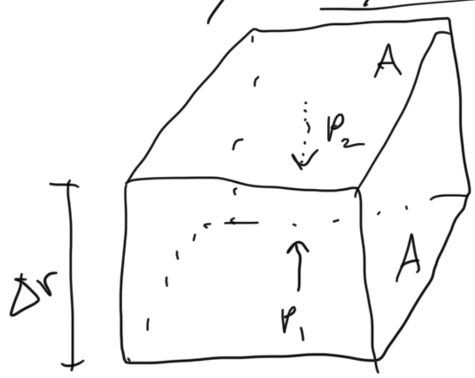
Stellar Atmospheres & Classification

Atmospheres in general (that of the Ear[th], Sun, etc.) have a structure determin[ed] by <u>hydrostatic equilibrium</u>



$$F_p = F_g$$

Pressure is a force per area

To keep the gas stationary, the <u>difference</u> in pressure across the box must equal F_g

$$\underbrace{(P_2 - P_1)}A = -\frac{G M(<r)\,\overbrace{\rho A \Delta r}^{m}}{r^2}$$

If box small enough

$$\Delta r$$
$$dP = -\frac{GM\rho}{r^2}\,dr$$

$$\frac{dP}{dr} = -\frac{GM(<r)\rho}{r^2}$$

pressure gradient

gravity

☆ For a spherical dist. w/ density that only depends on radius, all mass interior can be thought of as in the center, and the grav. force of all mass exterior cancel out, hence $M(<r)$

Luminosity Classes

- w/o distance hard to get class, but turns out <u>line widths</u> depend on class
- narrower lines in I (supergiant) than V (dwarf / MS): why?

Surface gravity, g, is ↓ b/c $R\uparrow$,

& thus the pressure is lower, so
lines have less pressure broadening

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} = -g\rho$$

Line comes from a shell dr that corresponds to the optical depth $\tau = 0 \to 1$, $\tau = n\sigma x$, or here, can write as $d\tau = -n\sigma\, dr$ (as $\tau\uparrow$, $r\downarrow$)

Can rewrite the cross-section as a quanti[ty] per mass (in this shell), called the opacity: $K = \frac{\sigma}{\text{mass}}$, or $\rho K = n\sigma$

$$d\tau = -\rho K\, dr$$

$$\frac{dP}{dr} = \frac{dP}{-d\tau/\rho K} = -g\rho$$

$$\frac{dP}{d\tau} = \frac{g}{K} \to P \sim \frac{g}{K}\tau$$

Since $\tau \sim 1$, $P \sim \frac{g}{K}$ & K is rough[ly] constant in stars, so $\downarrow g$, $\downarrow P$, narrow[er] lines

☆ Also, gas is stars generally follows the ideal gas law

$$P = nkT = \frac{\rho k T}{\mu m_p}$$

Density is $\frac{M}{V}$, but particles have diff. individual masses, so use 3 categories

$$\rho = \rho_H + \rho_{He} + \rho_{metal}$$

→ all elements w/ $Z > 2$ are called "metals", even though O, N, Ne, etc. are not technically metals

Express the ratio of each cat. by the total density, define

$$X \equiv \rho_H / \rho$$

$$Y \equiv \rho_{He} / \rho$$

$$Z \equiv \rho_{metal}/\rho = 1 - X - Y$$

$Z \sim$ metallicity

For the Sun, we observe

$$X_\odot = 0.734, \quad Y_\odot = 0.250, \quad Z_\odot = 0.016$$

This is very similar to the primordial abundance, $X \sim 0.75 / Y \sim 0.25 / Z = 0$

Often we care about # density

$$n = \frac{\rho}{\mu m_p}$$

$$\mu_H = \begin{cases} 1 & \text{neutral} \\ \frac{1}{2} & \text{ionized} \end{cases}, \quad \mu_{He} = \begin{cases} 4 & \text{netral} \\ 4/3 & \text{ionized} \end{cases}$$

$$\mu(\text{ionized}) = \frac{\rho}{n m_p}$$

$$= \left(2X + \frac{3}{4}Y + \frac{1}{2}Z\right)^{-1}$$

$$\mu_\odot(\text{ion.}) = 0.60$$

Stellar Interiors

Just like atmospheres, the interior structure is governed by HSE

$$\frac{dP}{dr} = - \frac{G M(<r) \rho(r)}{r^2}$$

By twisting calculus to our ends, can rewrite as

$$\frac{\Delta P}{\Delta r} \approx - \frac{G \langle M \rangle \langle \rho \rangle}{\langle r \rangle^2}$$

$$\langle M \rangle \sim \frac{M_\odot}{2}$$

$$\langle \rho \rangle \sim \frac{M_\odot}{\frac{4}{3}\pi R_\odot^3} = \rho_\odot = 1400 \frac{kg}{m^3}$$

$$\sim 1.4 \times \rho_{H_2O}$$

$$\langle r \rangle \sim \frac{R_\odot}{2}$$

$$\Delta P = P_{surf} - P_c = 0 - P_c$$

$$\Delta r = R_\odot - 0$$

$$P_c \approx \frac{8\pi}{3} G \rho_\odot^2 R_\odot^2$$

What about T_c? Can use

ideal gas law!

$$P(r) = \frac{\rho(r)\, k\, T(r)}{\mu m_p}$$

$$T_c = \frac{2 G M_\odot \mu_\odot m_p}{R_\odot k}$$

$\mu_\odot \approx 0.60$ $\boxed{T_c \approx 3 \times 10^7 \mathrm{K}}$

(Correct analysis gives $2\times \downarrow$ T_c)

For stars on the main sequence w/ $M < 1.66 M_\odot$

$$R/R_\odot = 1.06 (M/M_\odot)^{0.945}$$

$$\approx M/M_\odot$$

Since $T_c \propto \frac{M}{R}\mu$, + μ same for any star (since H dominates mass)

$$T_c \propto 0.6 \frac{M}{R} \propto 0.6 \frac{\cancel{M}}{\cancel{M}} \frac{M_\odot}{R_\odot}$$

$\propto$ const

This const. temperature determines the main sequence — what sets T_c?

Energy Generation

- in the 1800s, no nuclear physics (atomic structure unknown)
- gravity understood (Newtonian), so Helmholtz & Kelvin considered the formation of the Sun from a contracting gas cloud — its light would then be the conversion of grav. pot. E to heat

For a spherical object,

$$U = -q \frac{GM^2}{R} \quad \left(q \text{ depends on } \rho(r)\right)$$

This is the E that must be removed to collapse an object from $r=\infty$ to $r=R$ that has mass M

If the Sun's light comes from this energy, + we assume it has been emitting it at a constant rate...

$U \to J \qquad L \to J\,s^{-1}$

Can estimate the total lifetime of the Sun!

$$t_{KH} = \frac{U_\odot}{L_\odot} \approx \underline{50\ \mathrm{Myr}}$$

Geologists thought the Earth was older, but Kelvin used this estimate to cow them, setting back geology as a field for <u>decades</u> — almost 100 yr!

<u>Moral</u>: Physicists who don't respect the science of other fields are evil

↳ e.g., physicists making epidemiological models for COVID-19 spread at same university

In the 1930s, nuclear fusion [illegible] energy source recognized as energy)

$$4H \to 1\,He^4$$

$$\left.\begin{aligned} 4m_p &= 6.6905\times10^{-27}\,kg \\ 1m_{He} &= 6.6447\times10^{-27}\,kg \end{aligned}\right\} \begin{aligned} &0.0458\times10^{-27}\,kg \\ &\text{difference} \end{aligned}$$

Where does mass go?

$$E = mc^2!$$

$$E = \Delta m c^2 = 4.1\times10^{-12}\,J$$

Small amount of E, but Sun has lots of protons! Assuming it's entirely protons

$$N_H = \frac{M_\odot}{m_p} \approx \frac{2\times10^{30}\,kg}{1.67\times10^{-27}\,kg} \approx 10^{57}$$

If fuse all into He ($4H \to 1He$), then have Δm per reaction, & there are $\frac{N_H}{4}$ reactions, releasing

$$E_{fus} = \frac{N_H}{4}\Delta E = \frac{10^{57}}{4}\,4\times10^{-12}\,J = \underline{10^{45}\,J}$$

This is ~2000x more E than provided

by $U_\odot$, so $t_{fus} = \frac{E_{fus}}{L_\odot} \approx 100$ Gyr

Stars like the Sun only convert ~10% of their H to He, so actual lifetime is 10^{10} yr = 10 Gyr

Since $\tau \propto \frac{M}{L}$ & $L \propto M^4$,

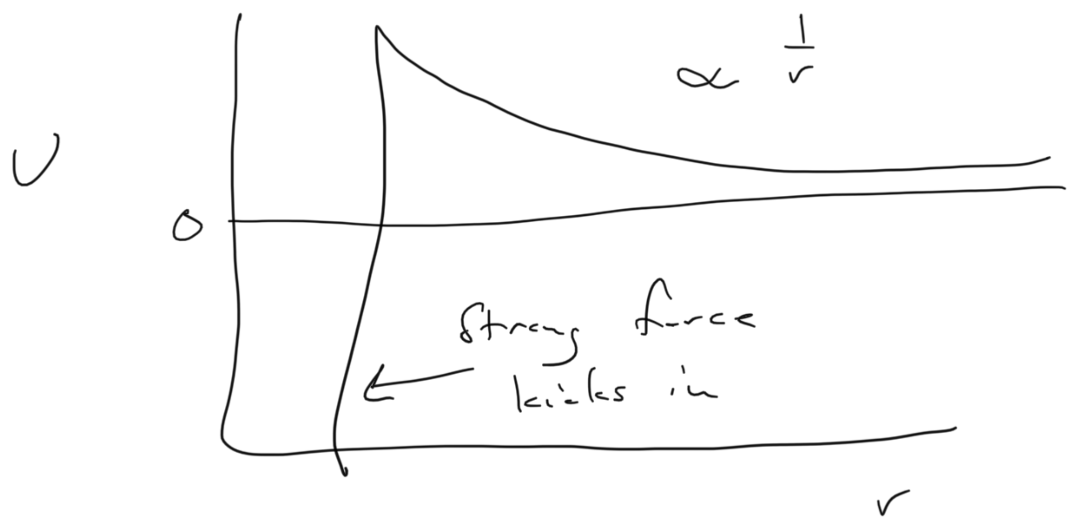
$$\tau \approx 10 \text{ Gyr} \left(\frac{M}{1 M_\odot}\right)^{-3}$$

for main sequence stars

Fusion reactions

How can 2 protons fuse? The repulsive Coulomb force is $\propto \frac{1}{r^2}$ — if protons were point-like, the force would repeal them always

But p^+ aren't — they're made of quarks governed by the strong force, which is VERY attractive force,



Need enough energy to get close enough for strong force to take over

$r \sim 10^{-15}$ m

$$U \simeq \frac{e^2}{4\pi\epsilon_0 r} \simeq 1.4 \text{ MeV}$$

Do protons in the center of the Sun have this much kinetic E?

$$\langle E \rangle = \frac{3}{2} k T_c \simeq 2 \text{ keV}$$

Nope! So how does fusion occur?

Particles are not just particles, but also waves (QM saves the day!)

$$\lambda_{prot} = \frac{h}{p}, \quad p = m_p v$$

$$\lambda_{prot} \approx 10^{-13}\,m$$

$$\sim 100\, r_{SF}$$

Essentially, the proton's position is uncertain, & has a chance of getting close enough even though classically that's impossible

<u>quantum tunneling</u>

So, how exactly do these reactions create He in the core?

$\boxed{\text{PP chain}}$ $\quad T_c < 1.8\times10^{7}\,K$ (eg, Sun)

$$p + p \rightarrow {}^{2}H + e^{+} + \nu_e$$

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p → n, positron carries away extra charge, neutrino to conserve e⁻ quantum #

energy from mass diff.

$${}^{2}H + p \rightarrow {}^{3}He + \gamma$$

$$^3H + p \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow \boxed{^4He} + p + p$$

$$4p \rightarrow {}^4He + 2\gamma + 2e^+ + 2\nu_e$$

$$e^+ + e^- \rightarrow 2\gamma$$ (antimatter annihilation

ν_e only interact via the weak force, which has a very ↓ cross-section, so Sun is essentially transparent to them & they escape

The γ-rays produced in the core take b/t 10,000-100,000 years to escape (b/c they scatter a lot — have a small m.f.p.). They are in LTE as they travel, so lose E as they travel, becoming visible at the surface.

photons ...

$H \to He$ releases converts 0.7% of the mass to energy

$He \to$ bigger elements releases more!

For more massive stars, where $T_c > 1.8 \times 10^7 K$, $H \to He$ via the

<u>CNO cycle</u> (C, N, O atoms used in intermediary steps, but no net change in their #s – they're catalysts)

CNO cycle gets 1 additional γ-ray

$$\underline{^{12}C} + \boxed{p} \to \underset{+\gamma}{^{13}N} \to \underset{+e^+ + \nu_e}{^{13}C} + \boxed{p} \to \underset{+\gamma}{^{14}N} + \boxed{p} \to$$

$$\underset{+\gamma}{^{15}O} \to \underset{+e^+ + \nu_e}{^{15}N} + \boxed{p} \to \underline{^{12}C} + {}^{4}He$$

To fuse $He \to C$, need the

<u>triple alpha process</u>

$T_c > 10^8 K$ as well as dense environment

$$^4He + {}^4He \rightarrow {}^8Be + \gamma$$

↳ decays quickly,

but if it hits another ^4He first

$$^4He + {}^8Be \rightarrow {}^{12}C + \gamma$$

$$3 He \rightarrow 1 C$$