ASTR 2500 - Solar System & Exoplanets

$$F = \frac{L}{A} = \frac{L}{4\pi d^2} \quad ; \quad \text{flux}$$

$$L_{BB} = A\,\sigma_{SB} T^4 = 4\pi R^2 \sigma_{SB} T^4 \quad ; \quad \text{BB Luminos.}$$

Sun produces $L_\odot$ energy / time

Maximum energy a planet can absorb / reflect is $F(d) \cdot$ cross-section (σ_p)

$$\frac{L_\odot}{4\pi d_p^2} \pi R_p^2 = \frac{1}{4} L_\odot \left(\frac{R_p}{d_p}\right)^2$$

Define albedo as fraction of light reflected: $A = \frac{E_{reflected}}{E_{incident}}$

Rate of energy absorption by planet is

$$W_p = F(d) \cdot \sigma_p (1-A)$$

$$= \frac{1}{4} L_\odot \left(\frac{R_p}{d}\right)^2 (1-A)$$

If planet emits absorbed radiation like a BB, $W_p = L_p = 4\pi R_p^2 \sigma_{SB} T_p^4$

$$= \frac{1}{4} L_\odot \left(\frac{R_p}{d_p}\right)^2 (1-A)$$

$$\boxed{T_p = \left(\frac{R_\odot}{d}\right)^{1/2} \left(\frac{1-A}{4}\right)^{1/4} T_\odot}$$

Earth: $T_p \approx 250\text{K} = -23°\text{C} = \underline{-9°\text{F}}$

Why isn't Earth this cold???

Why don't astronauts on the Moon freeze on the surface?

To do calculation, need the distance

- use radar: send pulse & wait for it to reflect & return
- use Kepler's 3rd law

$$\left(P/_{yr}\right)^2 = \left(a/_{AU}\right)^3$$

Other fundamental properties: mass, dens.

(infer what they're made of)

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

planet $\qquad$ satellite $m \ll M$

$$M_p \approx \frac{4\pi^2 a^3}{G p^2}$$

What about radius? Use angular size

$R_p \approx \theta d$; d is distance from us, but can figure out

The density is then $\rho = \frac{M}{V}$

$$\rho = \frac{M_p}{\frac{4}{3}\pi R_p^3}$$

Rocky : $\rho \approx 3000 - 5500 \text{ kg m}^{-3}$

Gas : $\rho = 700 - 2000 \text{ kg m}^{-3}$

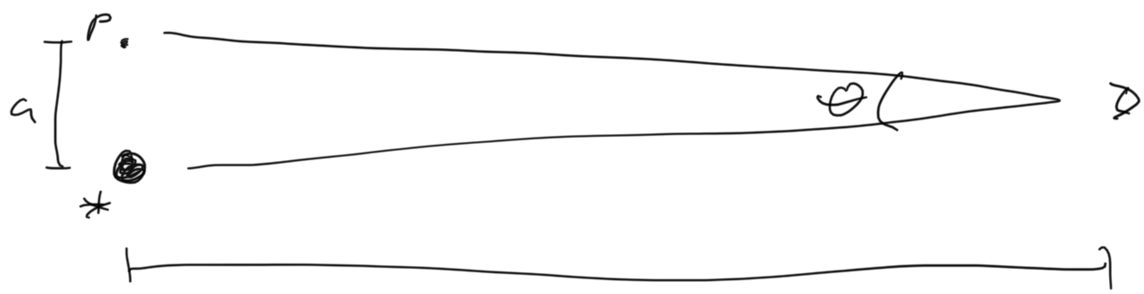
H_2O : $\rho \approx 1000 \text{ kg m}^{-3}$

Detecting Exoplanets

Why is it so hard to detect

☆ Why is it so ...
planets around other stars?

- stars are bright, planets reflect a small fraction of that light, & stars are far away, so the light is hard to distinguish



$d = a/\theta$ ← radians

$$\theta = \underline{0.004''} \left(\frac{m_p/M_*}{0.001}\right)\left(\frac{a}{5.2\,AU}\right)\left(\frac{d}{1.3\,pc}\right.$$

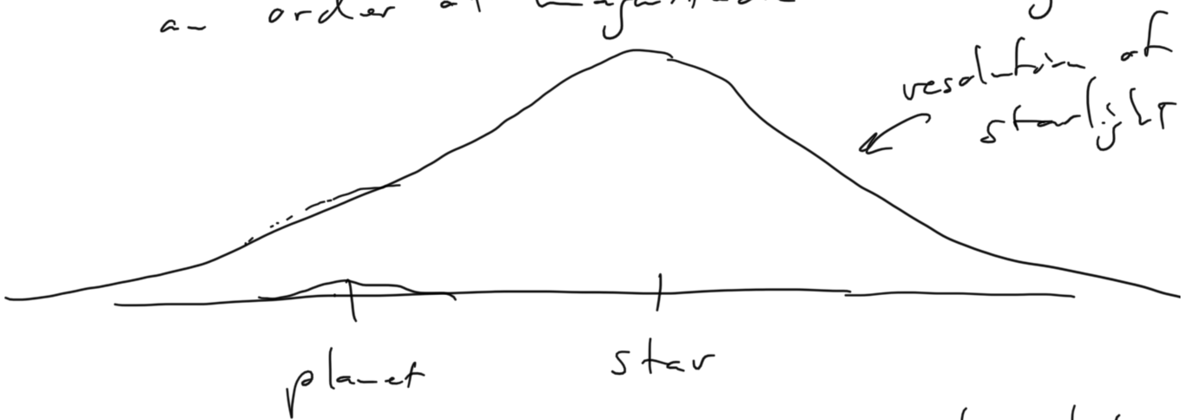
Jupiter's separation from Proxima Centauri

- On the Earth's surface, atmosphere blurs images (just like on a hot day in the desert), limiting $\theta \gtrsim 1''$

- In space, $\theta \gtrsim \frac{\lambda}{D}$; Hubble $D = 2.4m$

so $\theta \simeq \frac{400\times10}{2.4\,m} = 0.03$,

an order of magnitude too large



↳ in reality, starlight much brighter

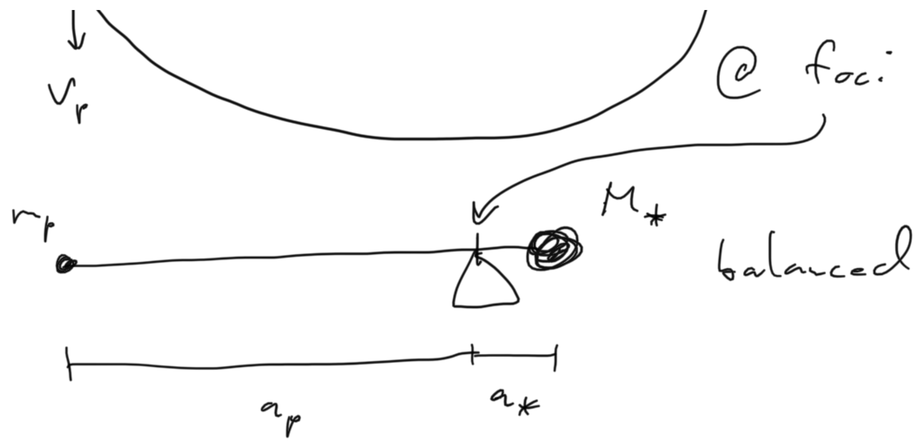
<u>Direct imaging</u> very challenging

<u>Radial velocities</u> also hard, but comparatively easier

↳ look @ how spectral lines shift over time due to the induced motion caused by gravity of a planet on its star

★ Stars are <u>NOT</u> @ the foci of elliptical orbits → they also orbit

Center of Mass (CoM)



$$\frac{m_p}{M_*} = \frac{a_*}{a_p}$$

Always keep the focus/CoM b/t them so their periods are the same

$$P = \frac{2\pi a_p}{v_p} = \frac{2\pi a_*}{v_*}$$ (circular vel. or avg.)

$$\frac{v_p}{v_*} = \frac{a_p}{a_*} = \frac{M_*}{m_p}$$

Radial velocities come from Doppler shifts:

$$\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$$

measure this

infer this

... velocity of the star,

Find the velocity

can infer the velocity of a planet

↳ estimate M_* somehow, use to get m_p!

Velocities are small: Jupiter is $1000\times \downarrow$ mass than $\odot$, $v_J = 13$ km/s

so $v_\odot = 13$ m/s

$$\frac{\Delta\lambda}{\lambda} = \frac{13\text{ m/s}}{3\times10^{8}\text{ m/s}} \sim 4\times10^{-8}$$

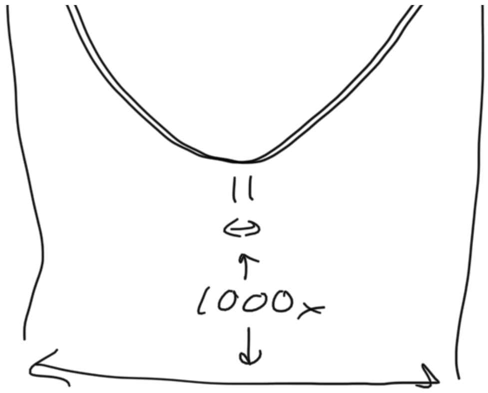
Measure λ to about ~~than~~ 1 part in 100 million

But the <u>width</u> of a line due to thermal motions is $\frac{\Delta\lambda}{\lambda} \simeq 3\times10^{-7}\left(\frac{T}{1\text{K}}\right)^{1/2}$

For H in a solar-like star, $T = 5780$K,

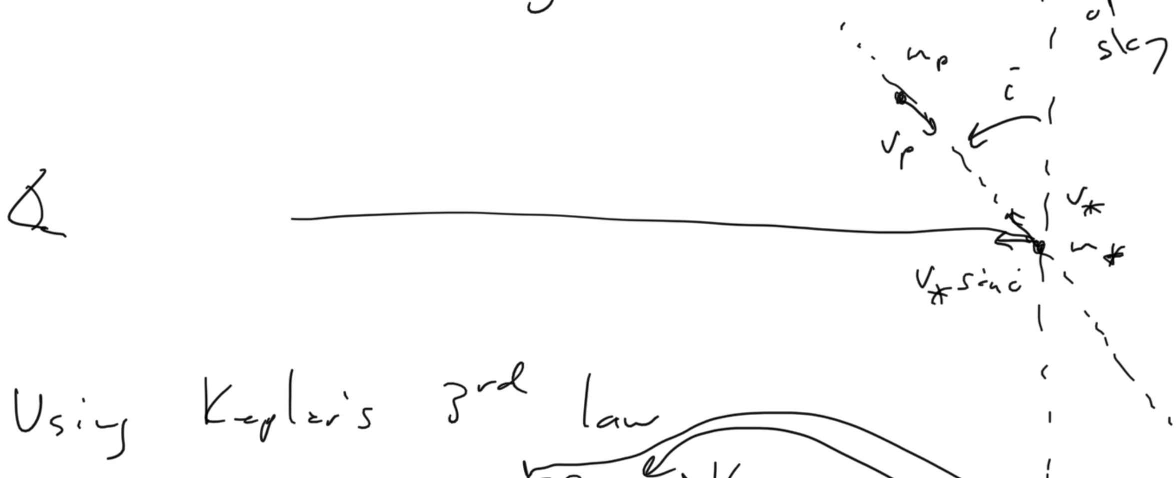
$\frac{\Delta\lambda}{\lambda} \sim 2\times10^{-5}$, 1000x wider than the shift

Very hard



But easier if planet more massive & close.

But, only measure velocity along the line-of-sight (l.o.s.)



Using Kepler's 3rd law

$$\underbrace{m_p \sin i}_{\text{calc. this}} \approx \left(\frac{M_*^2 P}{2\pi G} \right)^{1/3} \underbrace{v_* \sin i}_{\text{measure}}$$

infer

So, only get limit on mass of planet w/o some other way to constrain i

$$m_p \geq m_p \sin i$$

But, harder to dectect motions for

orientations w/ small

Selection Effects

- what you observe does not necessarily correspond to the true underlying population, but can be biased by the detection method

Radial velocity method favors massive planets near the star (can detect in a shorter time plus velocity larger)

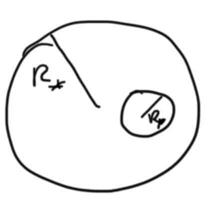
& orbital planes aligned w/ our l.o.s.

Transit Method (3rd way)

- see the flux from a star dip as a planet passes in front of it

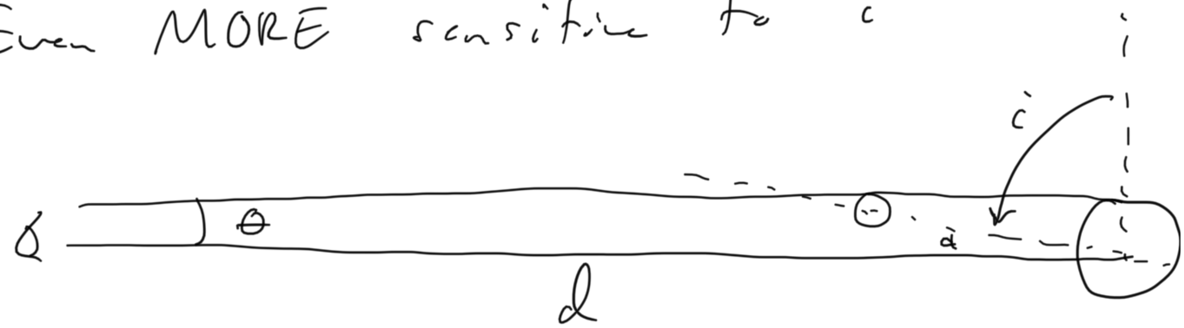
☆ What are some selection effects of this method?

Dips are small $\quad \dfrac{\delta F}{F} = \dfrac{\pi R_p^2}{\pi R_*^2}$



$R_J \sim 0.1 R_\odot$, $\quad \dfrac{\delta F}{F} \sim 0.01$

Even MORE sensitive to i



To see a transit, $\cos i \lesssim \frac{R_* + R_p}{a}$

BACK to SLIDES