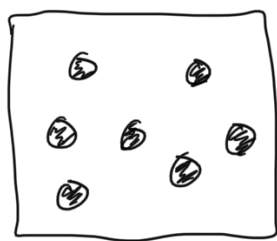
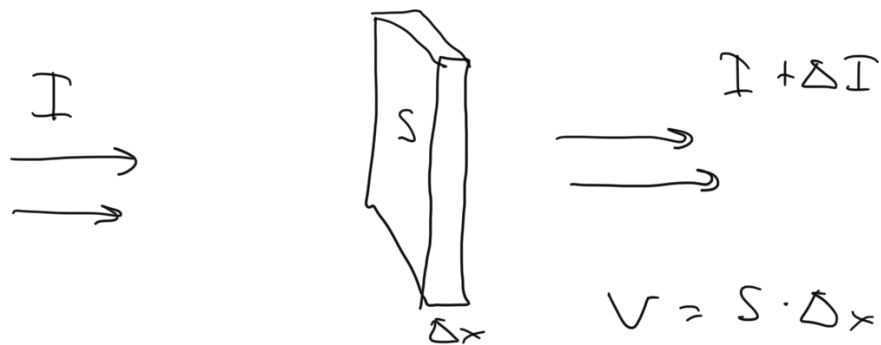


ASTR 2500 - Ch.5

Radiative Transfer

To understand the light we see, we need to know what happens to it on our way here \rightarrow universe is full of crap

Imagine light w/ some "intensity" I
 $\vec{I} \rightarrow \frac{\text{energy}}{\text{area} \cdot \text{time}}$ in same direction



Squish $\Delta x \rightarrow 0$, project cross-section of atoms onto surface S , each atom has area $\pi r^2 = \sigma$

so total cross-section is $\sigma_T = N\sigma$

If light is shining across ^{↖ # atoms} surface, but is absorbed wherever there are atoms,

$$\frac{\Delta I}{I} = -\frac{\sigma_T}{S}$$

Loss of energy just geometric.

But, fundamental absorbing element is the atom, so want in terms of σ

$$\frac{\Delta I}{I} = -\frac{N\sigma}{S} \rightarrow \text{what is } N?$$

Normally we know the density $n = \frac{N}{V}$ of a thing, so $N = nV = nS\Delta x$

$$\frac{\Delta I}{I} = -\frac{n\sigma}{S} \Delta x$$

Can make this a differential eqn.

$$\int_{I_0}^I \frac{dI}{I} = \int_0^x -n\sigma dx$$

$$\ln\left(\frac{I}{I_0}\right) = -n\sigma x$$

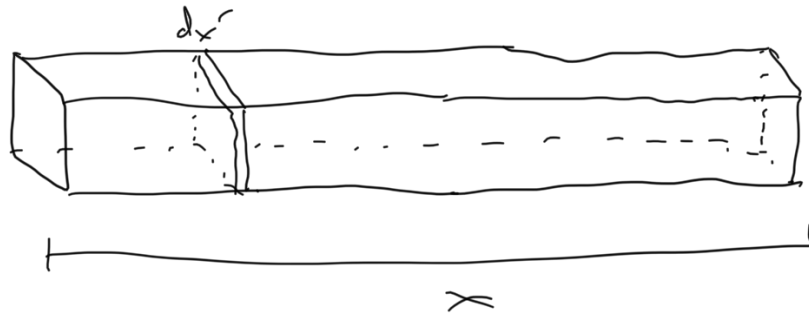
$$I(x) = I_0 e^{-n\sigma x}$$

Equ. of
Radiative
Transfer

Column Density

- integrated density along line-of-sight

$$N(x) \equiv \int_0^x n(x') dx' = n x$$



Optical Depth

Term in the exponent

$$\tau = n\sigma x = \sigma N(x)$$

$$I(x) = I_0 e^{-\tau(x)}$$

$\tau(x)$ $1 e\text{-folding}$

$$e \approx 2.72$$



Mean Free Path

- distance avg. photon travels before being absorbed / scattered : x when $\tau = 1$

$$\text{m.f.p. } \langle x \rangle = \frac{1}{n\sigma}$$

★ Real life example of optical depth

In general, processes are λ -dependent,

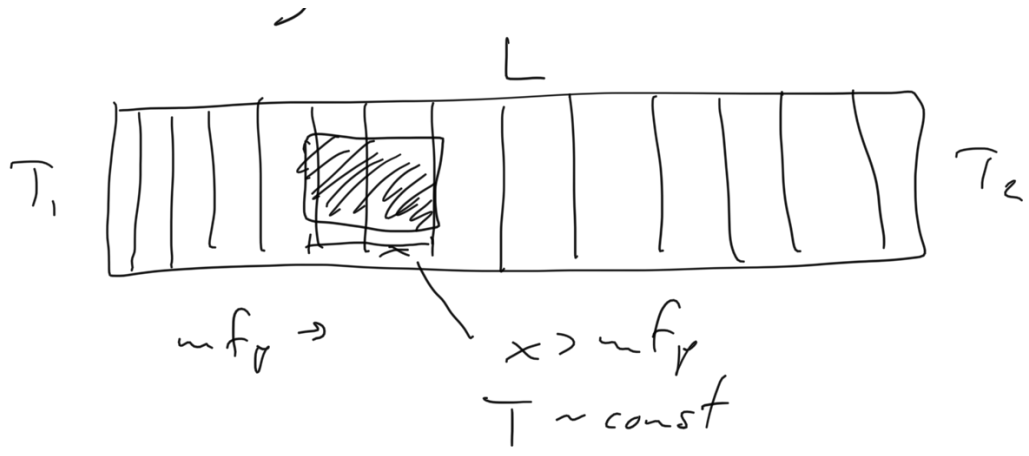
$$\text{so } I_{\nu} = I_{\nu 0} e^{-\tau_{\nu}(x)}$$

Local Thermodynamic Equilibrium

follows avg. statistics
Max-Boltz. dist.
etc.

no time depend

regions exist that are large enough where $x \gg \text{mfp}$, but small enough such that $\Delta T \ll T$



- particles share energy thru collisions, reach equilibrium

$$F(v) \propto v^2 e^{-mv^2/2kT}$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

Consider air, $T \sim 300K$ so $\frac{3}{2} kT \sim 0.04$

$$\begin{aligned} \text{rms } v &= \sqrt{\langle v^2 \rangle} = \sqrt{\frac{2 \cdot 0.04 \text{ eV}}{28 \times 10^{-9} \text{ eV}}} \cdot 3 \times 10^8 \text{ m/s} \\ &\sim 500 \text{ m/s (Nitrogen)} \end{aligned}$$

7 mph \sim 0.44 m/s \rightarrow 1100 mph!

During the storm, had 100 mph winds, still much lower velocity than molecules themselves \rightarrow wind vel. doesn't contribute much to air temperature

- n i i i i

mfp of air molecules is tiny: $\sim 100 \text{ \AA}$

★ What's the optical depth of air to light?

mfp optical light \gg mfp molecule.

\rightarrow not in LTE in room, considering air & light: optical photons don't tell us anything about thermodynamics of the air

LTE ① Photons & massive particles have a \uparrow # density

② System optically thick $\tau \gg 1$ for all λ 's

When true, the spectrum of light reaches an equilibrium dist. like the Max.-Boltz. dist. for particles
 \hookrightarrow called the Planck function

$$J_{\nu}(T) = \frac{2 h \nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

-

Spectrum of blackbody radiation
 (perfect absorber of energy)

Consider 2 limits

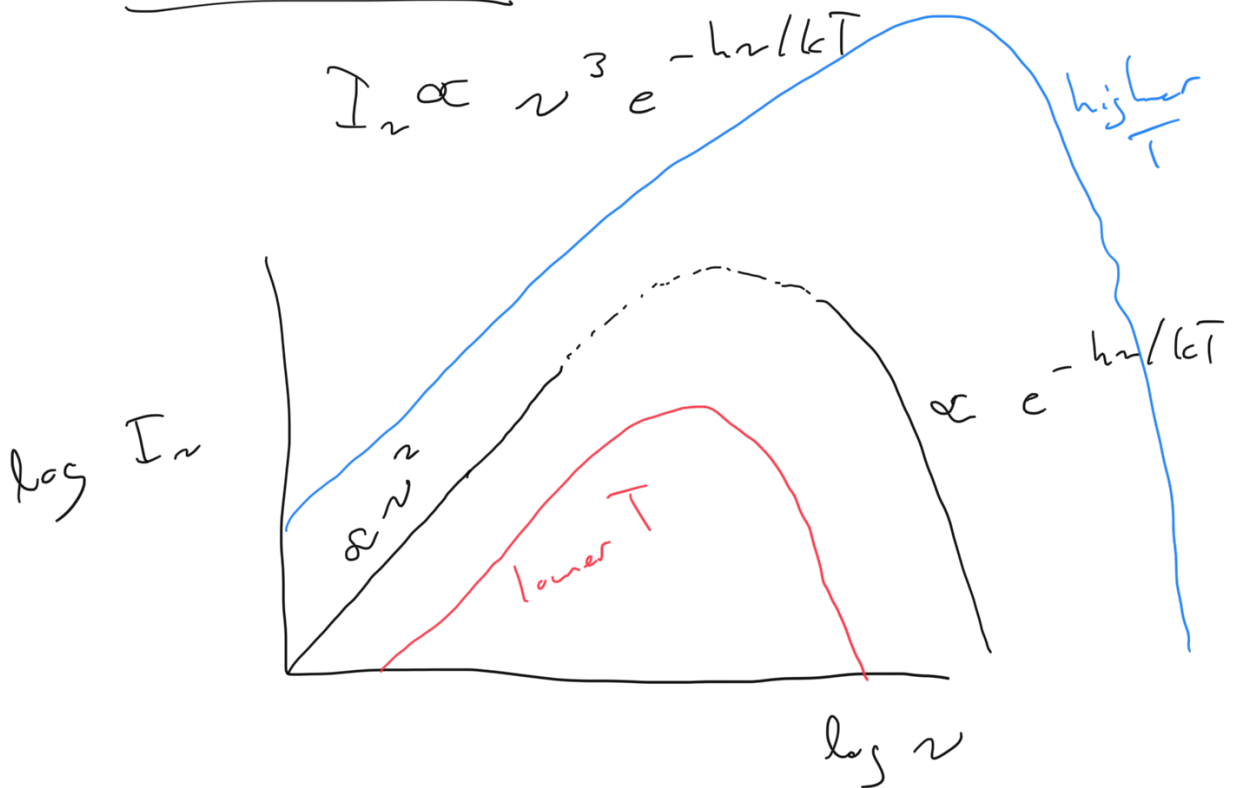
$$\left. \begin{array}{l} h\nu \ll kT : e^{h\nu/kT} \approx 1 + \frac{h\nu}{kT} \\ h\nu \gg kT : e^{h\nu/kT} \gg 1 \end{array} \right\}$$

Rayleigh - Jeans limit (low ν)

$$I_\nu \propto \frac{2kT}{c^2} \nu^2$$

Wien Limit (high ν)

$$I_\nu \propto \nu^3 e^{-h\nu/kT}$$



ν is a function of λ .

Can also express as function of λ

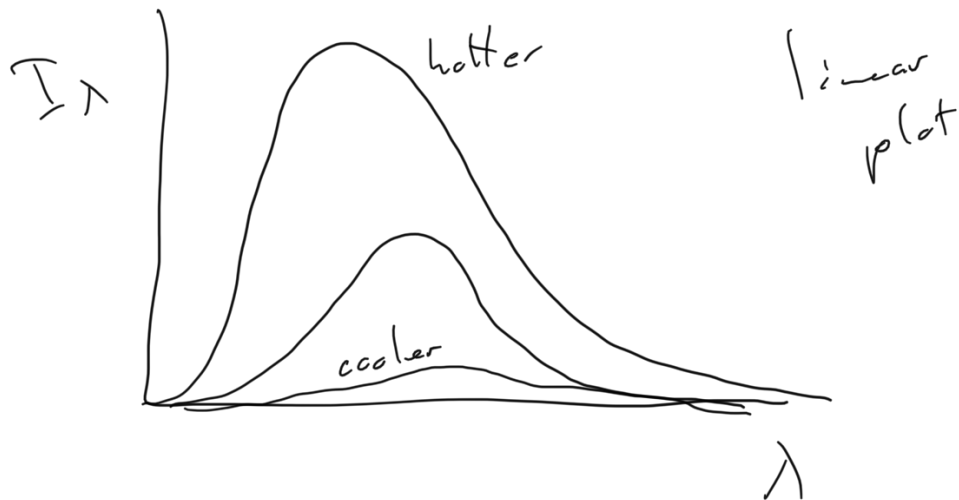
since $I_\nu d\nu (\nu \rightarrow \nu + d\nu) =$

$$I_\lambda d\lambda (\lambda \rightarrow \lambda + d\lambda)$$

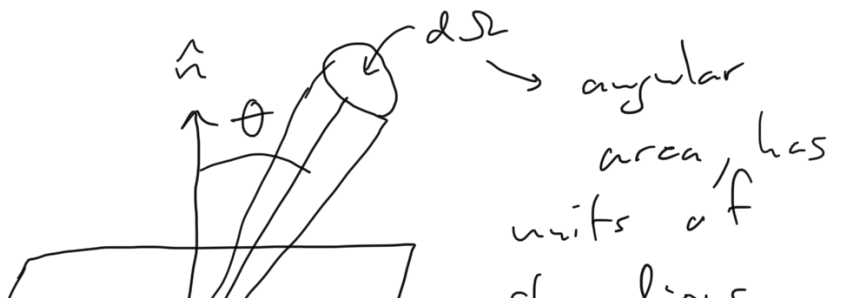
$$\text{and } \nu = \frac{c}{\lambda}, \quad d\nu = -\frac{c}{\lambda^2} d\lambda$$

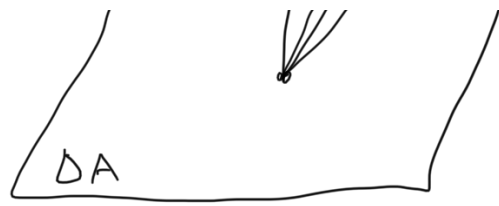
$$I_\lambda d\lambda = I_\nu d\nu = I_\nu \left| \frac{d\nu}{d\lambda} \right| d\lambda$$

$$I_\lambda d\lambda = \frac{2hc^2}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$



$$I \rightarrow \frac{\text{energy}}{\text{area} \cdot \text{time} \cdot \text{angle}} \quad (\text{freq. or wavelength})$$





steradians
 (a sphere has an angular area of 4π steradians)

Integrate over all freq. & all angles, get total E per area per time, i.e. the Flux

$$F = \frac{2\pi^5}{15} \frac{k^4}{c^2 h^3} T^4 \left(\frac{\text{energy}}{\text{area} \cdot \text{time}} \right)$$

σ_{SB}

Stefan-Boltzmann constant

$$\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$$

Tells us how much E ^{per time} a star is emitting per area of its surface, so need to integrate over its entire surface to get total E per time
 ↳ call this luminosity L

Assume star is spherical:

... .. m^2

$$L = F \cdot A = F \cdot 4\pi R$$

$$L = 4\pi R^2 \sigma_{\text{SB}} T^4$$

Usually compare to the Sun

$$R_{\odot} = 6.96 \times 10^8 \text{ m}$$

$$T_{\odot} = 5780 \text{ K}$$

$$L_{\odot} = 3.8 \times 10^{26} \text{ W}$$

Can rewrite eqn. as

$$L = L_{\odot} \left(\frac{R}{R_{\odot}} \right)^2 \left(\frac{T}{T_{\odot}} \right)^4$$

Similar to Kepler's law in how it's scaled to convenient units