Week 3 Thursday

Today's Agenda

- Review Tuesday's in class problems
- Let there be light (in lines)
- Lines shift (& practice problems)
- Lines broaden

Announcements / Reminders

- Friday office hours moved to 1-2pm this week only
- Read Chapter 5, 6.1, 6.4-7
- HW 2 due September 10th at 11:59pm via Canvas upload

- HW 3 now available, due Sept. 17th

A planet twice as massive as the Earth orbits a Sun-like star in a highly elliptical orbit.
At apogee, the planet is 8 AU from the star.
How long does it take the planet to complete 1 orbit around the star?

If the star were twice as massive as the Sun, what would its period be?

The escape velocity from the surface of the Earth is ~11 km/s and the escape velocity from the solar system at Earth's orbit is ~42 km/s.
How do these escape velocities compare to this planet at its orbital apogee?
Assume the star has twice the mass of the Sun.

If instead the planet's orbit is circular but has the same period as before, what is its orbital velocity?
What would be the escape velocity from the star system in the vicinity of the planet?
Assume the star has twice the mass of the Sun.

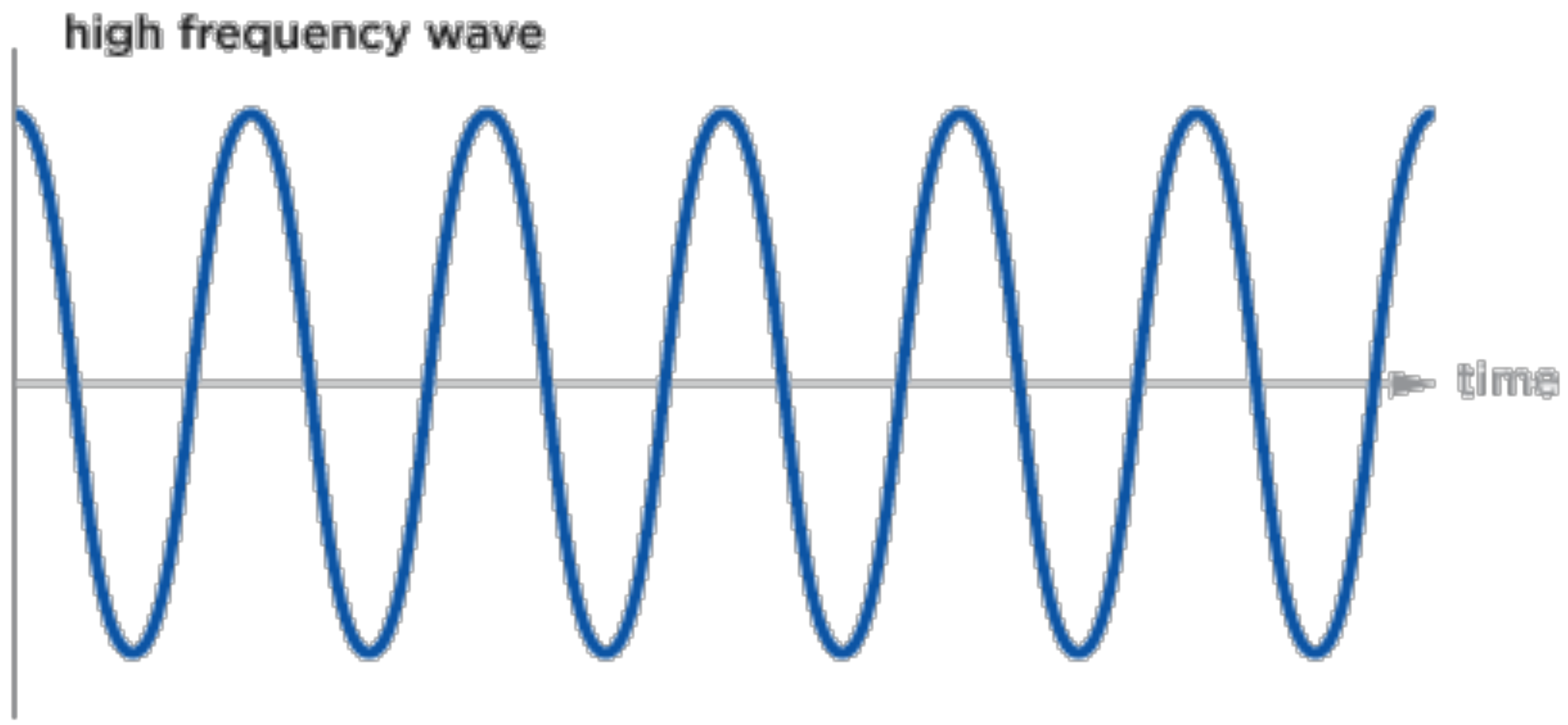
If ballistically launched from the surface of this planet, what initial velocity do you need to escape the star?

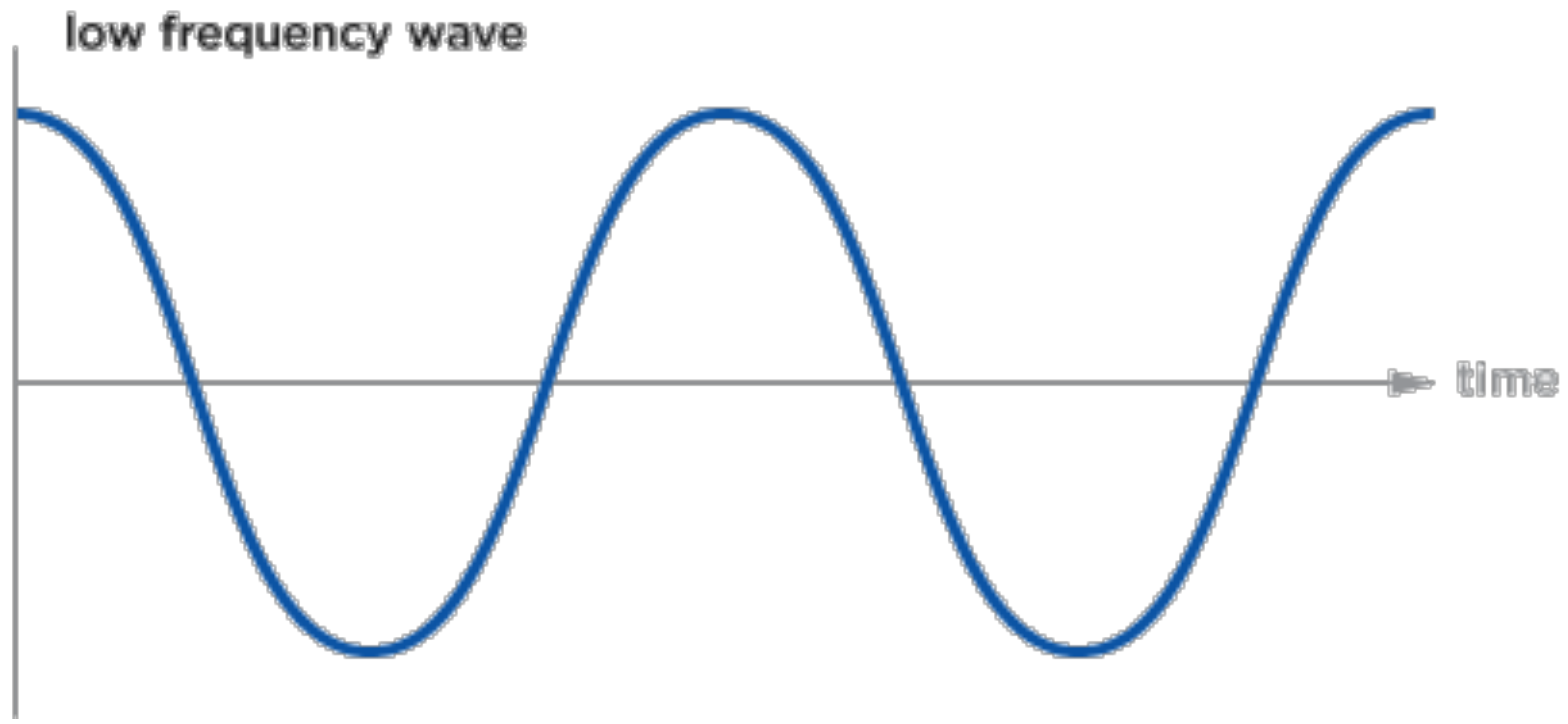
Chapter 5: Let there be *LIGHT!*

Cover today

- Review of atomic structure, energy exchange processes, and spectroscopy
- Radiative transfer
- Thermodynamic equilibrium
- Blackbody radiation
- Wien's Law

“Light” is electromagnetic radiation of any wavelength/frequency, not just what eyes see





Classically, can be thought of a wave traveling down an electric field line like an induced transverse wave down a rope.

In QM, quanta of the wave are called photons, which have energy and momenta determined by wavelength/frequency.

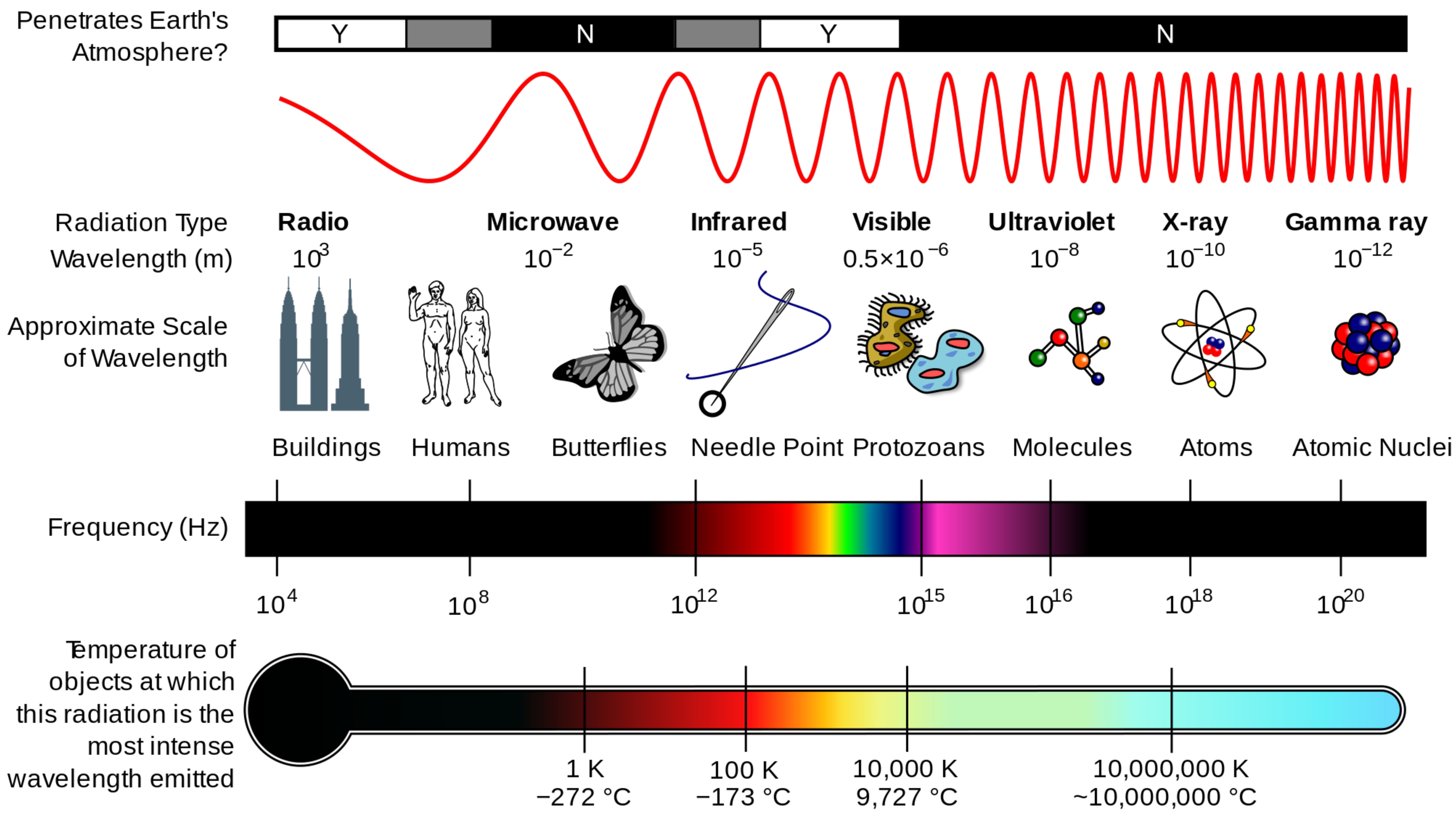
$$E = h\nu = \frac{hc}{\lambda}$$

$$h = 6.626 \times 10^{-34}\ \mathrm{J\ s} = 4.135 \times 10^{-15}\ \mathrm{eV\ s}$$

$$c = 2.998 \times 10^{8}\ \mathrm{m\ s^{-1}}$$

$$\hbar = \frac{h}{2\pi}$$

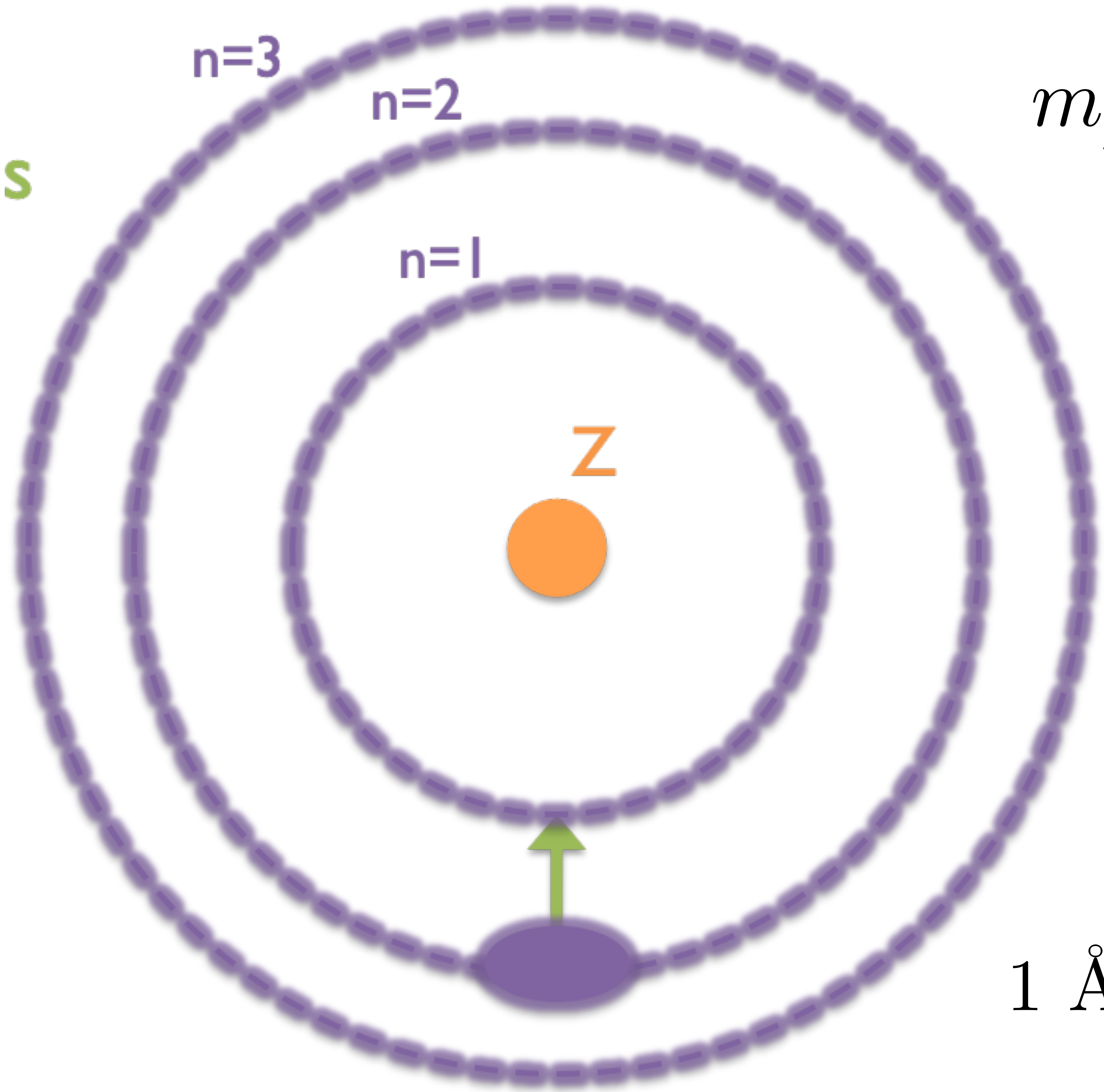
$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$


Penetrates Earth's Atmosphere?
Y
N
Y
N
Radiation Type
Radio
Microwave
Infrared
Visible
Ultraviolet
X-ray
Gamma ray
Wavelength (m)
10^{3}
10^{-2}
10^{-5}
0.5×10^{-6}
10^{-8}
10^{-10}
10^{-12}
Approximate Scale of Wavelength
Buildings
Humans
Butterflies
Needle Point
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^{4}
10^{8}
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of objects at which this radiation is the most intense wavelength emitted
1 K
−272 °C
100 K
−173 °C
10,000 K
9,727 °C
10,000,000 K
~10,000,000 °C

Atomic Structure (quantized energy levels)

Z protons

A−Z neutrons



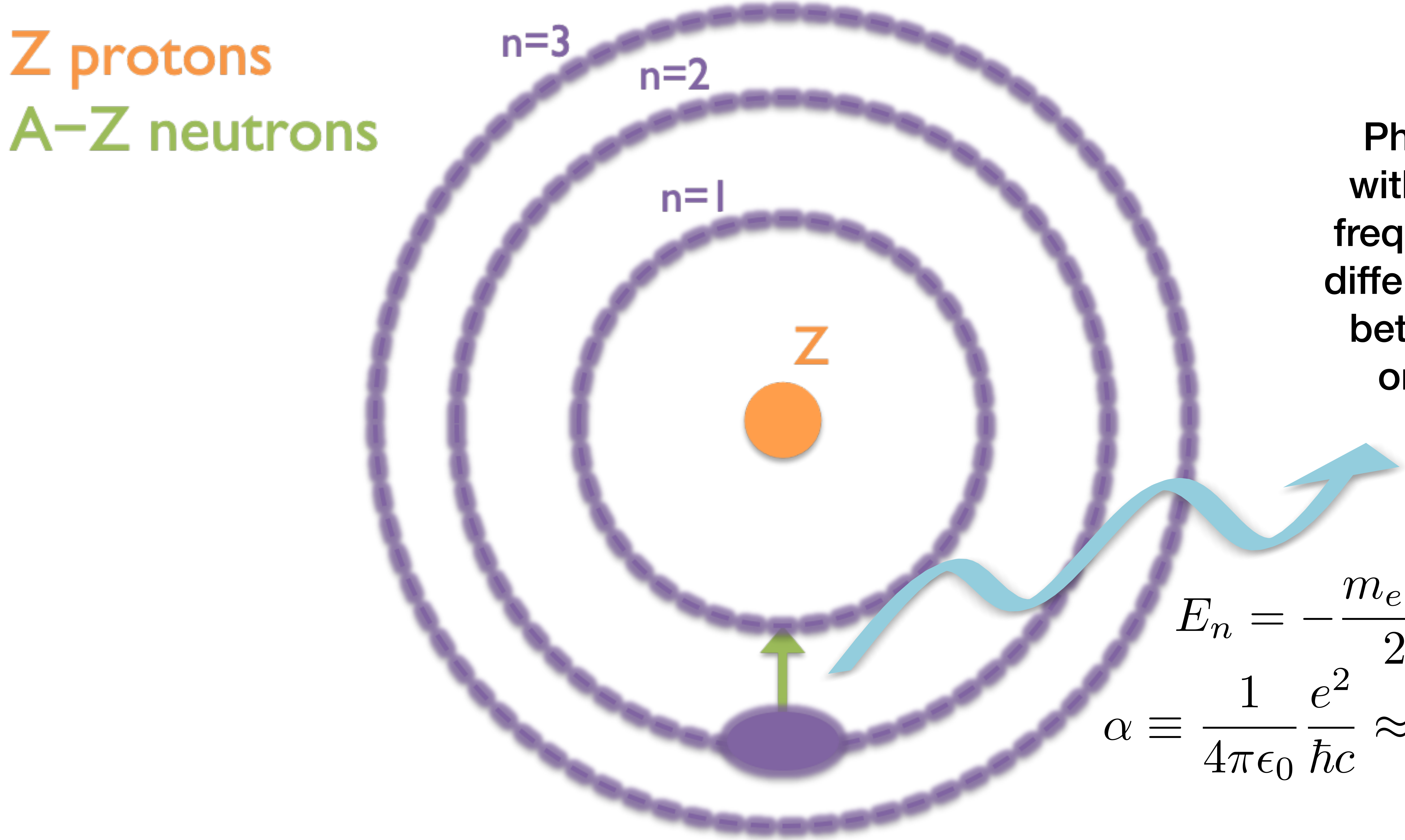
$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$m_e \approx m_p/1836$$

$$m_n \sim m_p$$

$$1 \text{ Å} = 0.1 \text{ nm} = 10^{-10} \text{ m}$$

Atomic Structure (quantized energy levels)



Photon emitted with wavelength/ frequency equal to difference in energy between allowed orbits (energy levels)

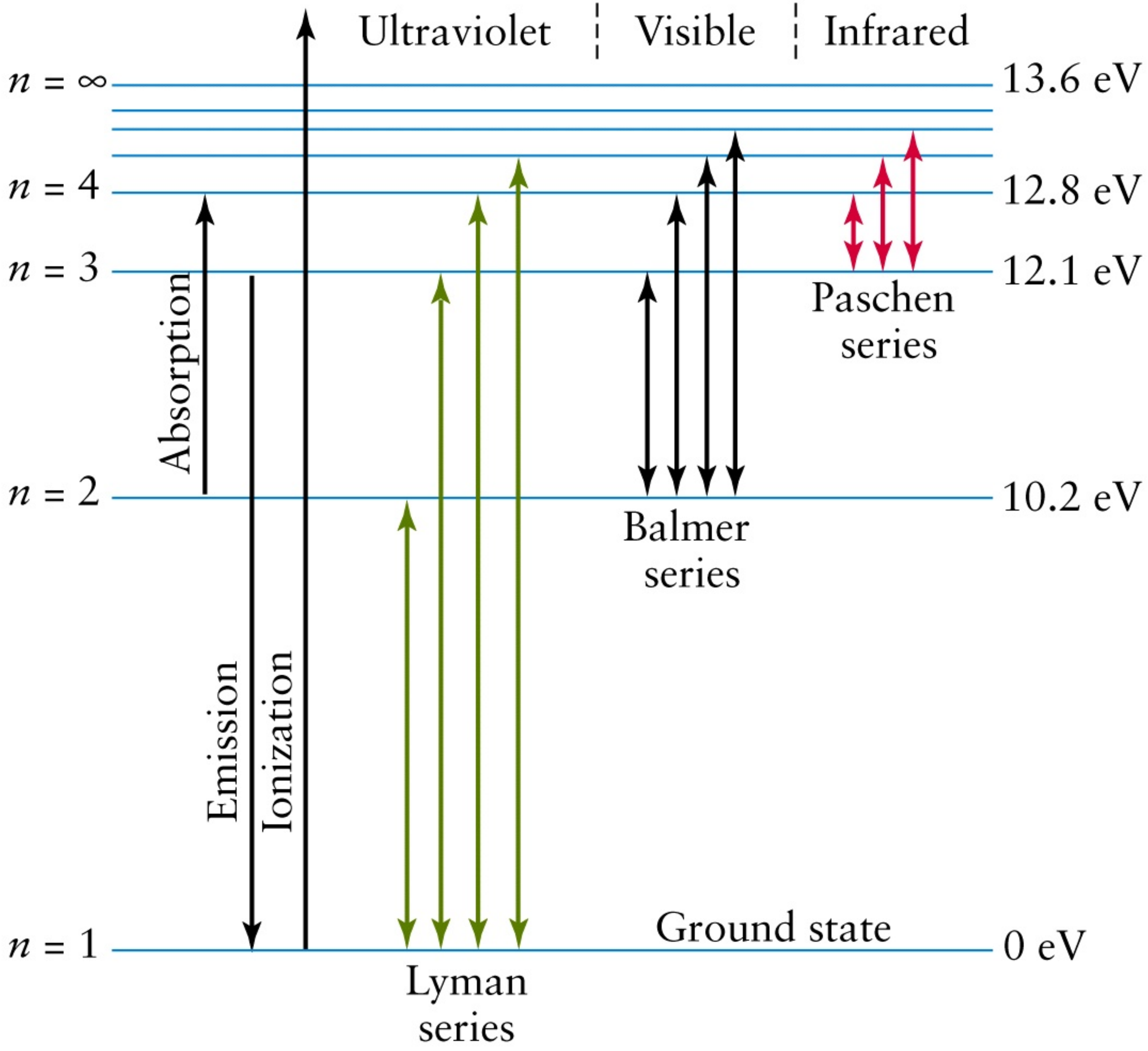
$$E_n = -\frac{m_e c^2}{2}\alpha^2 \frac{Z}{n^2}$$

$$\alpha \equiv \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx 7.30 \times 10^{-3} \approx \frac{1}{137}$$

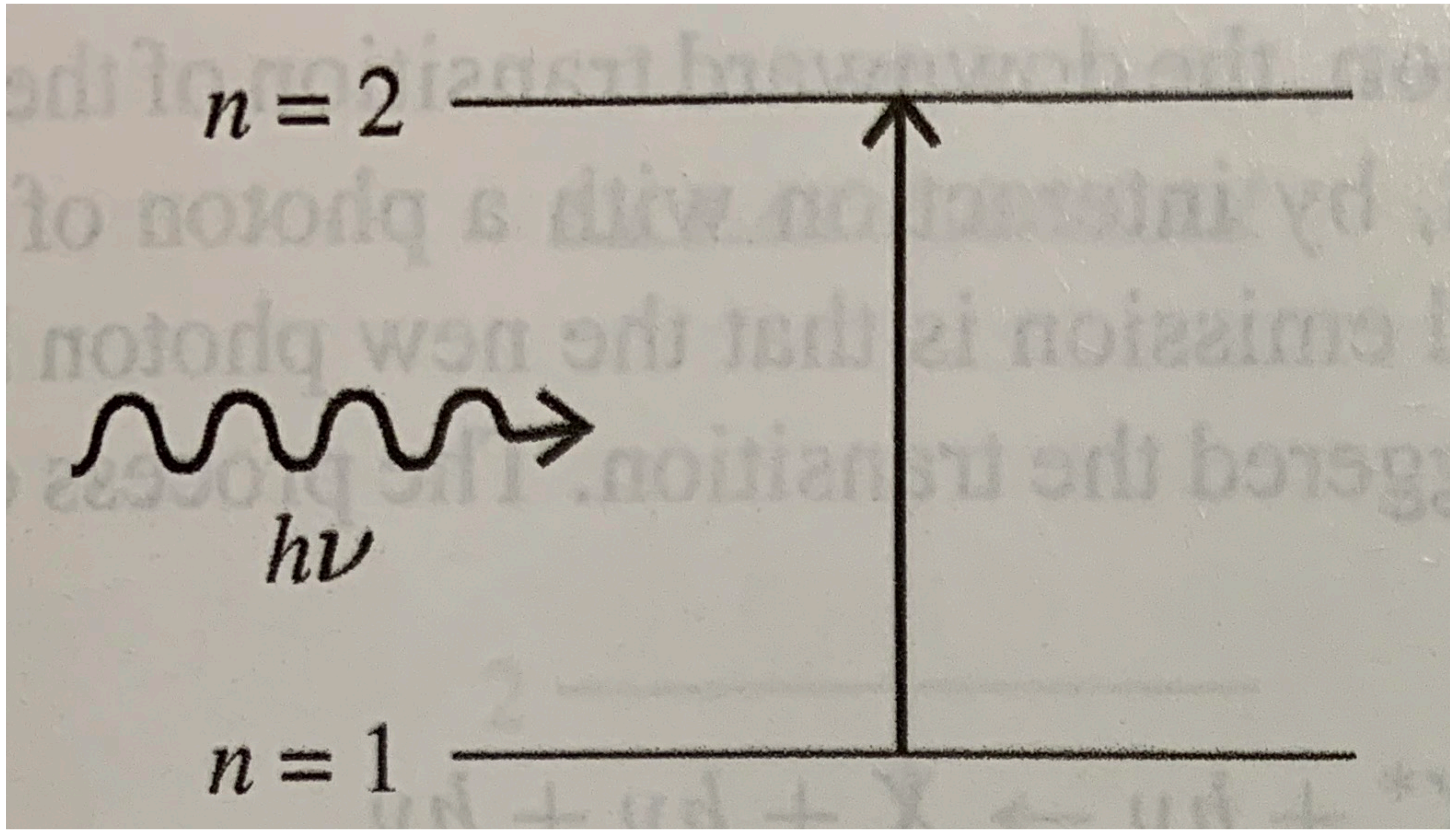
Energy Levels

$$\Delta E = E_n - E_{n'} = (13.6 \text{ eV}) \; Z^2 \left[\frac{1}{(n')^2} - \frac{1}{n^2} \right]$$

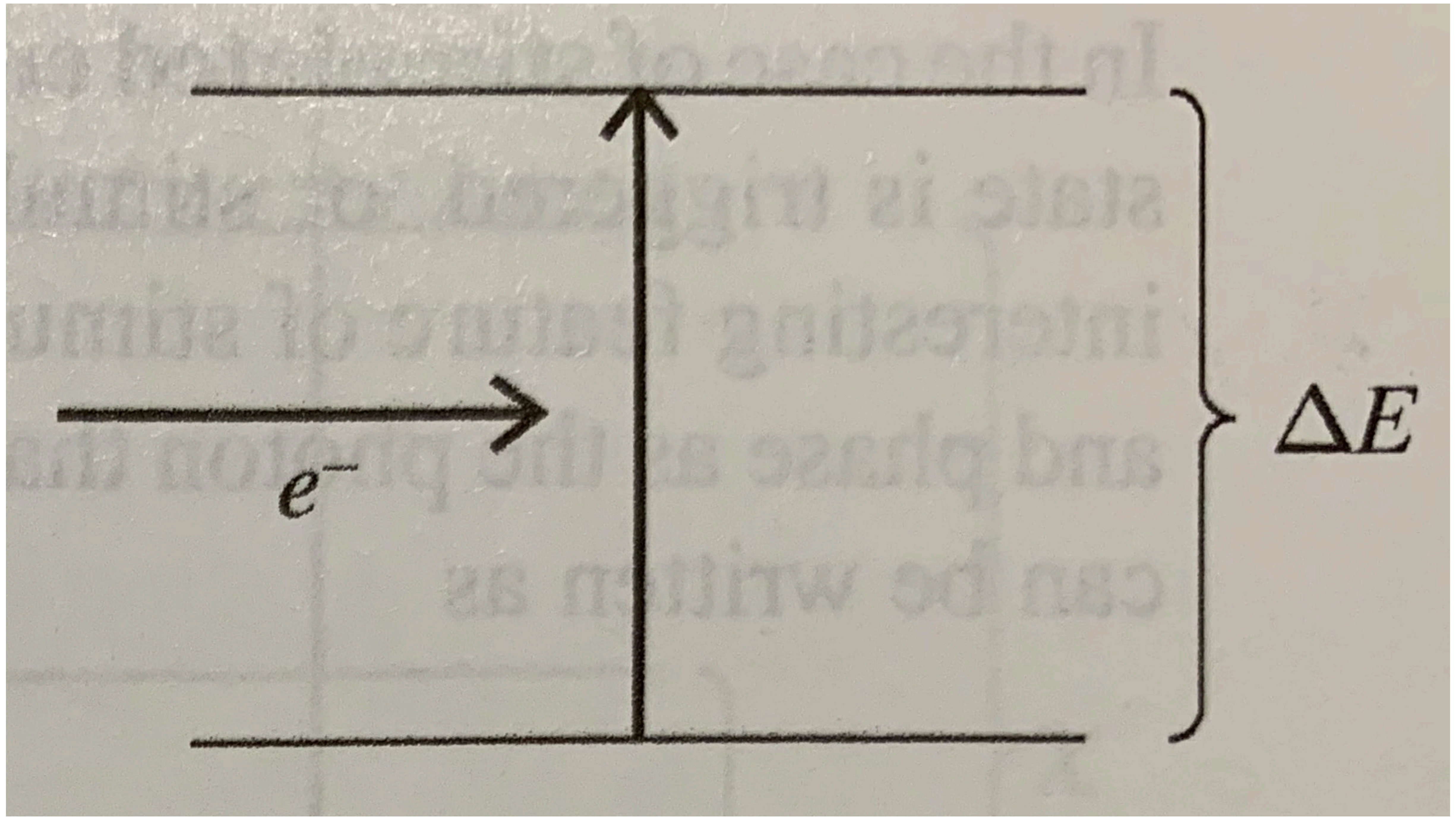
(Energies correspond to neutral hydrogen)



Absorption of Energy

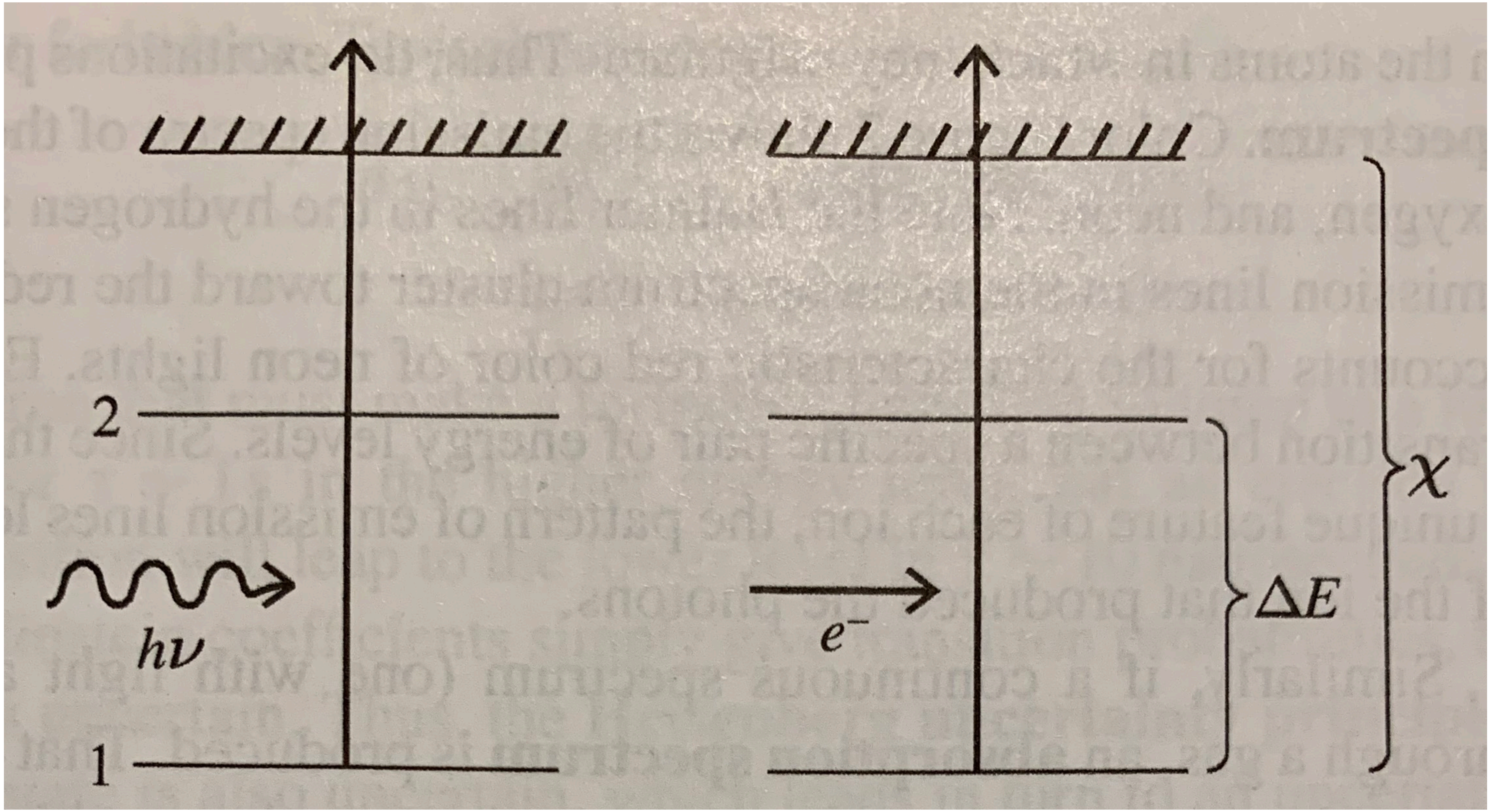


Photoexcitation



Collisional Excitation

Absorption of Energy



Photoionization

Collisional Ionization

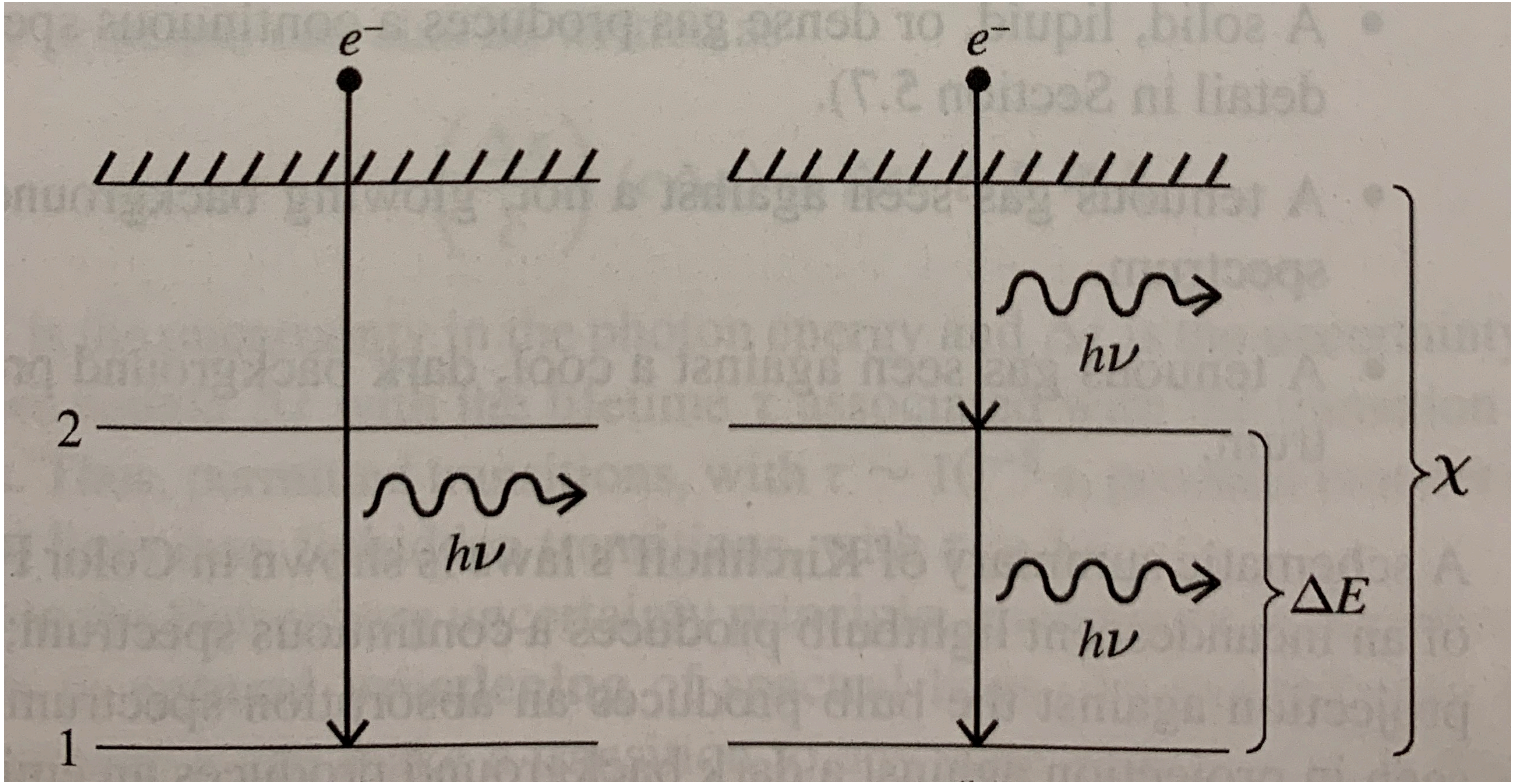
Emission of Energy

Stimulated Emission

Spontaneous Emission

Collisional De-excitation

Emission of Energy

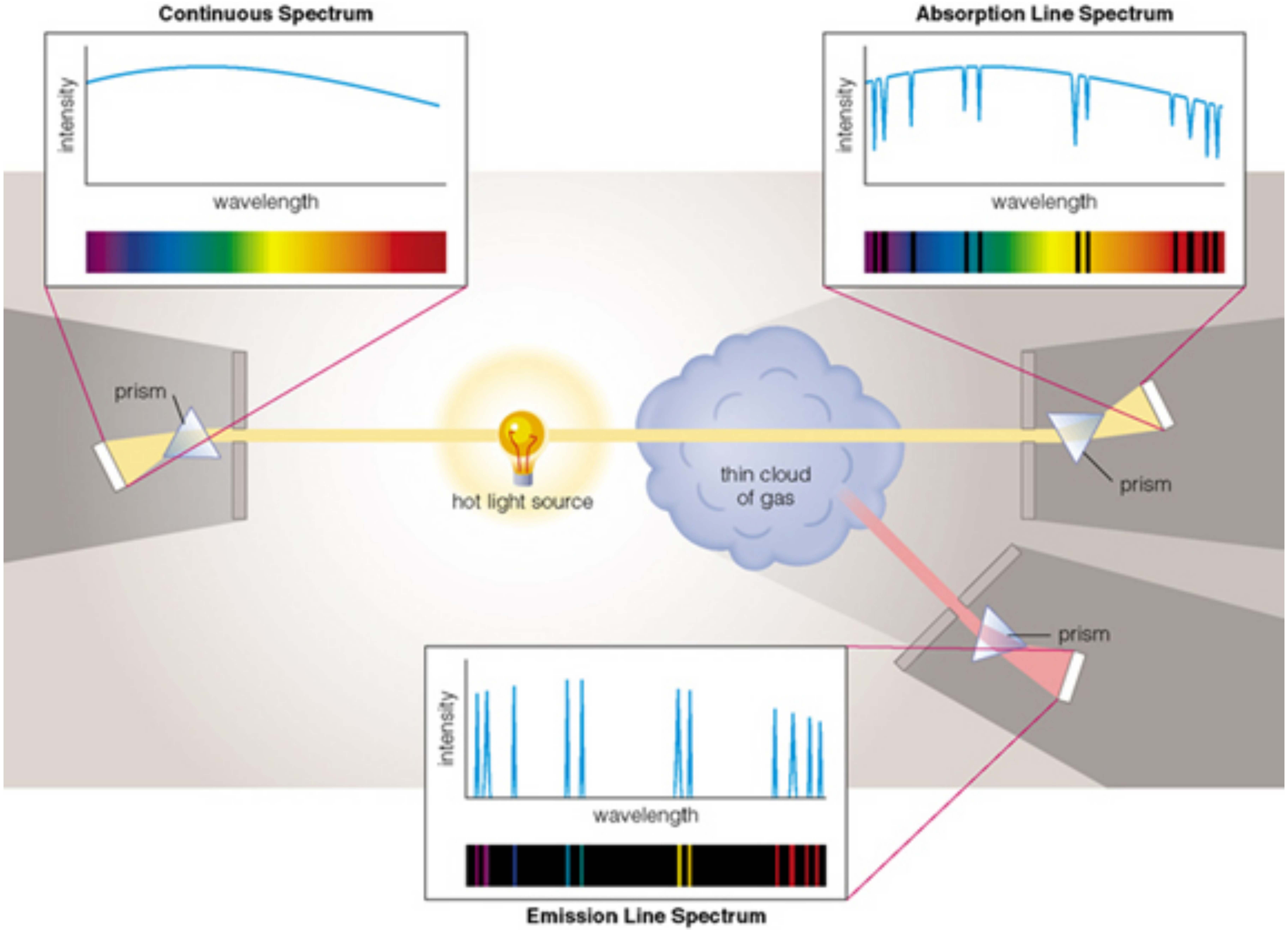


Radiative Recombination

Kirchoff's Laws

- A solid, liquid, or dense gas produces a continuous spectrum.
- A tenuous gas in front of a hot background produces an absorption spectrum.
- A tenuous gas in front of a cool background produces an emission spectrum.


Hot blackbody
Prism
Cloud of cooler gas
b
Absorption line spectrum
Prism
a
Continuous spectrum
c
Emission line spectrum

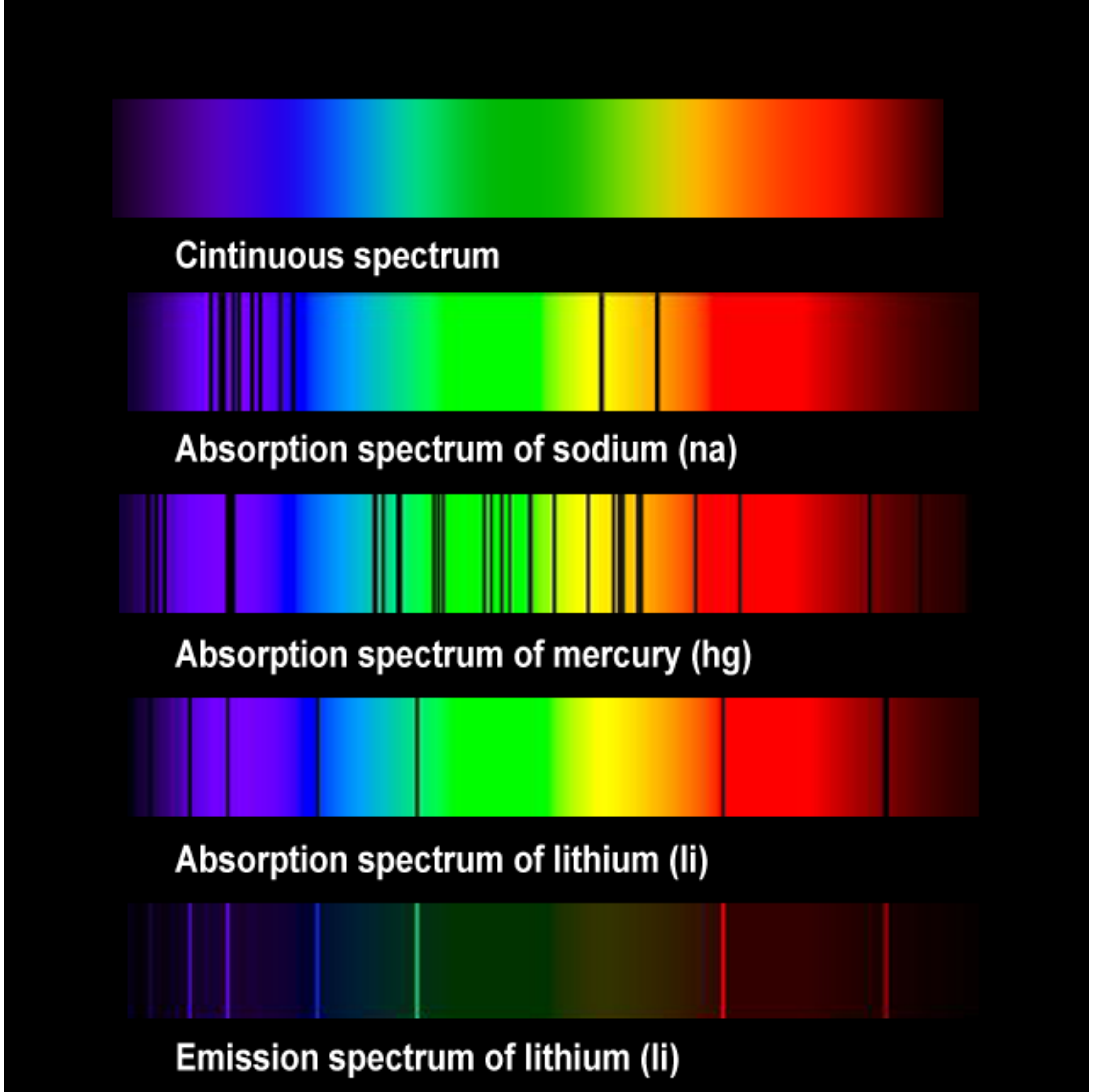

Continuous Spectrum
intensity
wavelength
Absorption Line Spectrum
intensity
wavelength
prism
hot light source
thin cloud
of gas
prism
prism
intensity
wavelength
Emission Line Spectrum

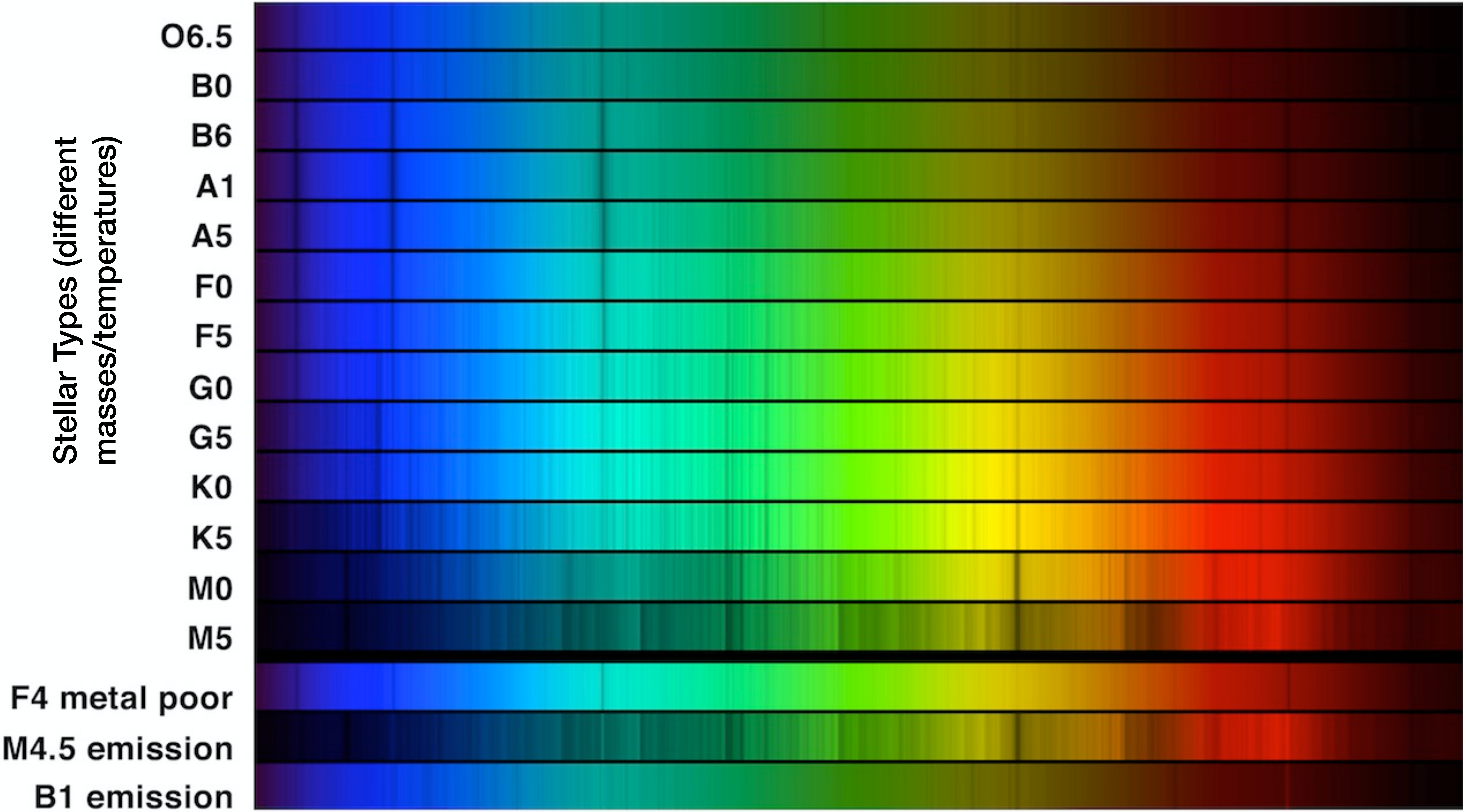
Spectra are like Fingerprints

They encode what and how much of an element is present in a gas (of a cloud, star, etc.), how hot it is, and whether it's being excited by something else

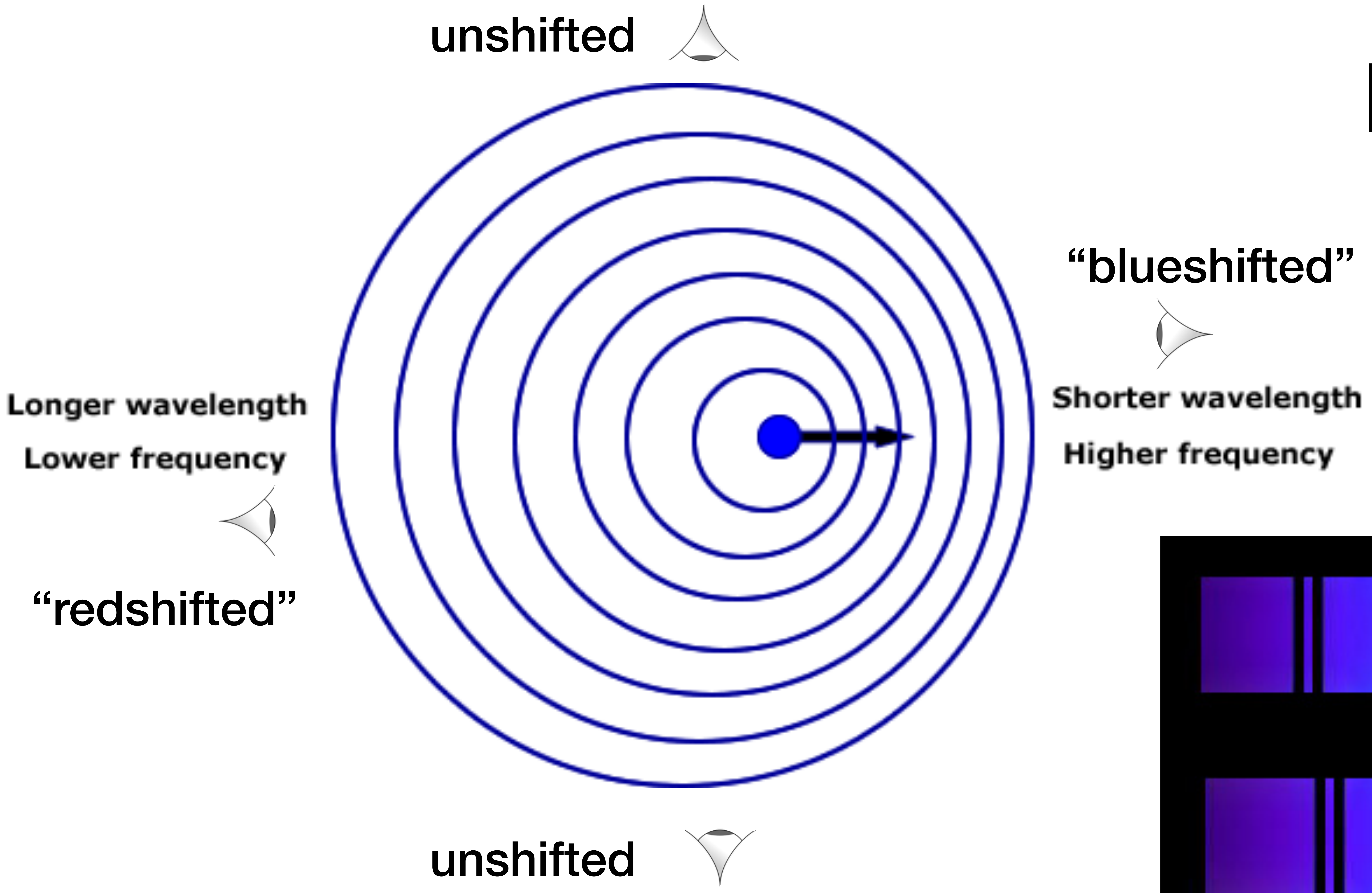
Each element has a unique pattern of lines, which can be seen in absorption or emission

$$\Delta E = E_n - E_{n'} = (13.6\ \mathrm{eV})\ Z^2 \left[\frac{1}{(n')^2} - \frac{1}{n^2}\right]$$

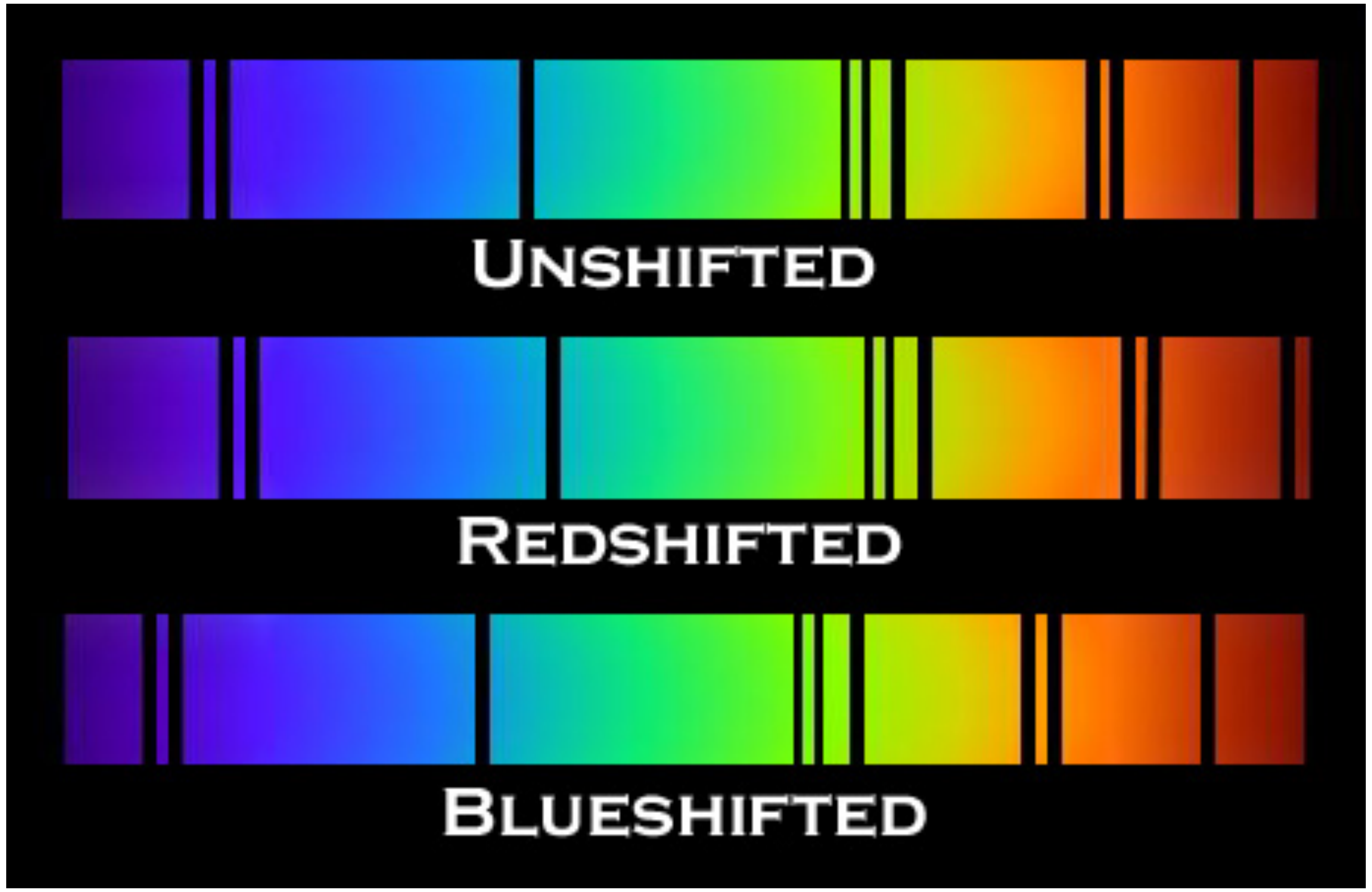



Stellar Types (different masses/temperatures)
O6.5
B0
B6
A1
A5
F0
F5
G0
G5
K0
K5
M0
M5
F4 metal poor
M4.5 emission
B1 emission

Doppler Shift



$$z = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu}$$



Allows us to infer motions
along the “line of sight”

Practice with redshift

- You measure the spectrum of a star and see an absorption line at a wavelength of 530 nm from an element with a laboratory absorption line at 540 nm.
  - What is the star's redshift? Is it moving toward or away from us?
  - How fast is it moving toward/away from us? What's its total velocity?

- You measure the spectrum of another star and see that same line at a frequency of 5.3×10^{14} Hz.
  - What is this star's redshift?
  - How fast is it moving toward/away from us?

Lines are not delta functions!

i.e., the difference b/t energy levels is NOT exact

Motion-induced Broadening (small Doppler shifts cause lines to appear more broad)

- Thermal Broadening
- Rotational Broadening
- Turbulent Broadening

Other Types of Broadening

- Natural Broadening
- Pressure Broadening
- Zeeman Broadening

Natural Broadening

$$\frac{dN_{\rm phot}}{dt} = n_2 A_{21}$$

$$A_{21} \sim 10^8 \ {\rm s}^{-1}$$ (permitted)

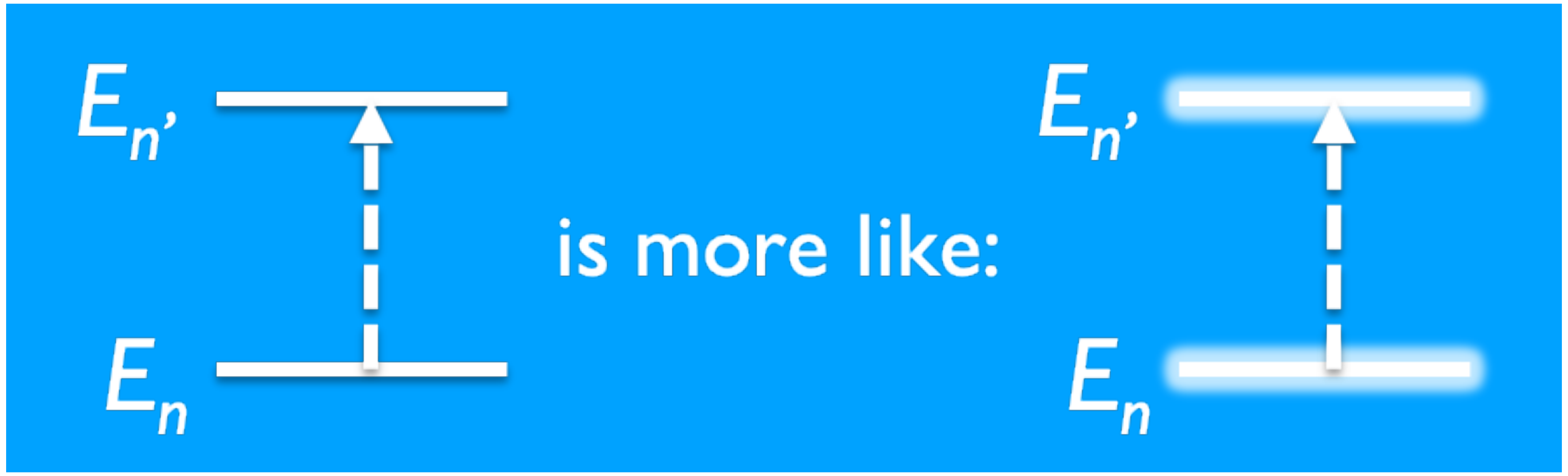
$$\sim 1 \ {\rm s}^{-1}$$ (forbidden)

Heisenberg uncertainty principle

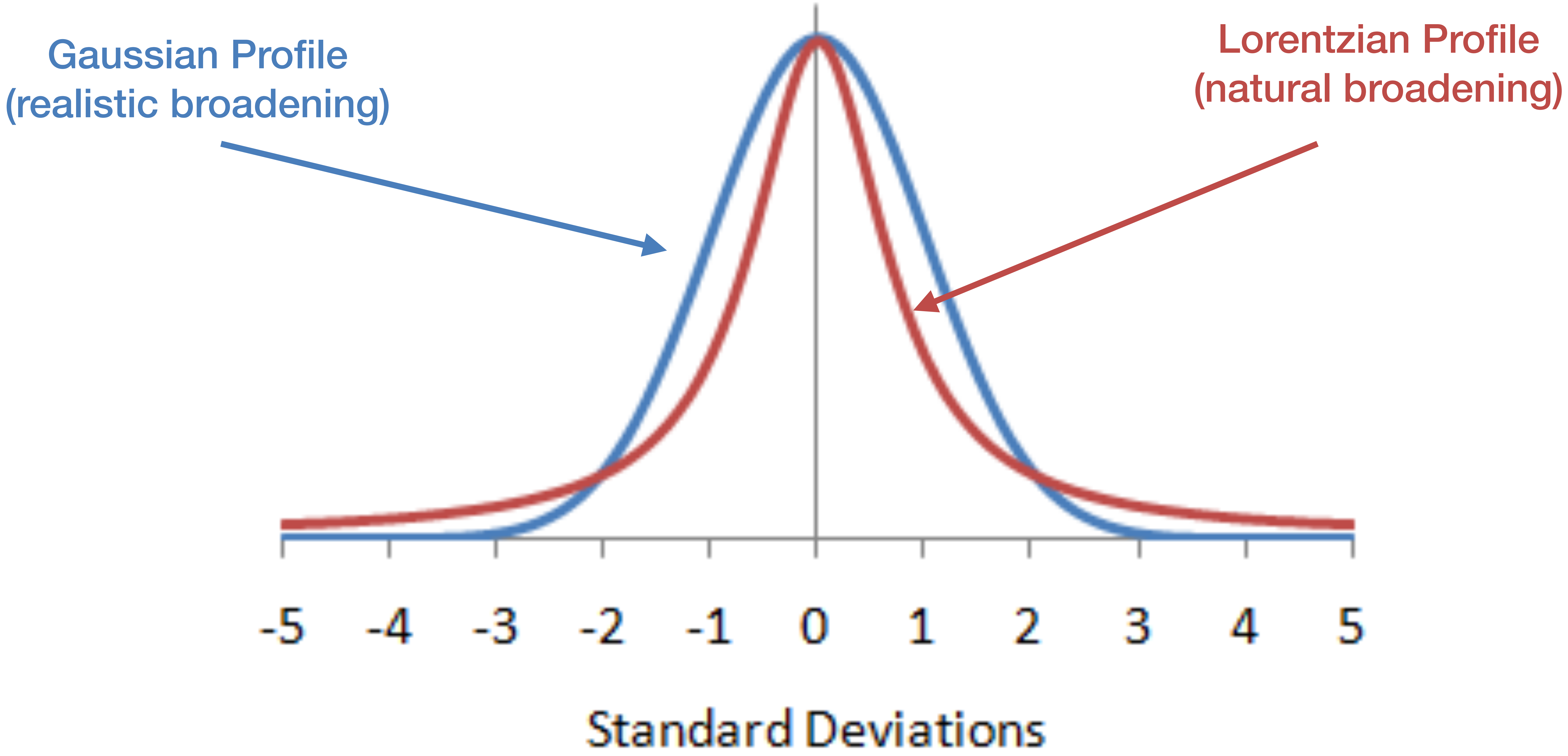
$$\Delta x \cdot \Delta p \gtrsim \hbar$$

$$(\frac{\Delta x}{c})(\Delta p \cdot c) \gtrsim \hbar$$

$$\Delta t {\cdot} \Delta E \gtrsim \hbar$$



Broadened Line Shapes



$$(\nu - \nu_0)/\sigma$$

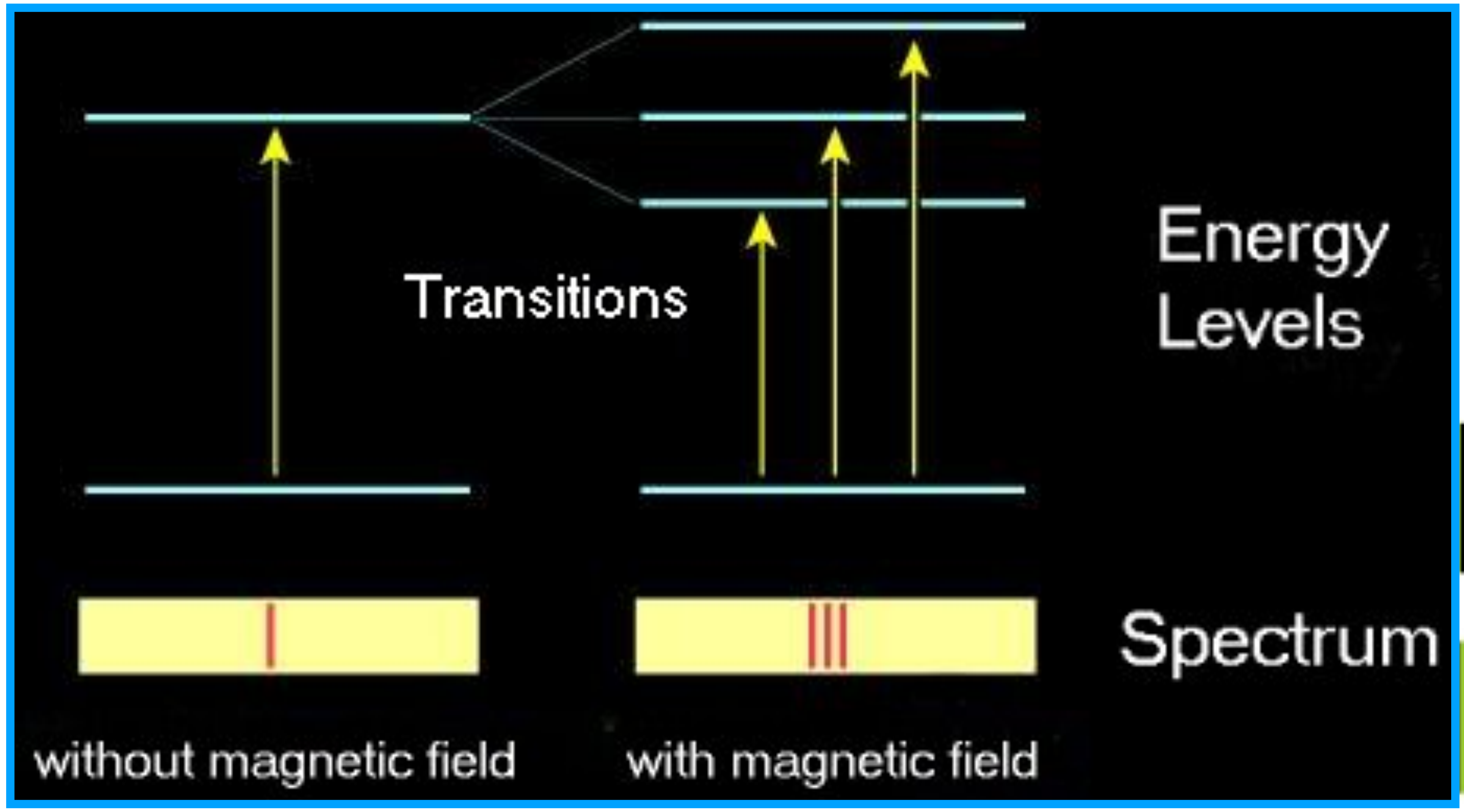
Lines are not delta functions!

i.e., the difference b/t energy levels is NOT exact

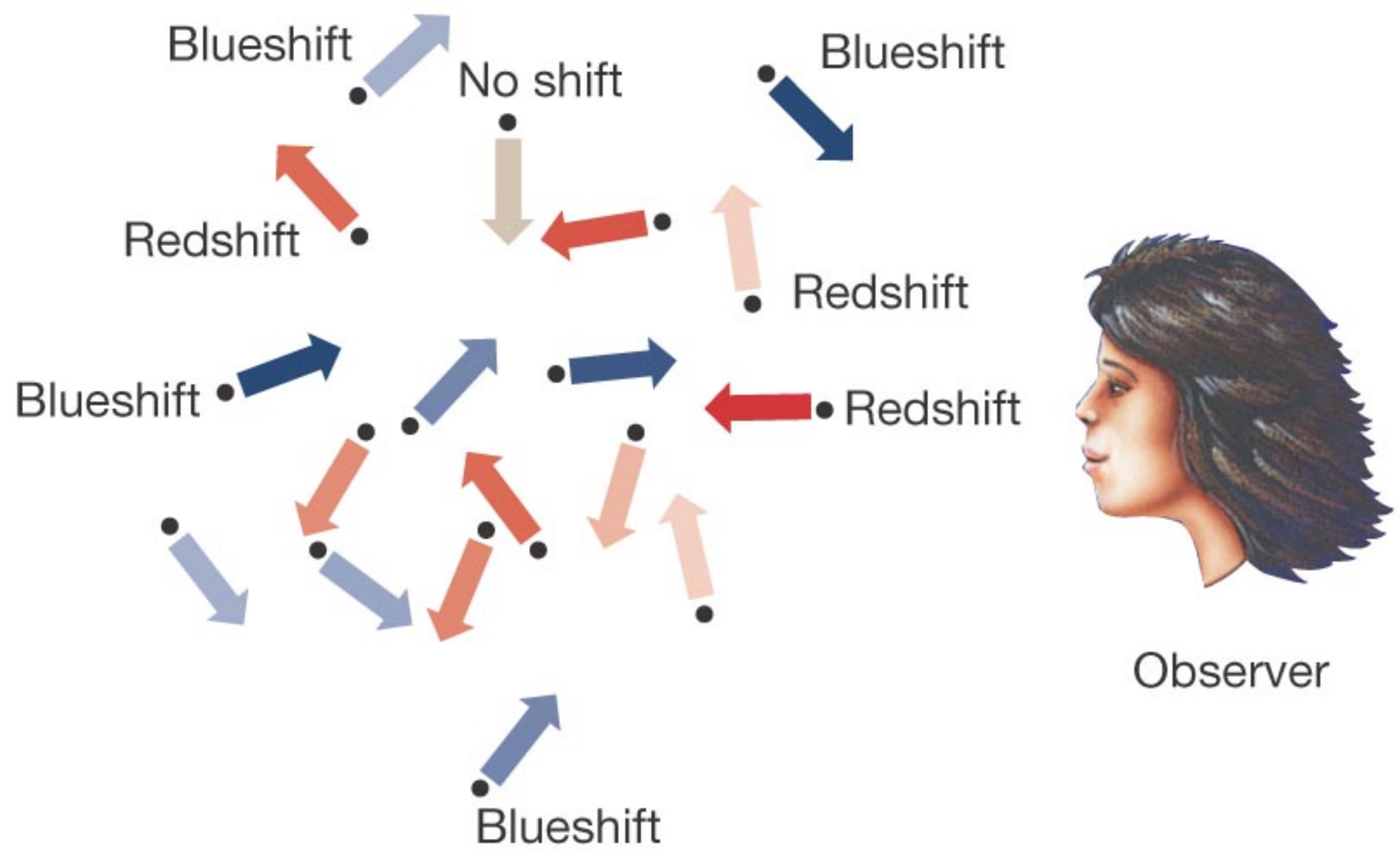
Dense environments, frequent collisions induce electric fields that modify energy levels

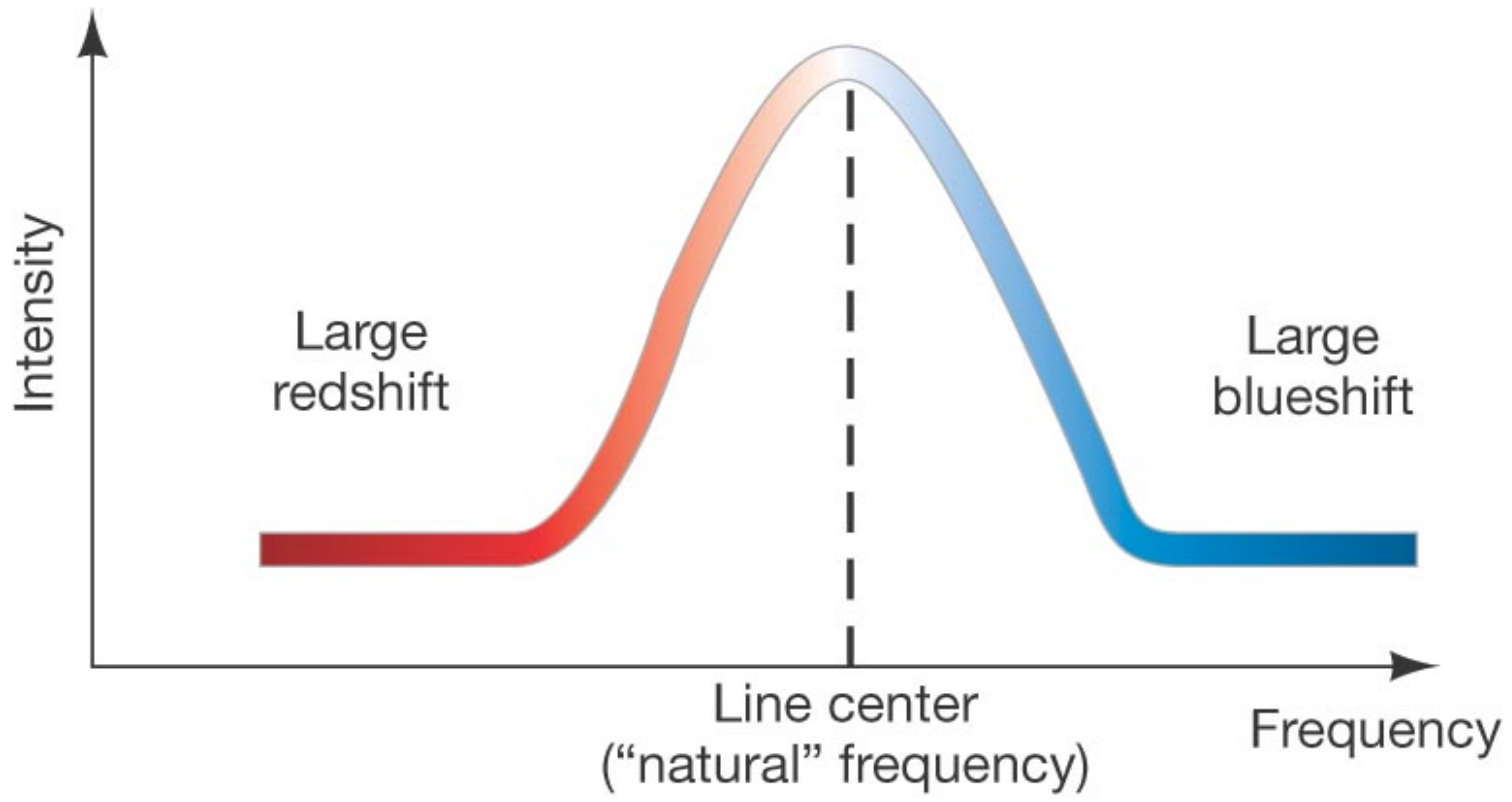
Other Types of Broadening

- Natural Broadening
- Pressure Broadening
- Zeeman Broadening



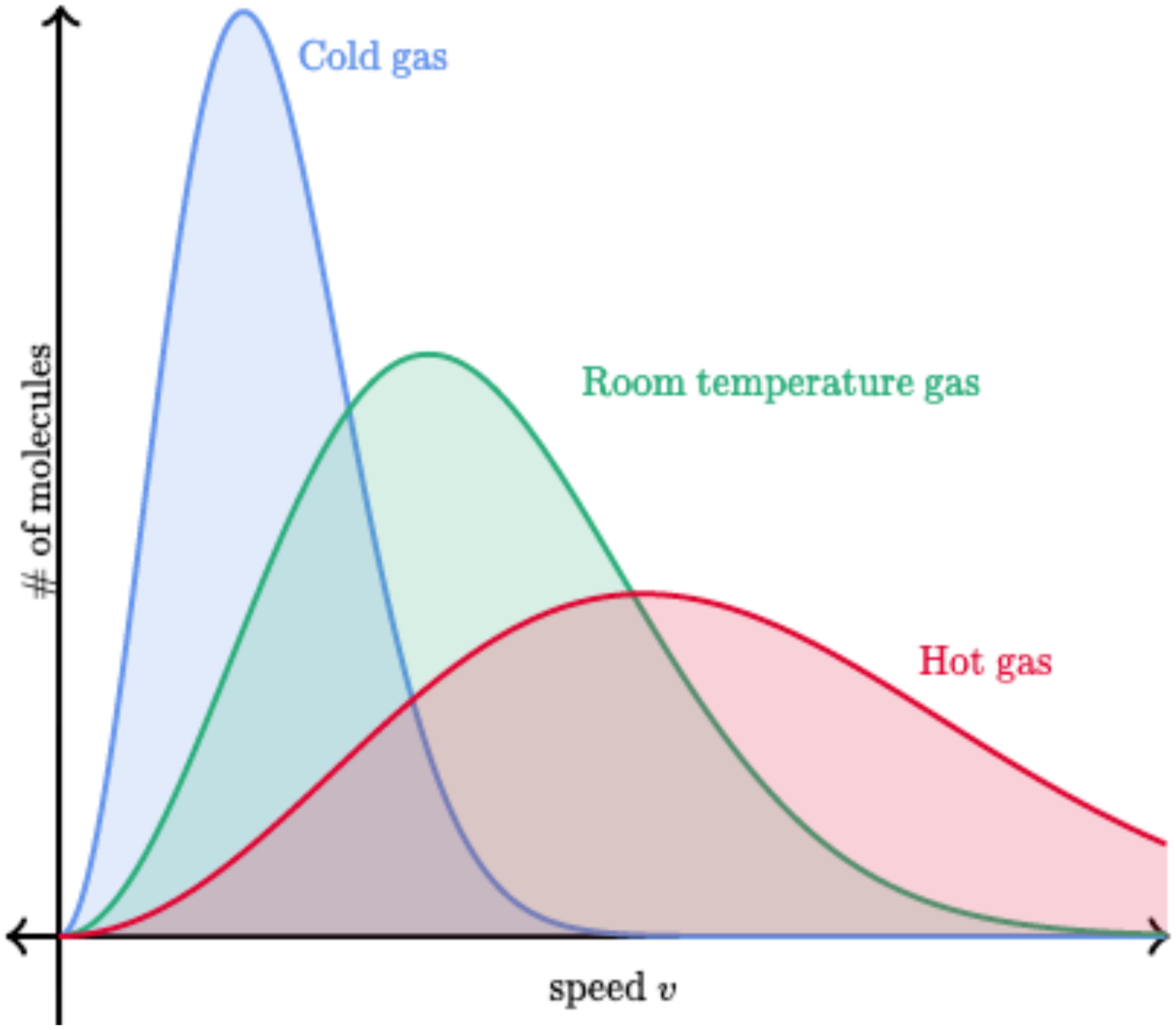
Doppler Broadening





Thermal Broadening

Velocity distribution of particles in thermal equilibrium have a Maxwell-Boltzmann distribution



$$F(v)dv = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

$$F(E)dE = F(v)\frac{dv}{dE} = \frac{2}{\sqrt{\pi}kT}\left(\frac{E}{kT}\right)^{1/2} \exp\left(-\frac{E}{kT}\right)$$

$$\langle x \rangle = \int x f(x) dx$$

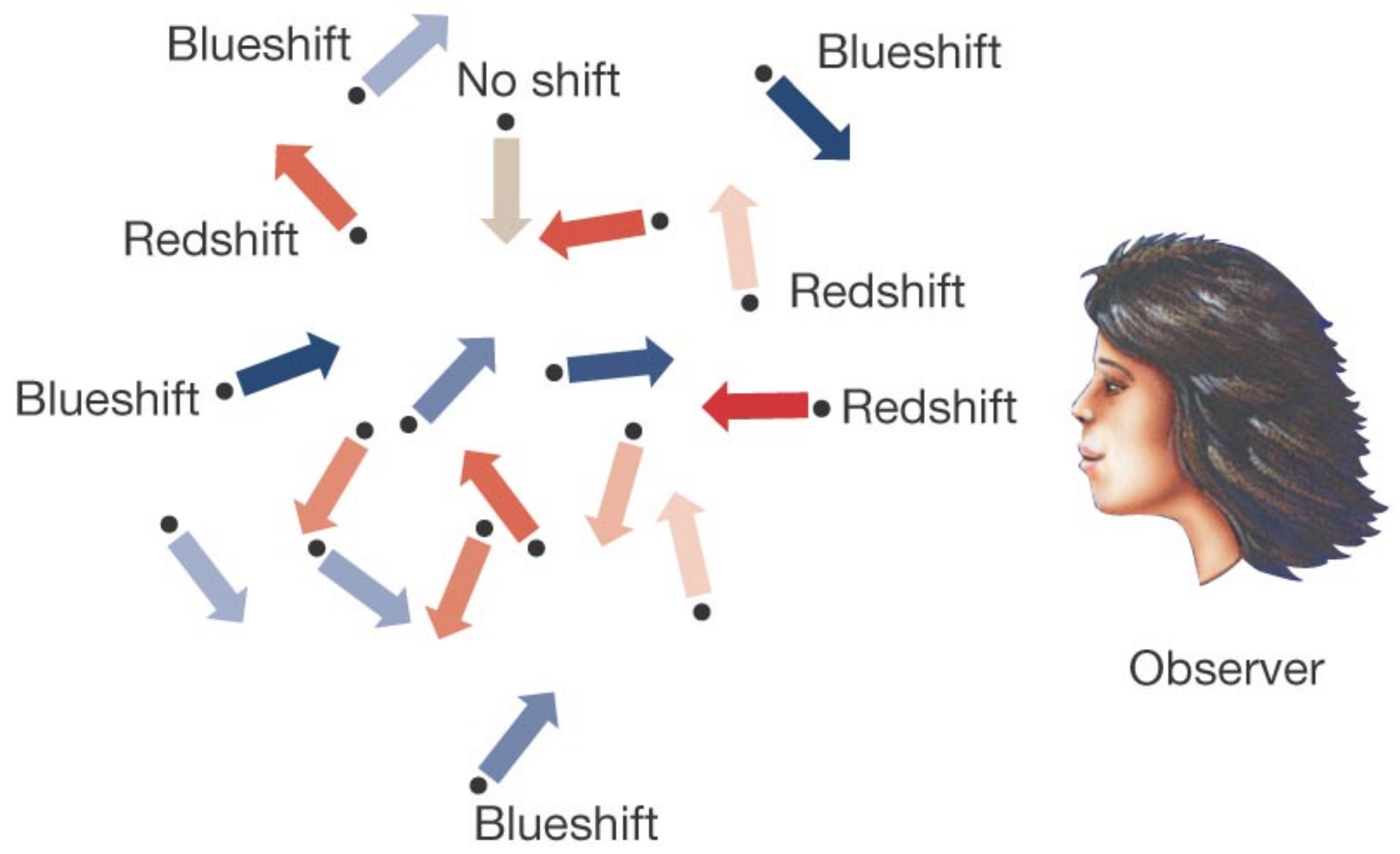
$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m}}$$

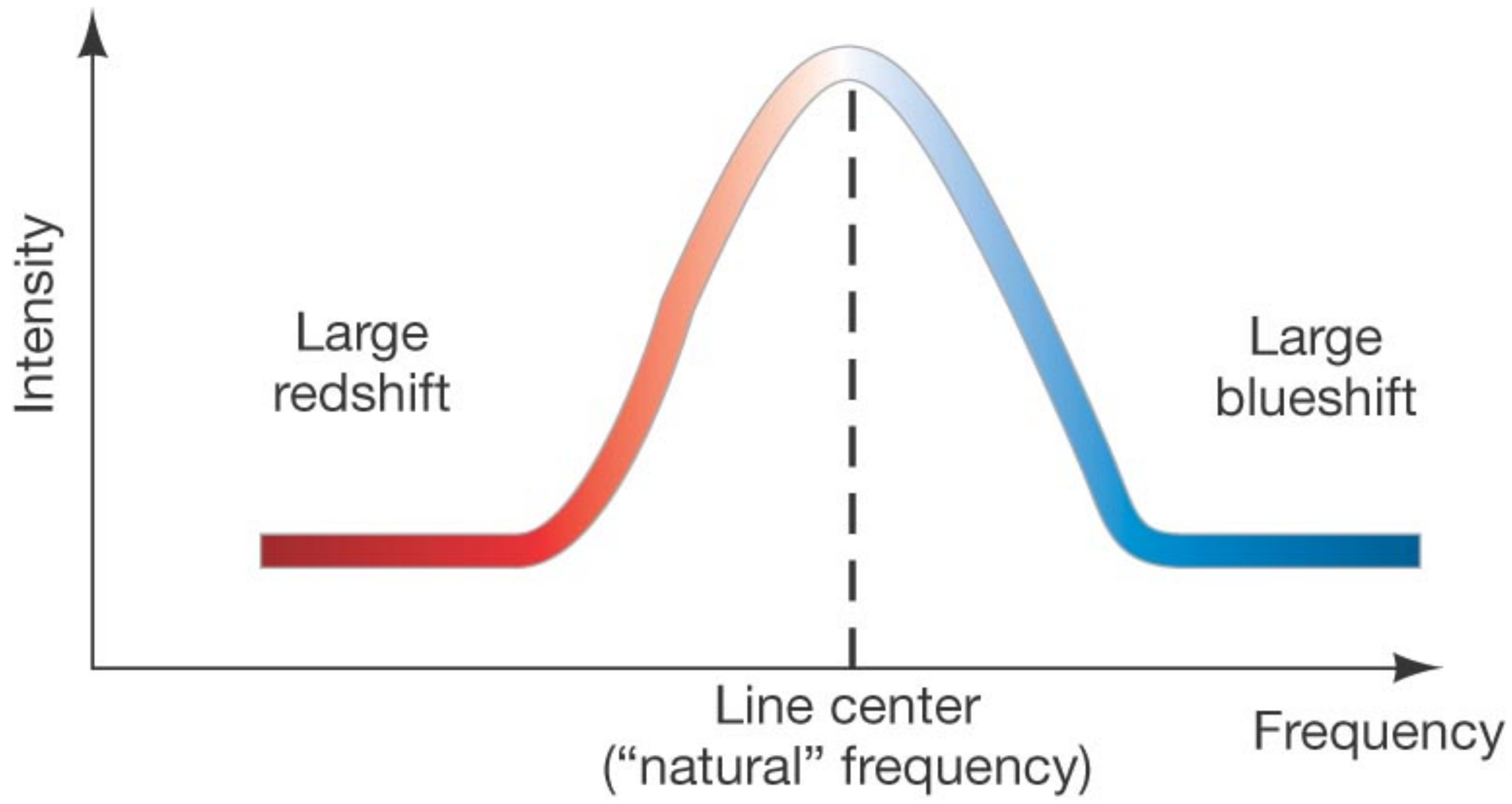
Avg. particle speed

$$\langle E \rangle = \frac{3}{2}kT$$

Avg. particle kinetic energy

Doppler Broadening





Thermal Broadening

line-of-sight "velocity dispersion"
(width of a Gaussian distribution)

$$\sigma_{\rm los} = \left(\frac{kT}{\mu m_p}\right)^{1/2} \approx 100 \text{ m s}^{-1} \left(\frac{T}{1\ K}\right)^{1/2} \mu^{-1/2} \longrightarrow \frac{\Delta\lambda}{\lambda} \approx \frac{\sigma_{\rm los}}{c}$$

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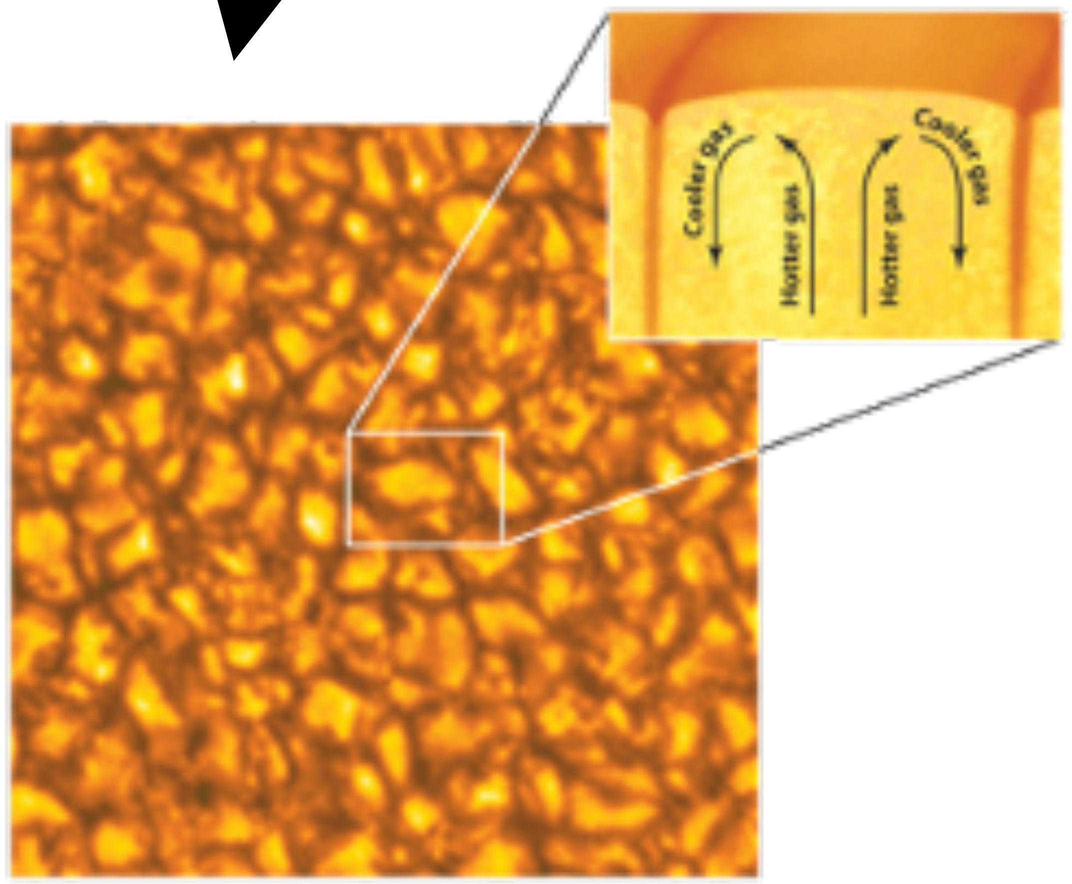

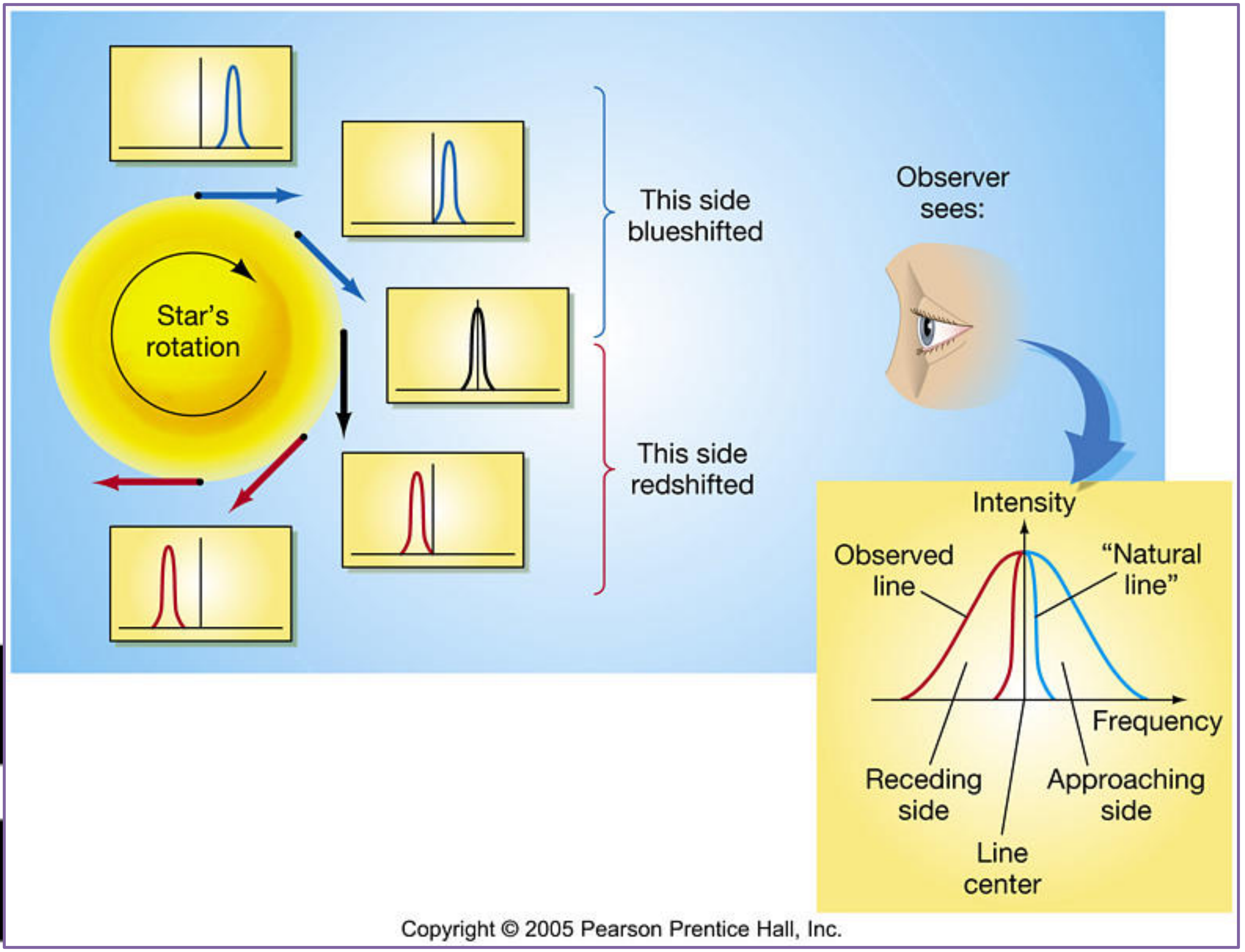

Radiative Transfer / mfp / optical depth / Blackbody Spectra