


$a(t)$

Our universe (we think)

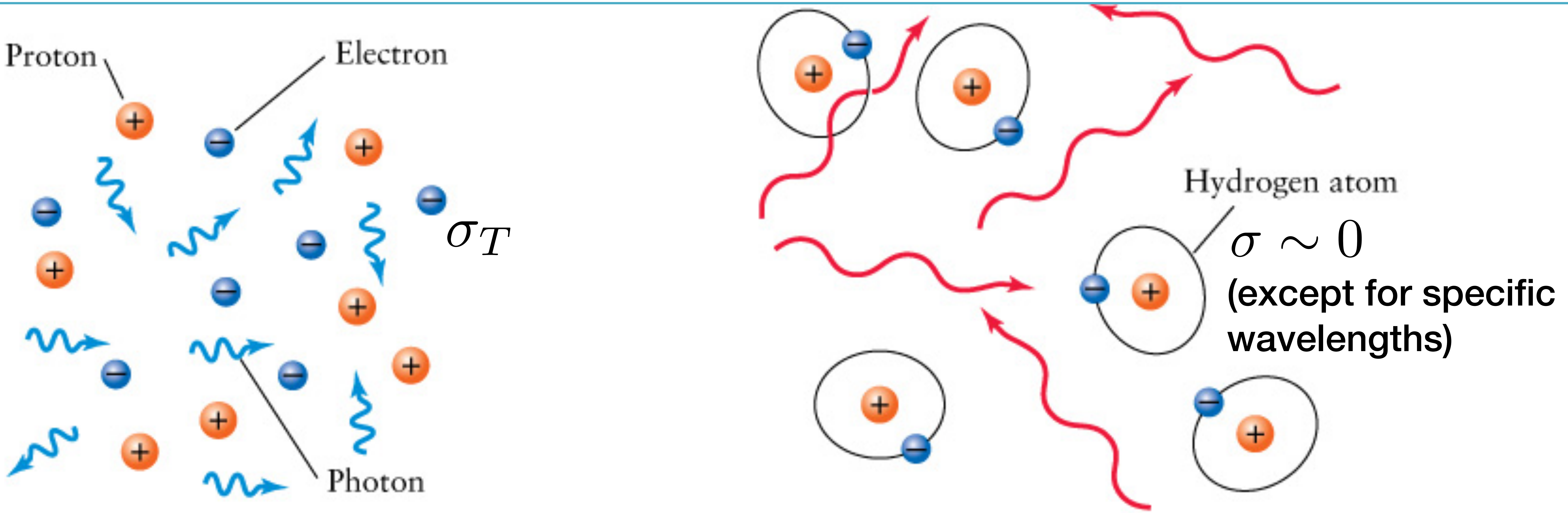
Fractions relative to the critical density

Many different kinds of observations agree



What happens if we roll back the clock to the early universe?

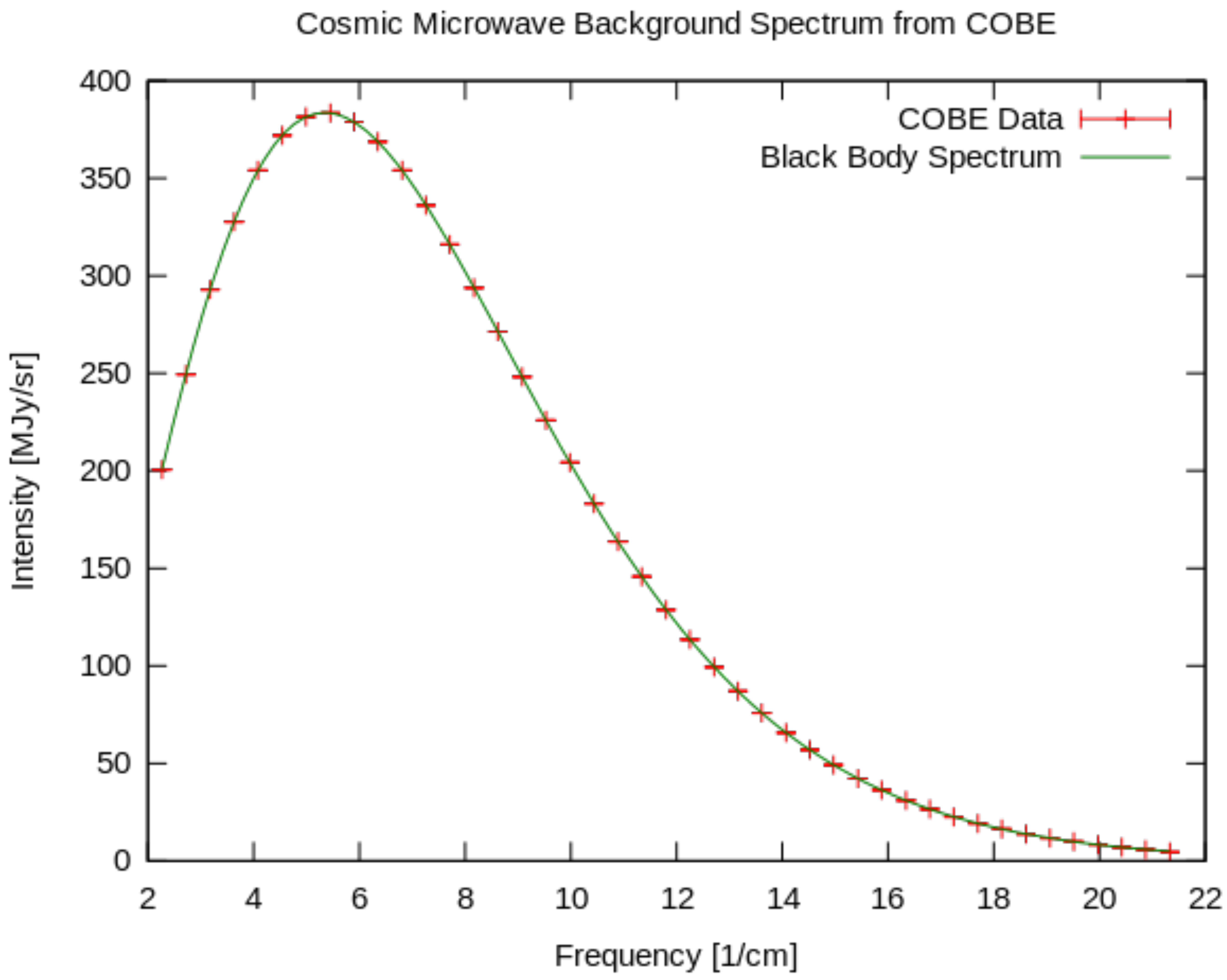
At some point in the past, universe so hot it is ionized —> opaque

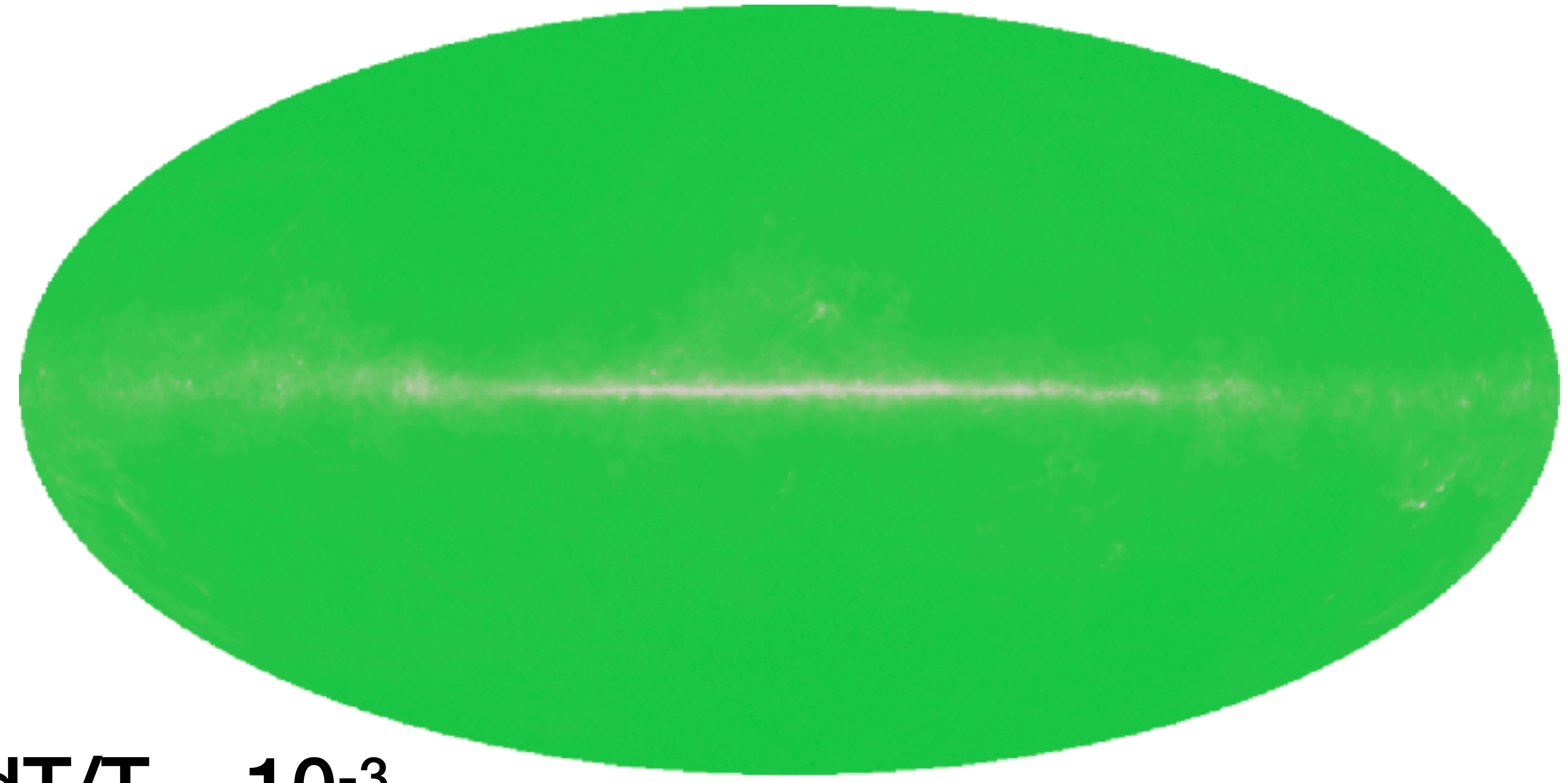


(a) Before recombination:
- Temperatures were so high that electrons and protons could not combine to form hydrogen atoms.
- The universe was opaque: Photons underwent frequent collisions with electrons.
- Matter and radiation were at the same temperature.

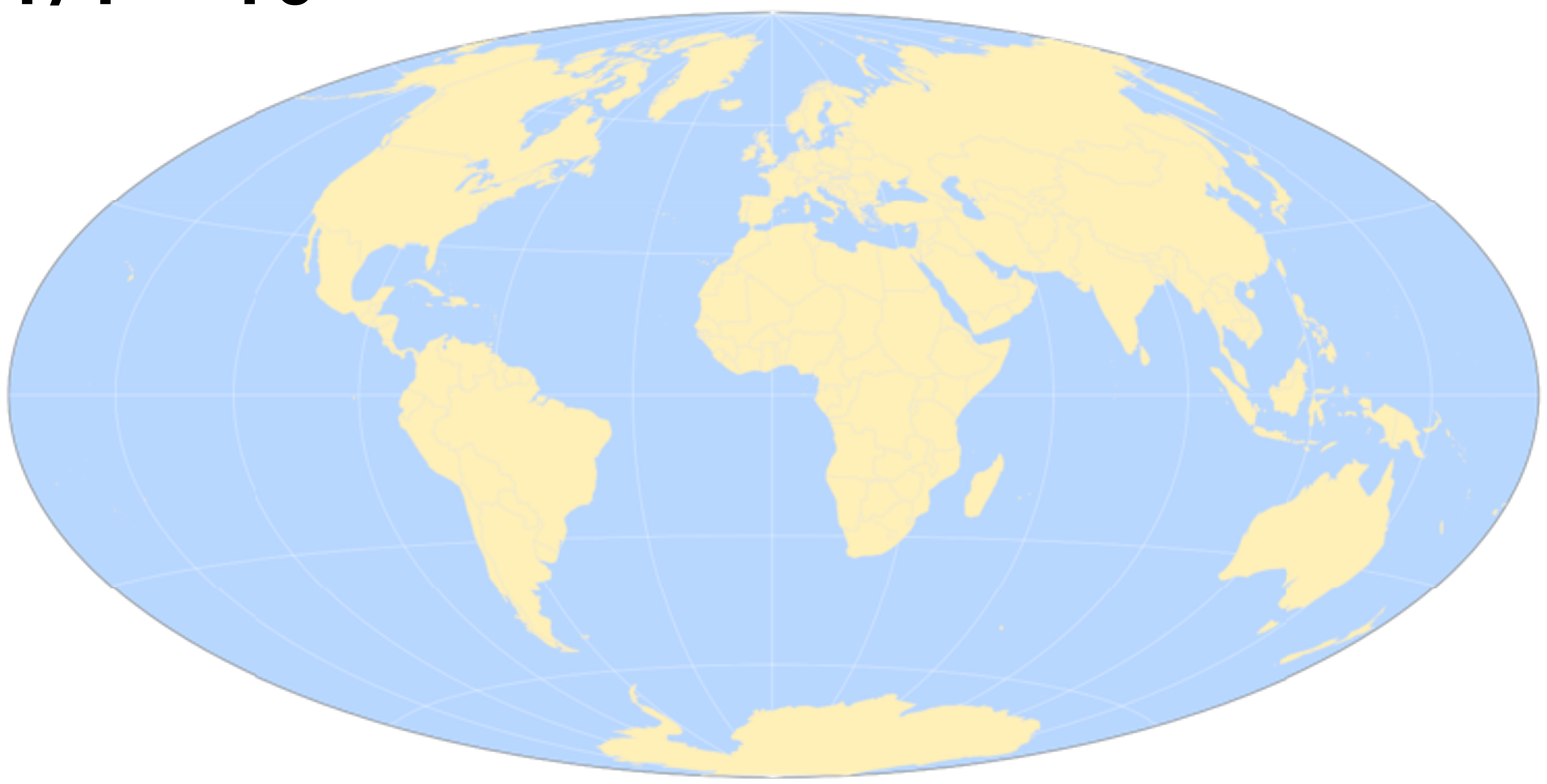
(b) After recombination:
- Temperatures became low enough for hydrogen atoms to form.
- The universe became transparent: Collisions between photons and atoms became infrequent.
- Matter and radiation were no longer at the same temperature.

Near perfect BB everywhere on the sky





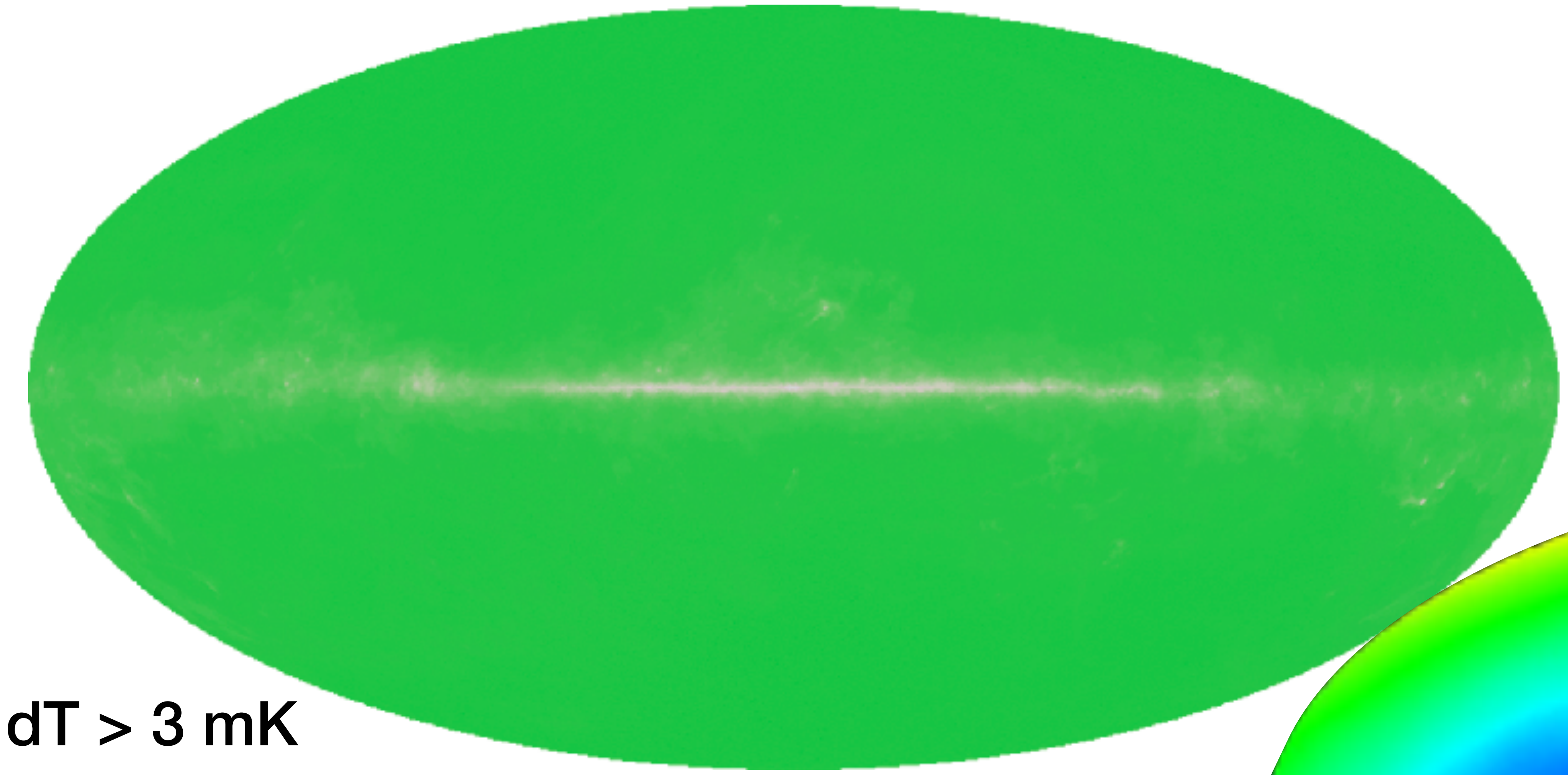
dT/T ~ 10^{-3}



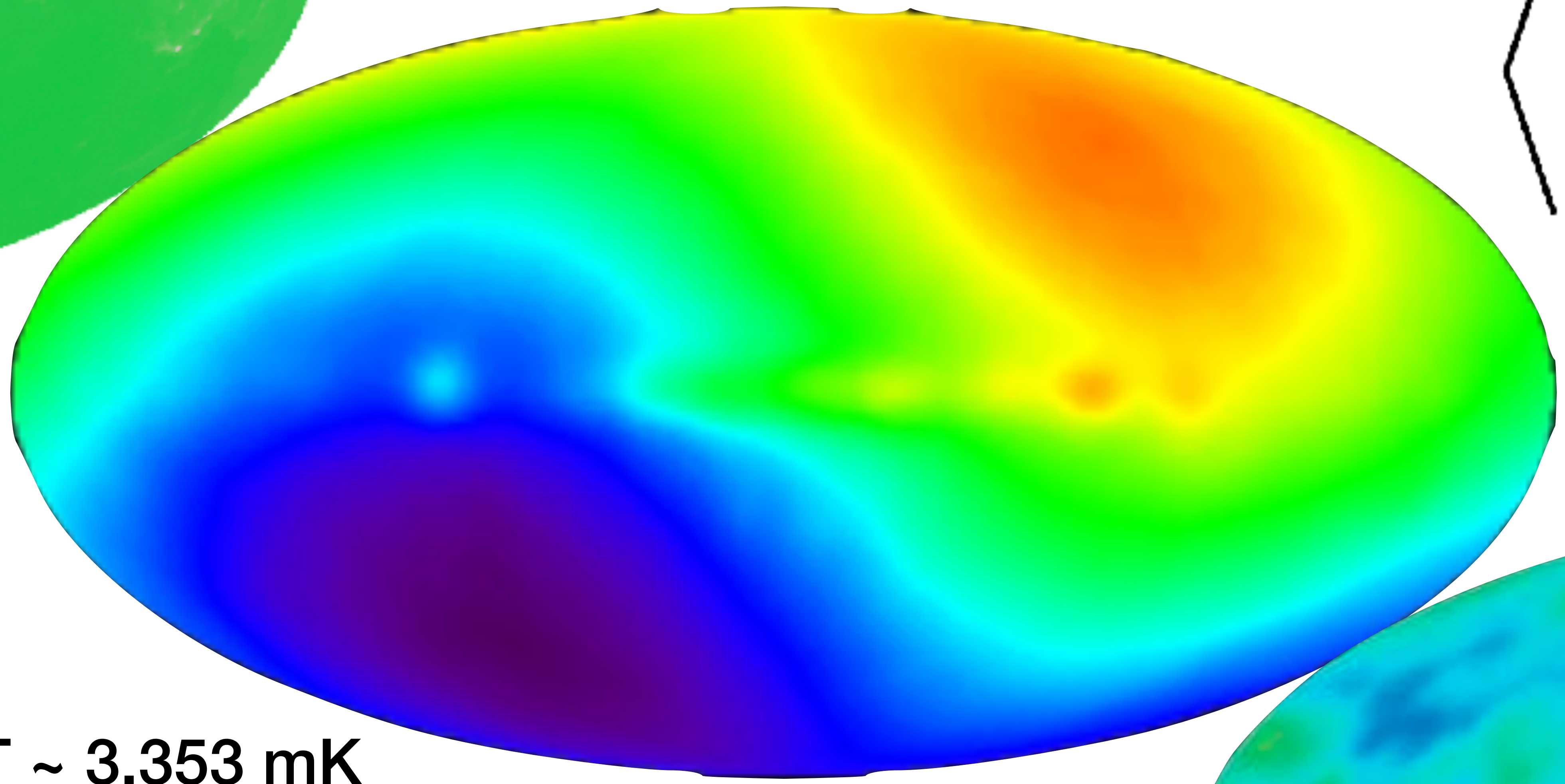
Spatial variations on different scales

$$\frac{\delta T}{T}(\theta, \phi) \equiv \frac{T(\theta, \phi) - \langle T \rangle}{T}$$

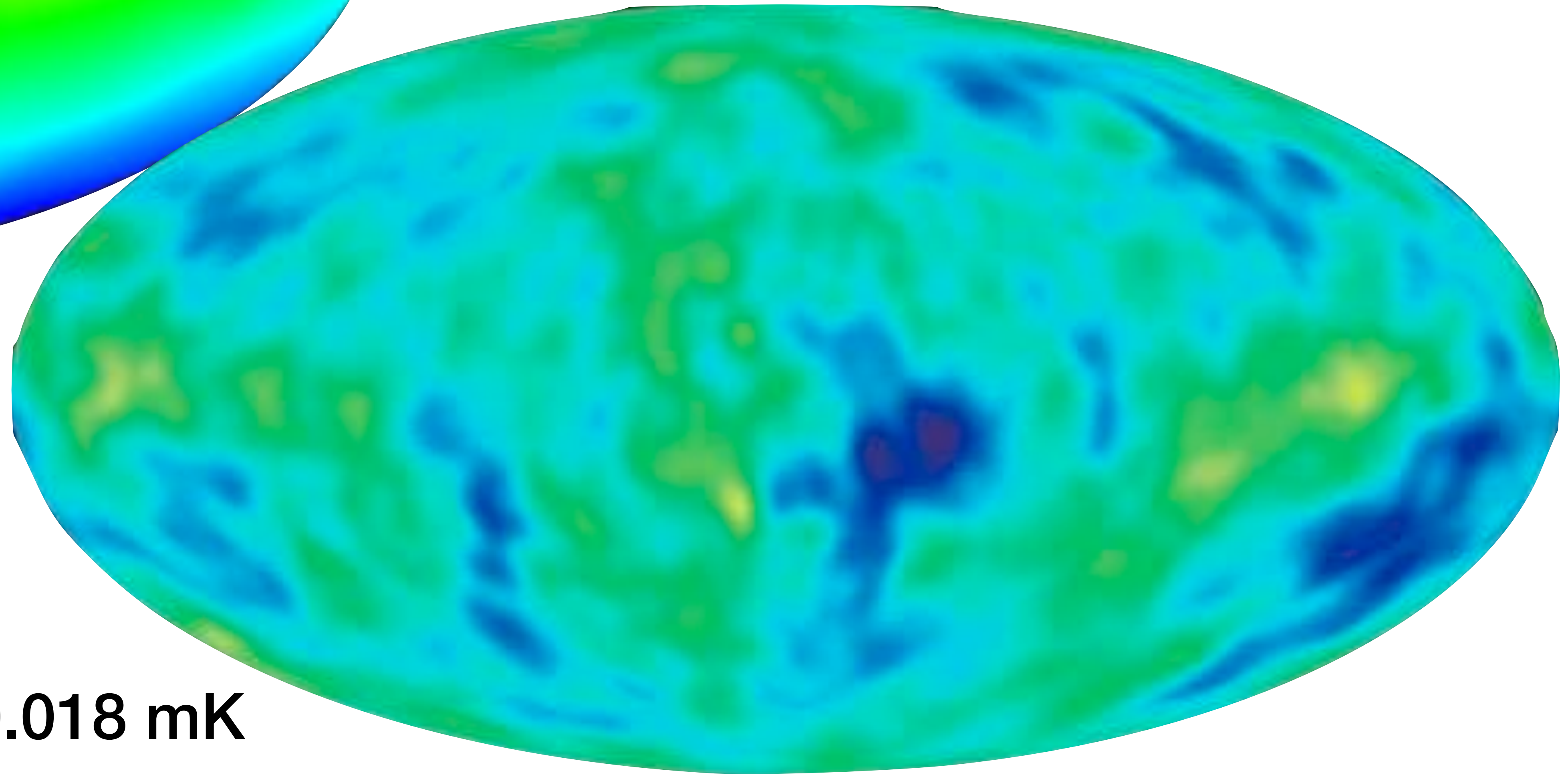
$$\left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5}$$



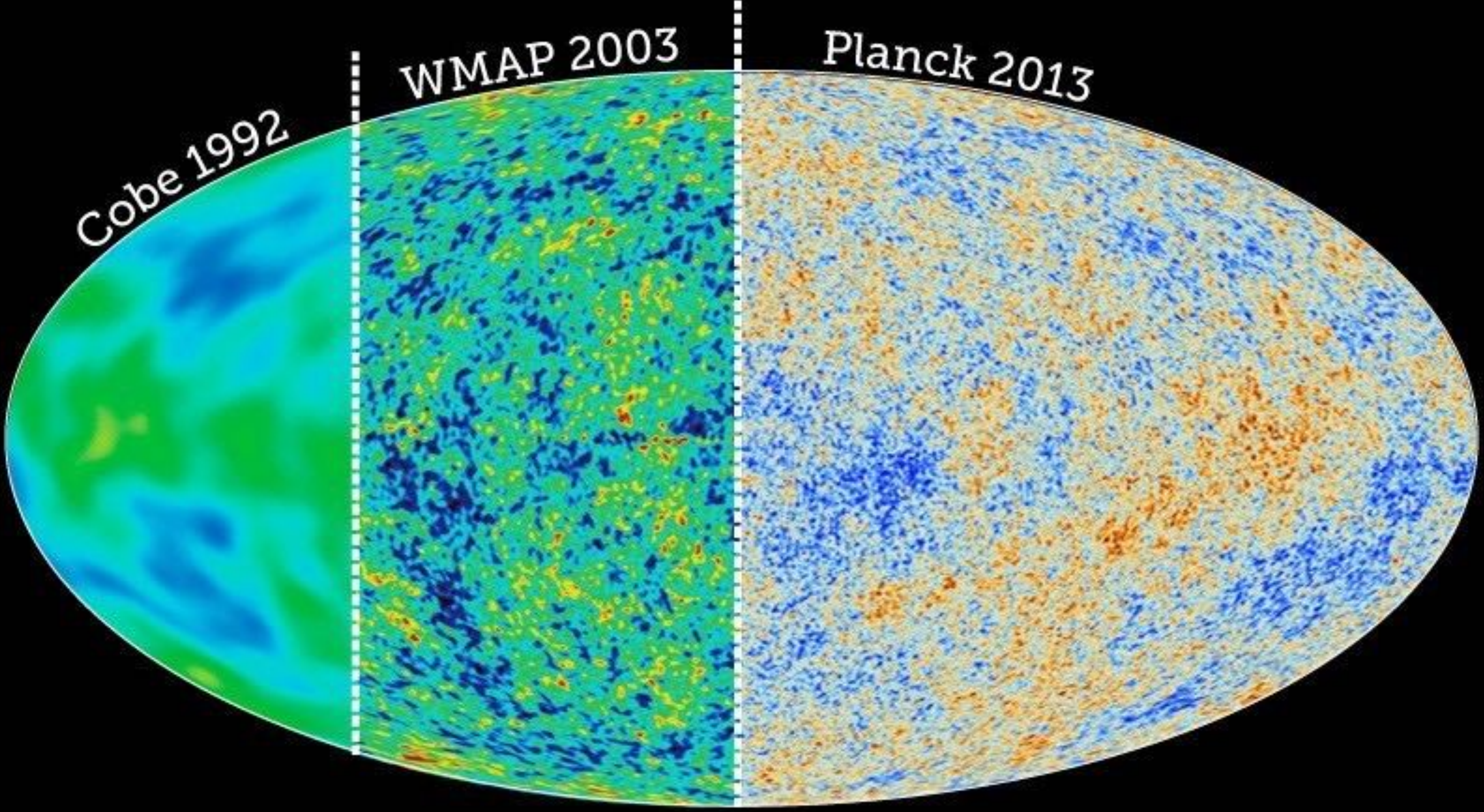
dT > 3 mK

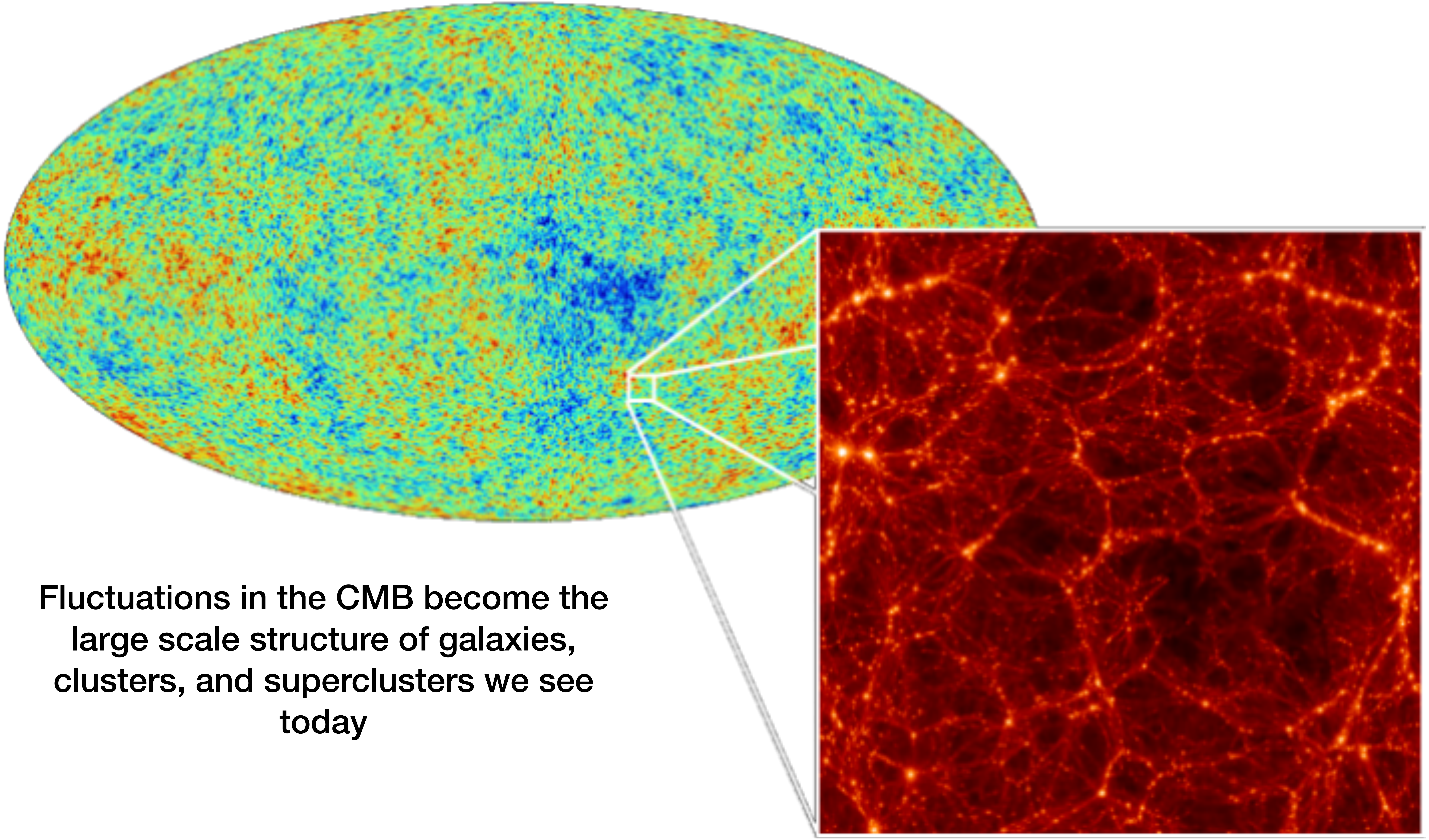


dT ~ 3.353 mK



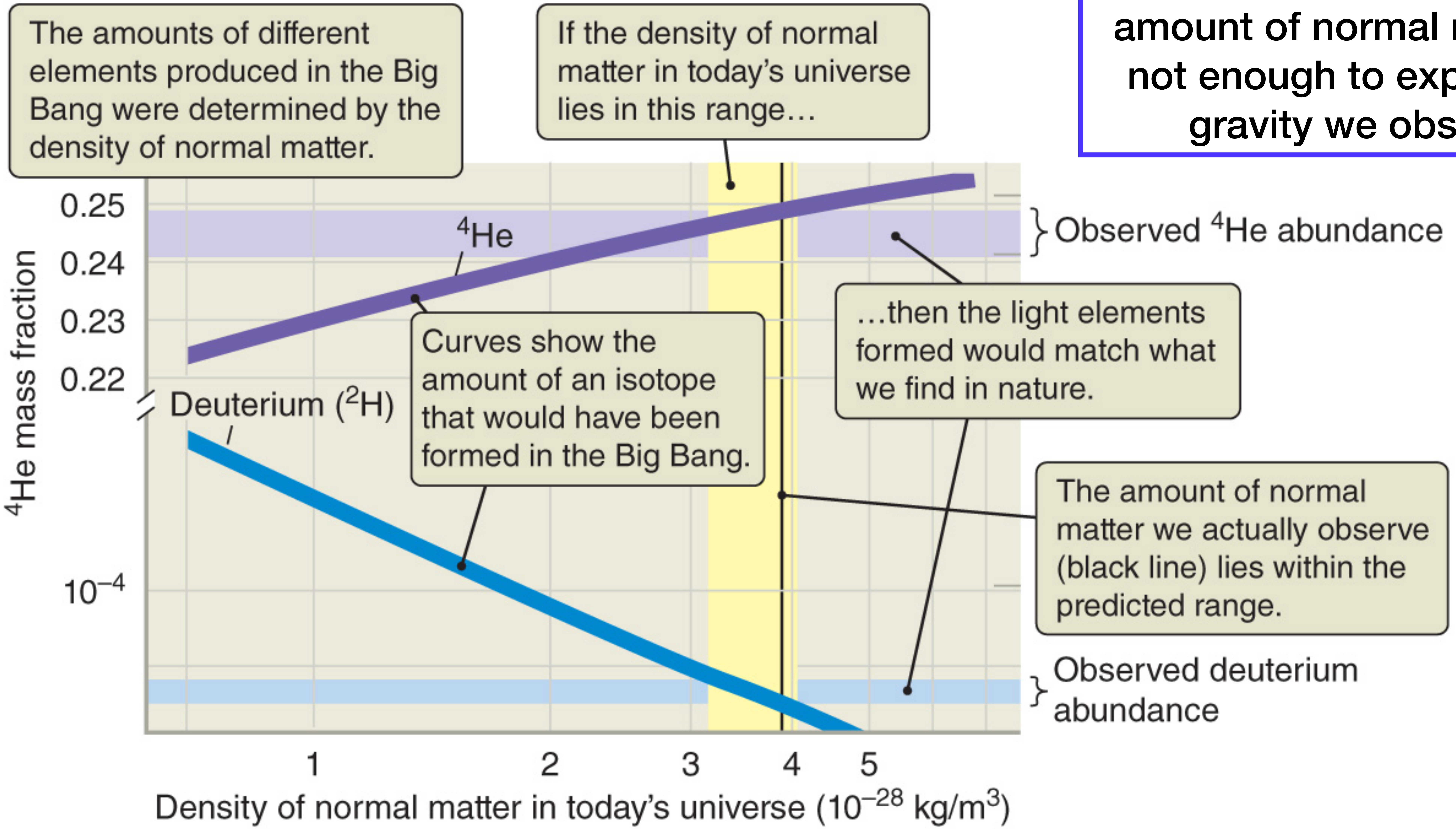
dT ~ 0.018 mK


Cobe 1992
WMAP 2003
Planck 2013



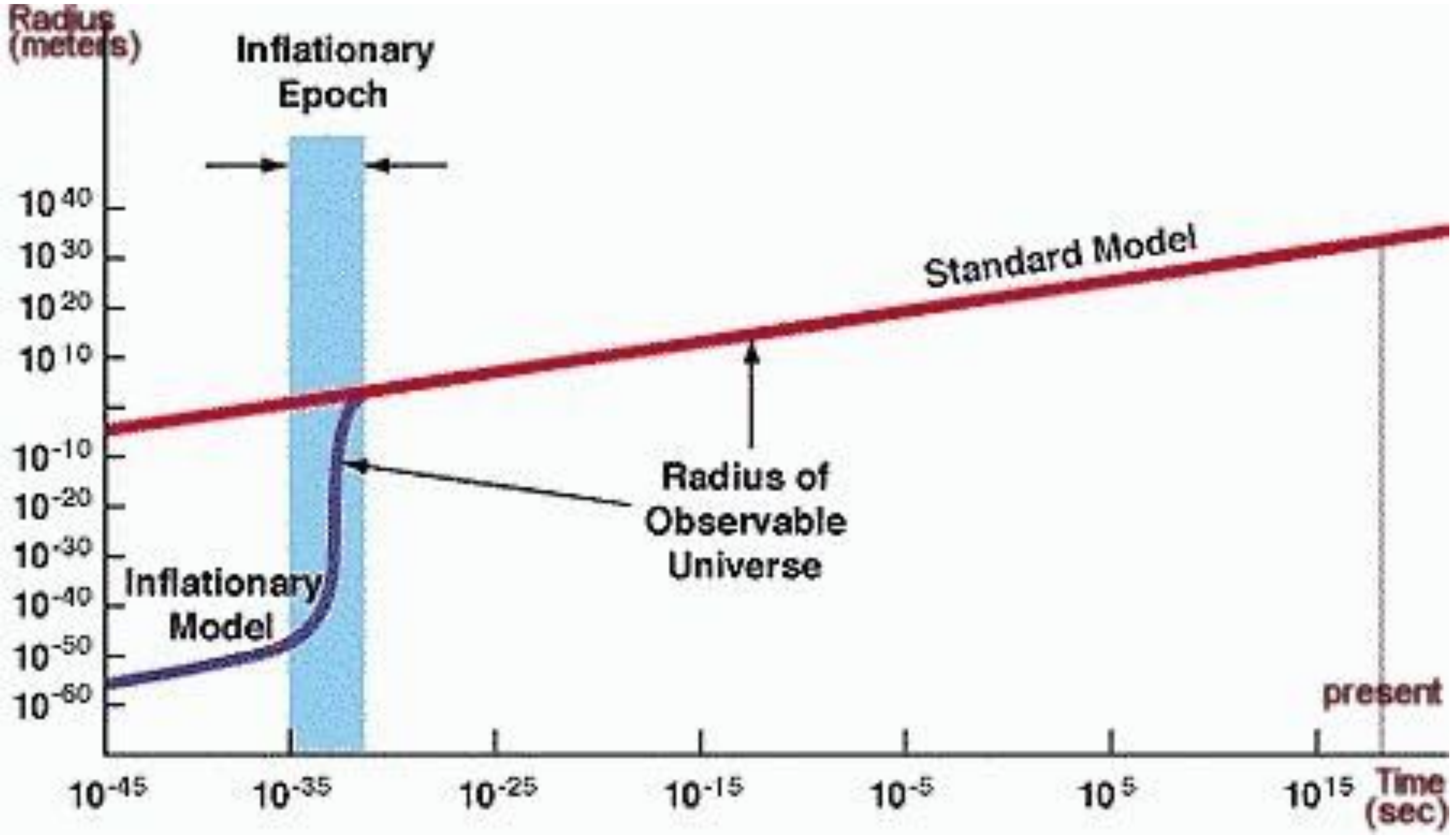
Fluctuations in the CMB become the large scale structure of galaxies, clusters, and superclusters we see today

Go further back, hot enough for H—>He fusion

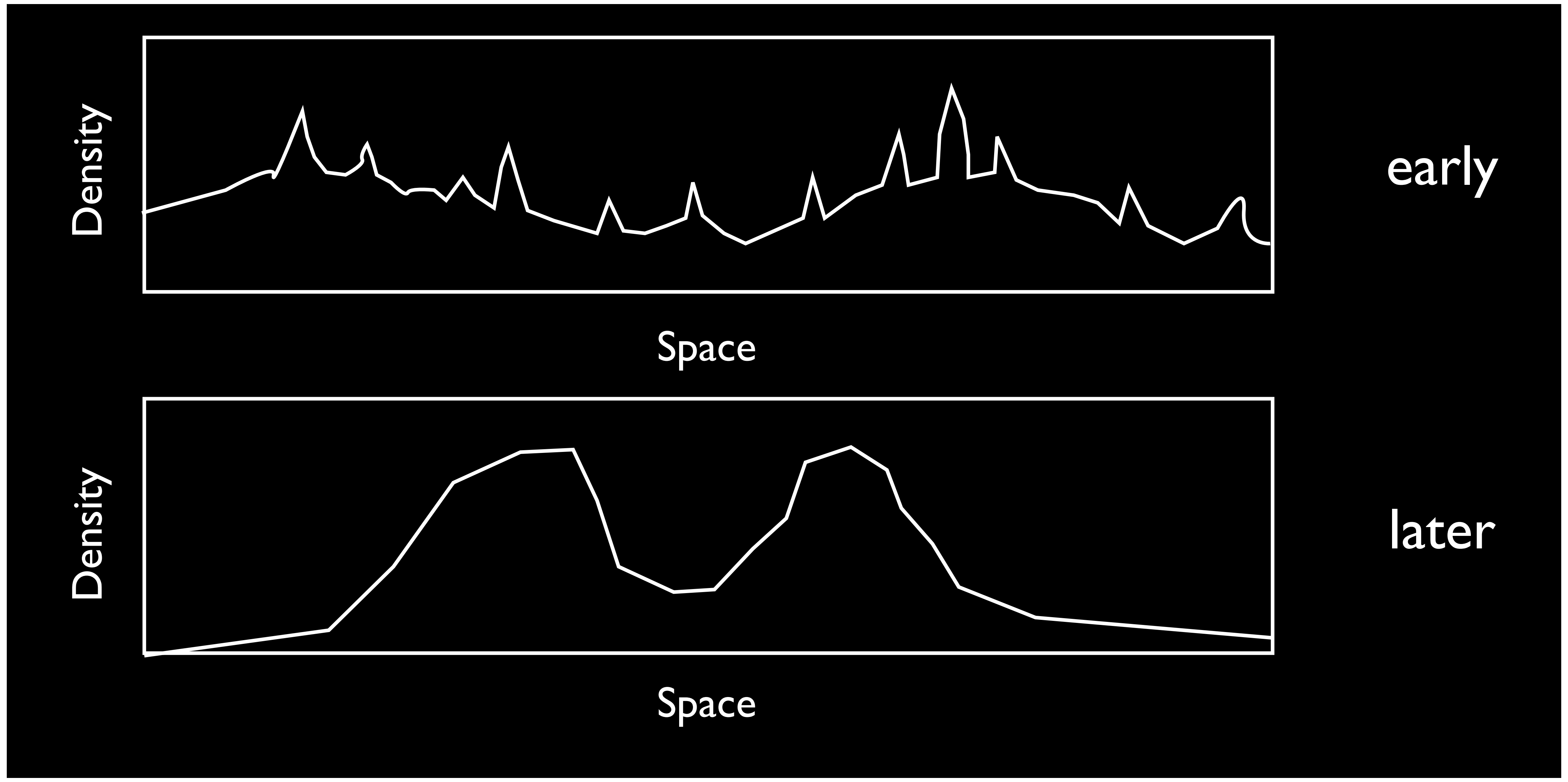


amount of normal matter is not enough to explain the gravity we observe

Way further back: origin of structure (inflation?)



Initial quantum density perturbations amplified by Inflation after the Big Bang.

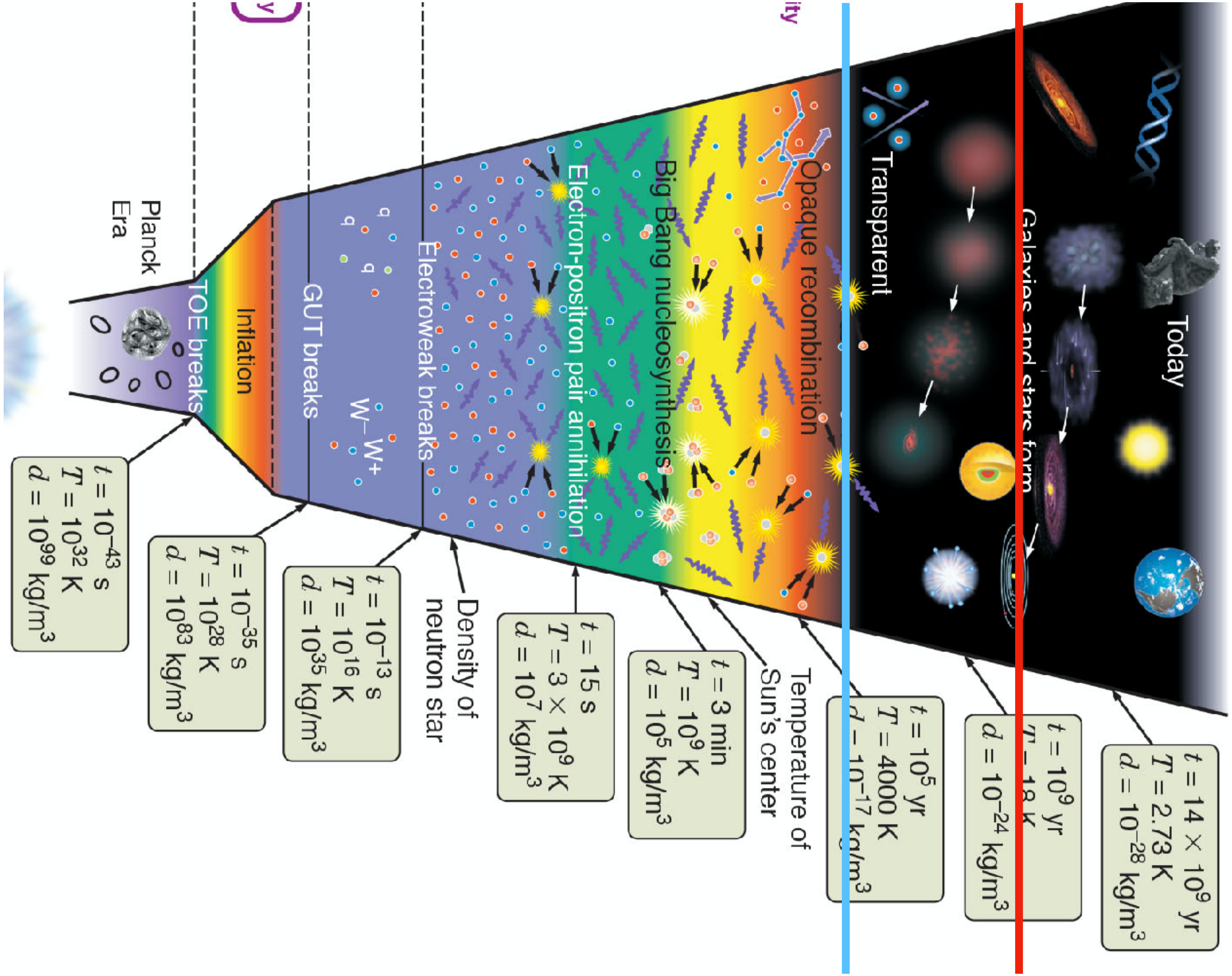


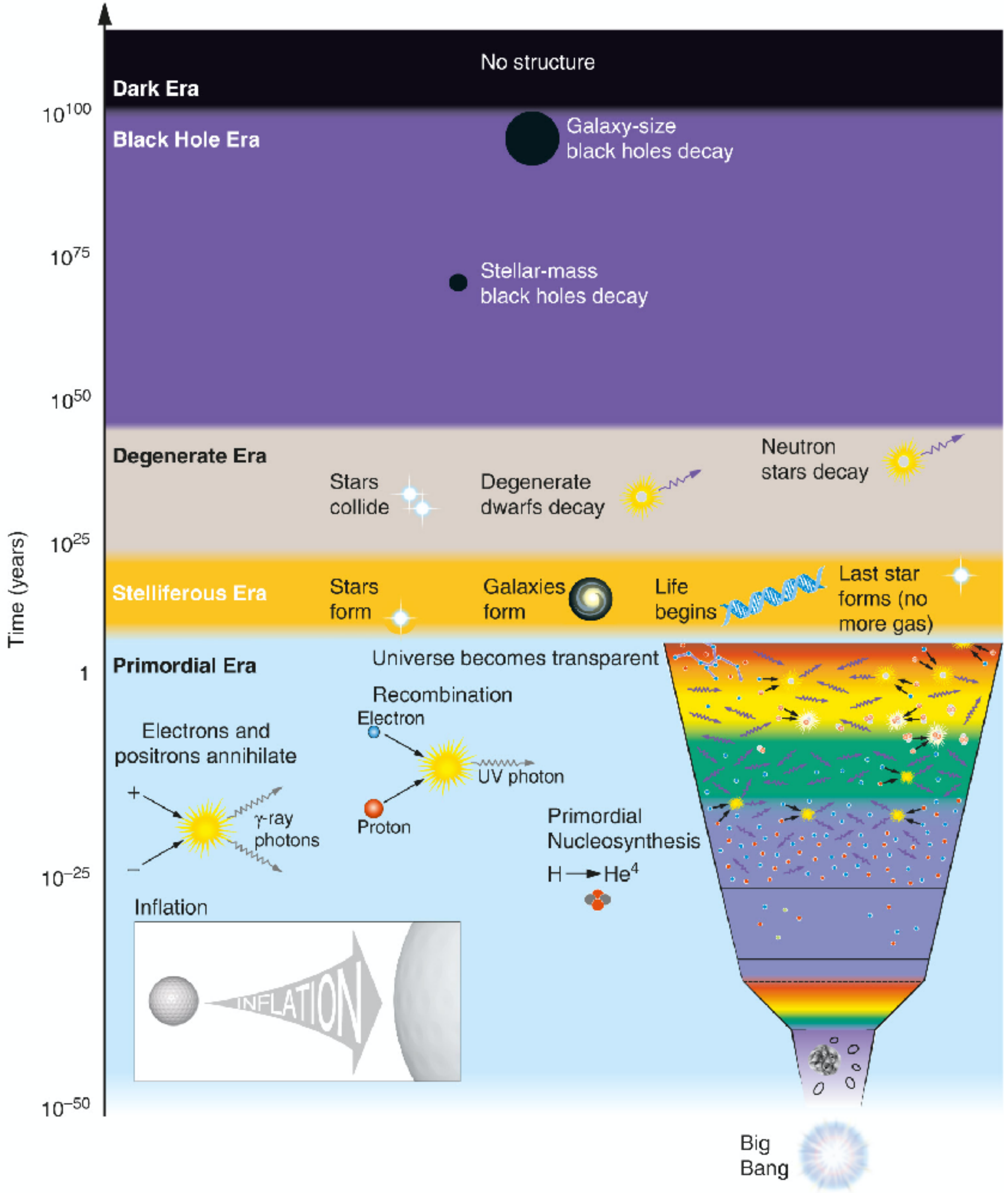
Called Hierarchical Structure Formation

Universe is opaque

Electrons & ions combine (recombination)

Universe gets ionized again (reionization)





The Deep Future (maybe?)

Primordial Era	10^5 yr
Stelliferous Era	10^{14} yr
Degenerate Era	10^{39} yr
Black Hole Era	10^{100} yr
Dark Era	infinity?