

## ASTR 2500 - Ch.13

# Stars !!!

Already know some measurements

$$\text{distance } d = \frac{1 \text{ pc}}{\pi''}$$

$$\text{flux \& luminosity } F = \frac{L}{4\pi d^2}$$

$L \rightarrow$  intrinsic brightness  
(absolute)

$F \rightarrow$  apparent brightness

Still use the magnitude system, recorded  
by Hipparchus 2200 years ago <sup>00</sup> <sub>mm</sub>

Visible stars were put into 6 brightness  
bins, brightest mag = 1, faintest mag =

Turns out our eyes (also ears) work on  
a logarithmic scale, not linear scale

(so something that looks 2x as  
bright is actually emitting > 2x

bright  
as many photos)

In mid-1800s, system was quantified  
(when photography made measurements  
more precise)

→ difference of  $\Delta m$  → 100x rat.  
of flux

$$m_2 - m_1 = 5 \quad \leftarrow \text{equivalent}$$

$$\frac{F_1}{F_2} = 100$$

$$\text{if } \Delta m = 1, \quad \frac{F_1}{F_2} = (100)^{\frac{1}{5}}$$

$$\Delta m = 2, \quad \frac{F_1}{F_2} = (100)^{\frac{2}{5}} \quad \text{etc.}$$

$$\text{in general, } \frac{F_1}{F_2} = 100^{(m_2 - m_1)/5}$$
$$= 10^{2(m_2 - m_1)/5}$$

$$\log \left( \frac{F_1}{F_2} \right) = \frac{2}{5} (m_2 - m_1)$$

$$\boxed{m_2 - m_1 = 2.5 \log \left( \frac{F_1}{F_2} \right)}$$

$\overbrace{\hspace{10em}}^{\text{apparent magnitudes}}$   
 relative system, need a reference mag  
 $m$  means fainter  
 $m = m_i - 2.5 \log F + 2.5 \log F_i$   
 $\swarrow$  reference value

Use the star Vega (could use anything; not even a real star)

$$\text{Vega: } m_i = 0, \quad C = 2.5 \log F_{\text{Vega}}$$

$$m = C - 2.5 \log F$$

It's a flux, so depends on distance

$$m = C - 2.5 \log \left( \frac{L}{4\pi d^2} \right)$$

$$= C - 2.5 (\log L - \log 4\pi d^2)$$

$$= C - 2.5 (\log L - 2 \log d - \log 4\pi)$$

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The magnitude equivalent of the luminosity needs a distance ref., so we use 10 pc b/c the system is stupid anyway why not

Absolute magnitude

$$M = C - 2.5(\log L - 2 \log(10)) - \log^4 i$$

$$m = C - 2.5(\log L - 2 \log d - \log^4 i)$$

Subtracting these,

$$m - M = 5 \log d - 5 \log 10 \text{ pc}$$

$$\boxed{m - M = 5 \log \left( \frac{d}{10 \text{ pc}} \right)}$$

distance modulus

Convenient reference is the Sun

$$m_{\odot} = -26.75, \quad M_{\odot} = 4.83$$

In practice, flux is a function of  $\lambda$  and measurements are

or ...  
w/in some range  $\lambda_1 \rightarrow \lambda_2$   
i.e., our eyes are sensitive  
to light  $4000\text{\AA} < \lambda < 7000\text{\AA}$

Can define the total flux as the  
integrated light over all  $\lambda$ , called

$$\text{bolometric flux } F_{\text{bol}} = \int_0^{\infty} F_{\lambda} d\lambda$$

$$\text{bolometric mag } m_{\text{bol}} = C_{\text{bol}} - 2.5 \log F_{\text{bol}}$$

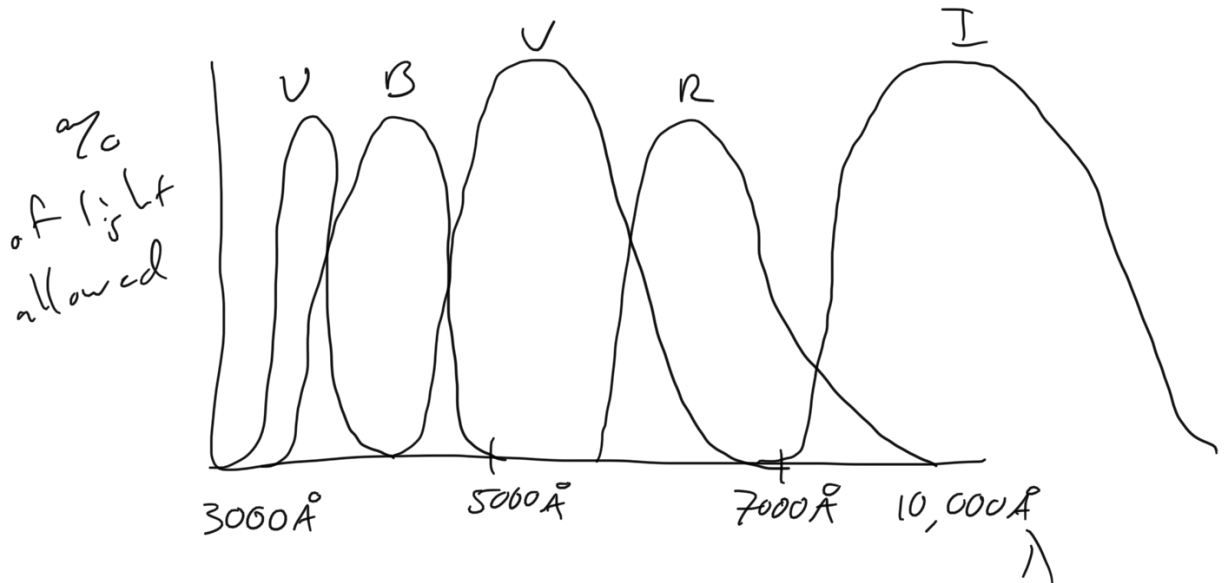
define  $M_{0,\text{bol}} = 4.74$  ↗

$$M_{\text{bol}} = 4.74 - 2.5 \log (L/L_{\odot})$$

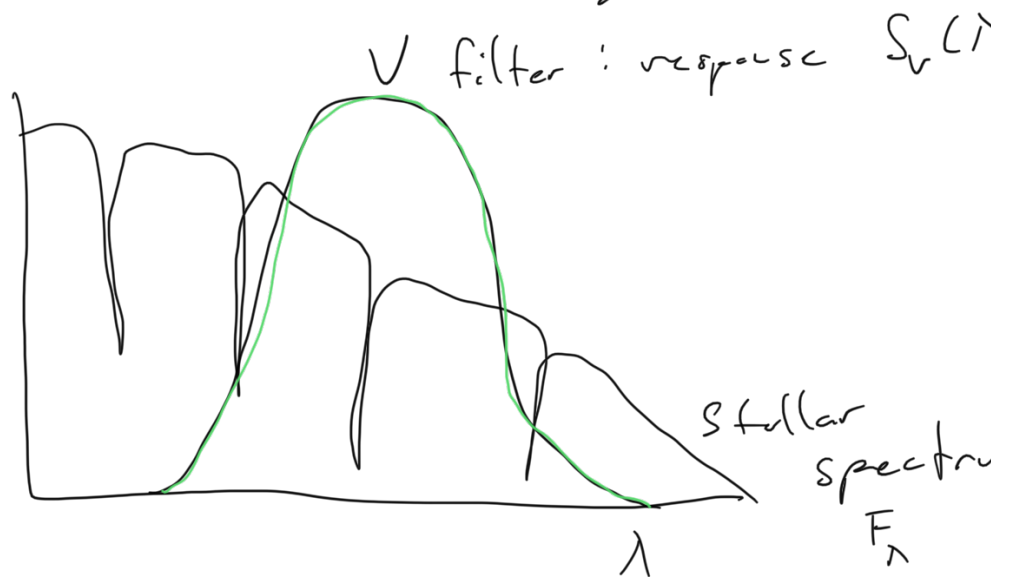
In practice, no physical detector is  
sensitive to all  $\lambda$ 's; need diff.  
technology @ diff.  $\lambda$ 's

Also, lose info if you just count  
photons & don't record their exact  
 $\lambda$ , but hard to do that & make an  
i-are, so must choose

"- J -"  
 Images → "filter" the light so  $\lambda$ 's  
 are restricted to a more  
 narrow band



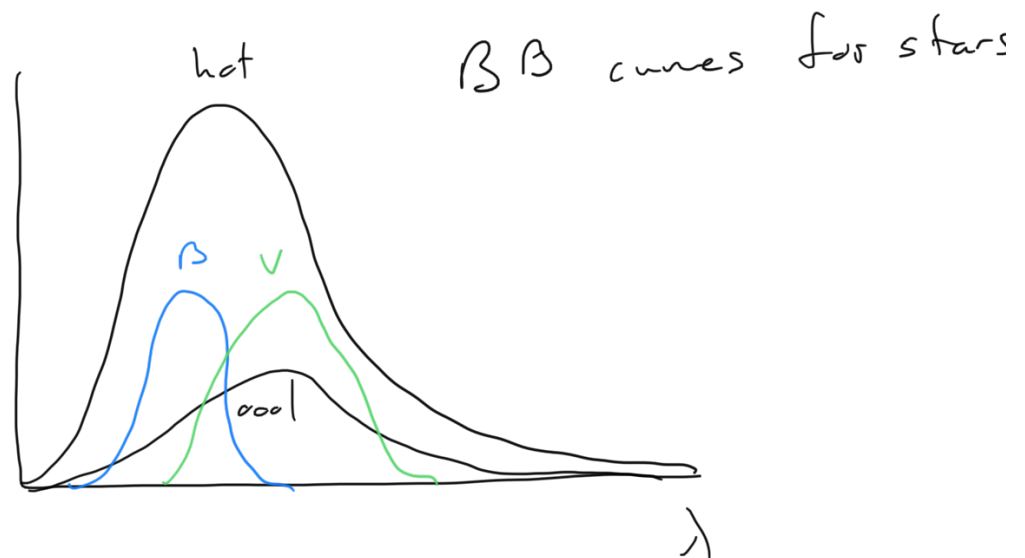
Johnson - Cousins system



$$F_V = \int_0^{\infty} F_{\lambda} S_V(\lambda) d\lambda$$

$$m_V = C_V - 2.5 \log F_V$$

similar expressions for other filters



$B - V \rightarrow$  color, ratio of fluxes  
hot stars have more blue light rel.  
to green light than cooler star

$$B - V = m_B - m_V$$

Define Vega to have  $B - V = 0$

$$T_{\text{Vega}} \approx 10,000\text{K}, \text{ so}$$

$B - V < 0$  (brighter in B rel.  
to V, blue, hotter)

$$\hookrightarrow T > 10,000\text{K}$$

$$B - V > 0, T < 10,000\text{K}$$

In practice, stars aren't perfect  
BBs, but we observe that

$$T \approx \frac{9000K}{(B-V) + 0.93}$$

Can convert filter mag.s to bol.  
mag.s using a bolometric correct.

$$BC = m_{bol} - m_V = M_{bol} - M_V$$

Stellar radii are hard to measure,  
but when we can, can estimate  
its  $T$

$$L = 4\pi R^2 \sigma_{SB} T^4$$

BB relation, which won't apply perfectly  
to stars, but can use it to  
define an "effective temperature"

$$T_{eff} = \left( \frac{L}{4\pi R^2 \sigma_{SB}} \right)^{1/4}$$

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Masses are harder, but as we've  
seen, they can be estimated from  
Kaler's



star-planet systems

3<sup>rd</sup> law  $M_A + M_B = \frac{4\pi^2}{G} \frac{a^3}{P^2}$

Lucky for us, MOST stars are in binary systems & we can use the same techniques for stars as planet. and it's generally easier since stars are brighter / more massive / bigger than planets

→ how we know so much about the properties of stars

3 types (just like planets)

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Visual : can see both (usually ↑ a)

Spectroscopic : see 1 star, but 1 or 2 sets of lines shifting over time

Eclipsing : see 1 star, but flux regularly dips as they pass in front of each other

Visual, can directly measure  $a$  &  $P$ , which

∴  $M_A + M_B = \frac{4\pi^2}{G} \frac{a^3}{P^2}$  ( $a =$

gives  $\frac{M_A + M_B}{M_0} = \frac{1}{\rho^2} a_A + a_B$

+ individual masses come from COM

$$\frac{M_A}{M_B} = \frac{a_B}{a_A}$$

Spectroscopic, measure velocities, but includes an unknown inclination  $i$

$$\frac{M_B}{(1 + M_A/M_B)^2} \sin^3 i = \frac{\rho}{2\pi G} (v_A \sin i)^3$$

↑  
unmeasurable

↑  
measured quantities

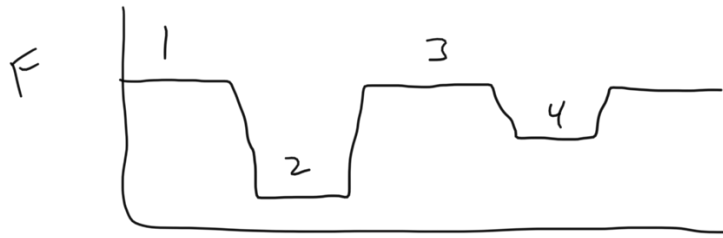
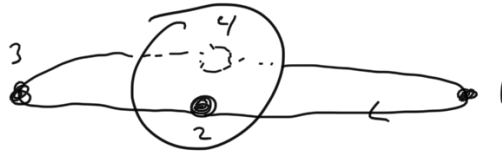
Left side constrains masses given a definite value on right side, call this the "mass function"

$$f(M_A, M_B) = \frac{M_B^3 \sin^3 i}{(M_A + M_B)^2}$$

This reduces to equation we found

for exoplanets if  $M_A \gg M_B$

Eclipsing



Assuming bigger star is hotter, 2  
will be the bigger dip b/c

$$L = 4\pi R^2 \sigma_{SB} T^4$$

↖ more important factor

How does  $R$ ,  $T$ , & lifetime  
depend on mass?

$$\frac{R}{R_0} = \begin{cases} 1.06 (M/M_0)^{0.945} & M < 1.6 \\ 1.33 (M/M_0)^{0.555} & M > 1.6 \\ & M_0 \end{cases}$$

$$\frac{L}{L_0} = \begin{cases} 0.35 (M/M_0)^{2.62} & M < 0.7M \\ 1.02 (M/M_0)^{3.92} & M > 0.7M \end{cases}$$

lifetime of a star is determined by  
its mass &  $L$  & rate fuel burned  
 $\hookrightarrow \propto \text{fuel}$

$$\tau \propto M/L \propto \begin{cases} M^{-1.62} & M < 0.71 \\ M^{-2.92} & M > 0.71 \end{cases}$$