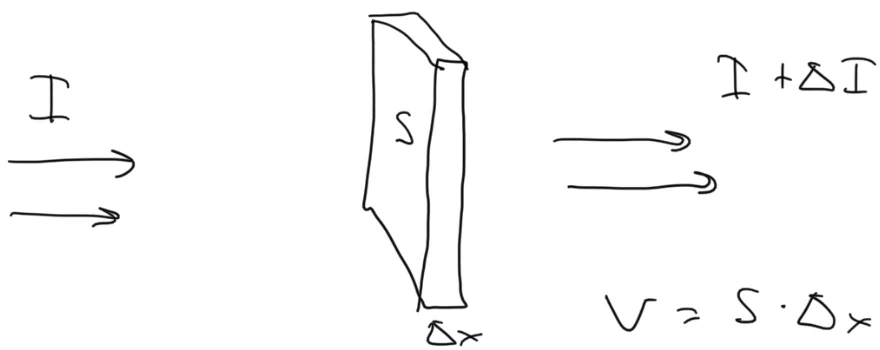
ASTR 3070 - Week 04a (Ch.5: RadTrans/OptDepth/BB)

Radiative Transfer

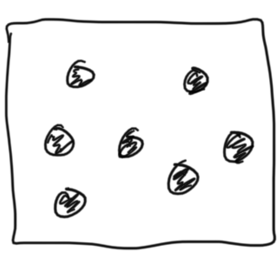
To understand the light we see, we need to know what happens to it on our way here → universe is full of crap

Imagine light w/some "intensity" I

$$\vec{I} \rightarrow \frac{\text{energy}}{\text{area} \cdot \text{time}} \quad \text{in some direction}$$



$V = S \cdot \Delta x$



Squish $\Delta x \rightarrow 0$, project cross-section of atoms onto surface S, each atom has area $\pi r^2 = \sigma$

so total cross-section is $\sigma_{Tot} = N\sigma$ ← # atoms

If light is shining across surface, but is absorbed wherever there are atoms,

$$\frac{\Delta I}{I} = -\frac{\sigma_{Tot}}{S} = -\frac{N\sigma}{S}$$

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Loss of energy just geometric.

But, fundamental absorbing element is the atom, so want in terms of σ

$$\frac{\Delta I}{I} = -\frac{N\sigma}{S} \Rightarrow \text{what is } N?$$

Normally we know the density $n = \frac{N}{V}$ of a thing, so $N = nV = nS\Delta x$

$$\frac{\Delta I}{I} = -\frac{n\cancel{S}\sigma}{\cancel{S}}\Delta x$$

Can make this a differential eqn.

$$\int^{I} \frac{dI}{I} = \int^{x} -n\sigma\, dx$$

$/_{I_0}$ $\perp$ $/_0$

$$\ln\left(\frac{I}{I_0}\right) = -n\sigma x$$

$$\boxed{I(x) = I_0 e^{-n\sigma x}}$$

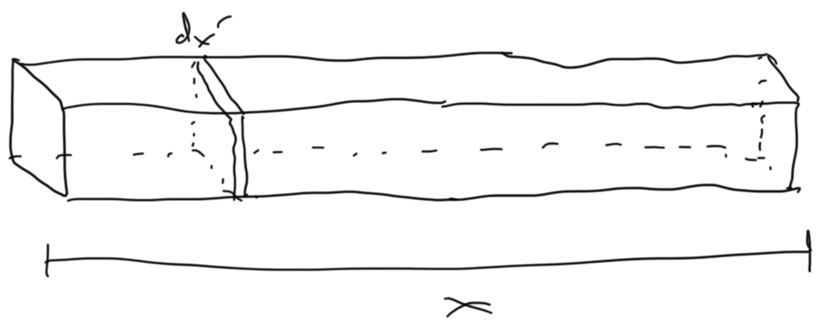
Eqn. of Radiative Transfer

Column Density

optical depth 1st

- integrated density along line-of-sight

$$N(x) \equiv \int_0^x n(x')dx' = nx$$



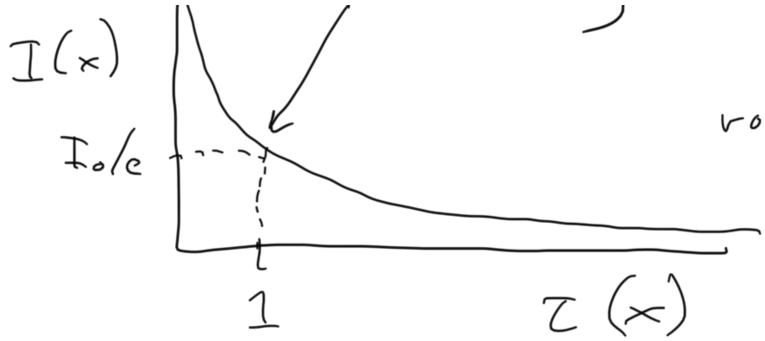
Optical Depth

Term in the exponent

$$\tau = n\sigma x = \sigma N(x)$$

$$I(x) = I_0 e^{-\tau(x)}$$

I_0 τ 1 e-folding



$e \approx 2.72$

roughly $\frac{1}{3}$ of photons left when $\tau = 1$

Mean Free Path

- distance avg. photon travels before being absorbed / scattered : x when $\tau = 1$

m.f.p. $\langle x \rangle = \dfrac{1}{n\sigma}$

☆ Real life example of optical depth

In general, processes are λ-dependent,

so $\boxed{I_\nu = I_{\nu,0}\, e^{-\tau_\nu(x)}}$

Local Thermodynamic Equilibrium

Thermodynamic → follows avg. statistics, Max-Boltz. dist. etc.

Equilibrium → no time dependence

Local → regions exist that are large enough where $x \gg$ mfp, but small enough such that $\Delta T \ll T$

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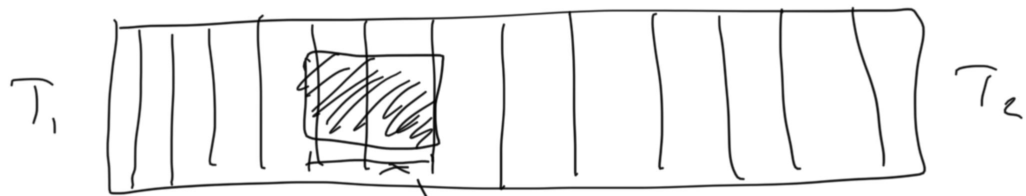


$mfp \rightarrow$

$x > mfp$

$T \sim const$

- particles share energy thru collisions, reach equilibrium

$$F(v) \propto v^2 e^{-mv^2/2kT}$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$$

Consider air, $T \sim 300K$ so $\frac{3}{2}kT \sim 0.04$

$$\& \; V_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{2 \cdot 0.04\, eV}{28 \times 10^9\, eV}} \cdot 3 \times 10^8\, m/s$$

$\sim 500\, m/s$ (Nitrogen)

1 mph $\sim$ 0.44 m/s $\rightarrow$ 1100 mph!

During the storm, had 100 mph winds, still much lower velocity than molecu[les] themselves $\rightarrow$ wind vel. doesn't contribute much to air temperature continuous

mfp of air molecules is tiny: ~ 400Å

★ What's the optical depth of air to light?

mfp optical light >> mfp molecule

→ not in LTE in room, considering air + light: optical photons don't tell us anything about thermodynam. of the air

$\boxed{\text{LTE}}$ ① Photons + massive particles have a ↑ # density

② System optically thick

$\tau \gg 1$ for all λ's

When true, the spectrum of light reaches an equilibrium dist. like the Max.-Boltz. dist. for particles

→ called the <u>Planck function</u>

$$\boxed{J_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}}$$

Spectrum of blackbody radiation

(perfect absorber of energy)

Consider 2 limits

$h\nu \ll kT$: $e^{h\nu/kT} \approx 1 + \frac{h\nu}{kT}$

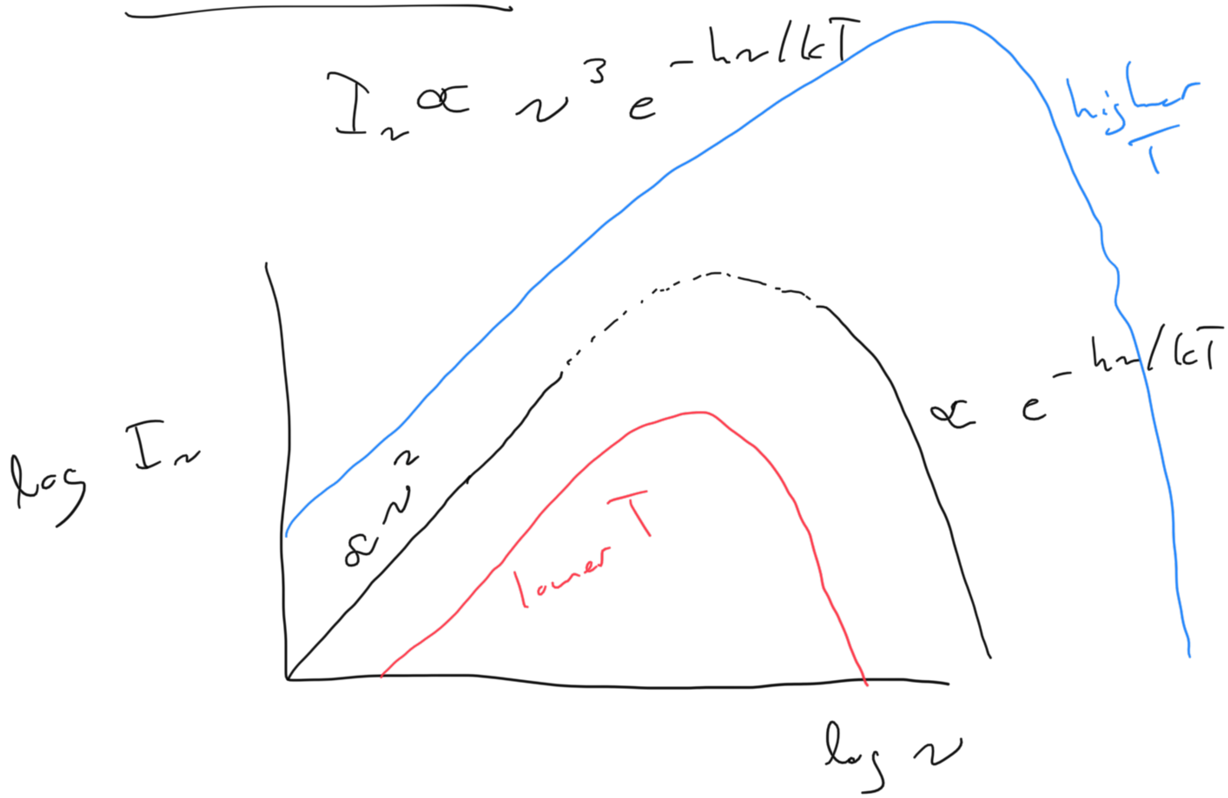
$h\nu \gg kT$: $e^{h\nu/kT} \gg 1$

Rayleigh-Jeans limit (low ν)

$$I_\nu \propto \frac{2kT}{c^2}\nu^2$$

Wien Limit (high ν)

$$I_\nu \propto \nu^3 e^{-h\nu/kT}$$



Can also express as function of λ,

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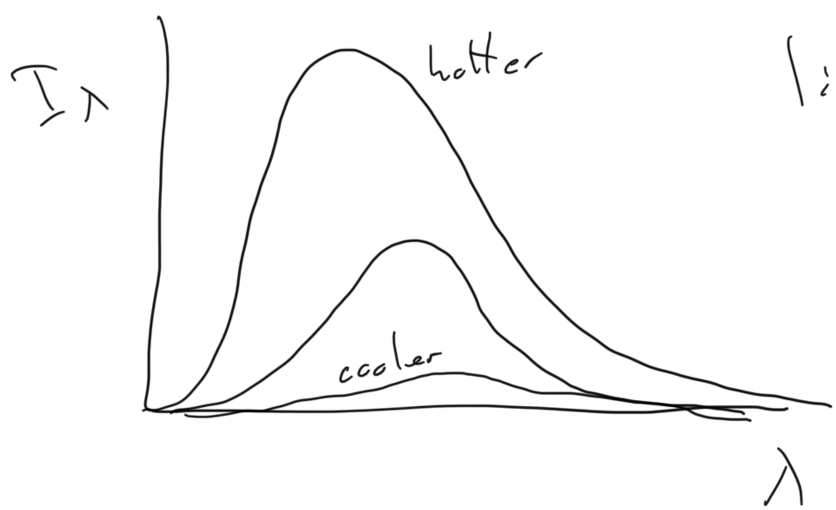
since $I_\nu d\nu \, (\nu \to \nu + d\nu) =$

$$I_\lambda d\lambda \, (\lambda \to \lambda + d\lambda)$$

& $\nu = \frac{c}{\lambda}$, $\quad d\nu = -\frac{c}{\lambda^2} d\lambda$

$$I_\lambda d\lambda = I_\nu d\nu = I_\nu \left| \frac{d\nu}{d\lambda} \right| d\lambda$$

$$\boxed{I_\lambda d\lambda = \frac{2hc^2}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}}$$



linear plot

$$I \to \frac{\text{energy}}{\text{area} \cdot \text{time} \cdot \text{angle} \cdot (\text{freq. or wavelength})}$$

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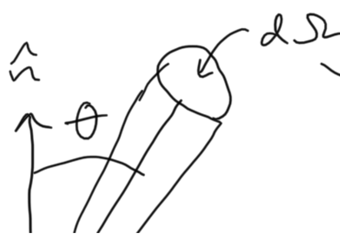
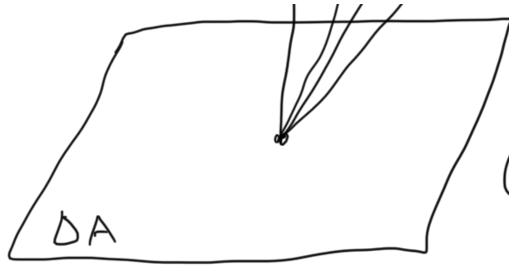


→ angular area, has units of



steradians

(a sphere has an angular area of 4π steradians)

Integrate over all freq. & all angles, get total E per area per time, i.e. the Flux

$$F = \underbrace{\frac{2\pi^5}{15}\frac{k^4}{c^2h^3}}_{\sigma_{SB}} T^4 \quad \left(\frac{\text{energy}}{\text{area}\cdot\text{time}}\right)$$

Stephan-Boltzmann constant

$$\sigma_{SB} = 5.67\times10^{-8}\ J\ s^{-1}\ m^{-2}\ K^{-4}$$

Tells us how much E per time a star is emitting per area of its surface, so need to integrate over its entire surface to get total E per t

↳ call this luminosity L

Assume star is spherical:

$$L = F \cdot A = F \cdot 4\pi R^2$$

$$\boxed{\boxed{L = 4\pi R^2 \sigma_{SB} T^4}}$$

Usually compare to the Sun

$R_\odot = 6.96 \times 10^8 m$

$T_\odot = 5780 K$

$L_\odot = 3.8 \times 10^{26} W$

Can rewrite eqn. as

$$L = 1 L_\odot \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

Similar to Kepler's law in how it's scaled to convenient units

If class boring...

Problem-solving break

Walk thru HW2:4

$\sqrt{1 + \frac{v}{c}}$

$$E = \gamma mc^2 \quad / \quad \lambda = \lambda_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

vs. $\frac{v}{c} = \frac{\lambda-\lambda_0}{\lambda_0}$

$v \ll c$, equ.s equivalent

$$\lambda = \lambda_0 \left(1+\frac{v}{c}\right)^{\frac{1}{2}}\left(1-\frac{v}{c}\right)^{-\frac{1}{2}}$$

Taylor expand each term

$$\left(1+\frac{1}{2}\frac{v}{c}+\dots\right)\left(1+\frac{1}{2}\frac{v}{c}+\dots\right)$$

$$\left(1+\frac{1}{2}\frac{v}{c}+\frac{1}{2}\frac{v}{c}+\underbrace{\cancel{\frac{1}{4}\frac{v^2}{c^2}}}_{\text{small}}+\dots\right)$$

$$\lambda = \lambda_0\left(1+\frac{v}{c}\right) = \lambda_0 + \lambda_0\frac{v}{c}$$

$$\lambda - \lambda_0 = \lambda_0 \frac{v}{c} \rightarrow \boxed{\frac{v}{c} = \frac{\lambda-\lambda_0}{\lambda_0}}$$

Ever wonder why $KE = \frac{1}{2}mv^2$ ← why this?

$$E = mc^2 \rightarrow E_{tot} = \gamma mc^2$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Taylor expand $\approx 1 + \frac{1}{2}\frac{v^2}{c^2} + \dots$

$$E = \gamma mc^2 \approx mc^2\left(1+\frac{1}{2}\frac{v^2}{c^2}+\dots\right)$$

$$= mc^2 + \frac{1}{2} mv^2$$

rest mass E

kinetic E when $v \ll c$!