Week 4 Thursday

Today's Agenda

- Radiative Transfer
- Group Problem
- Equilibrium / blackbody spectrum
- Telescopes

Announcements / Reminders

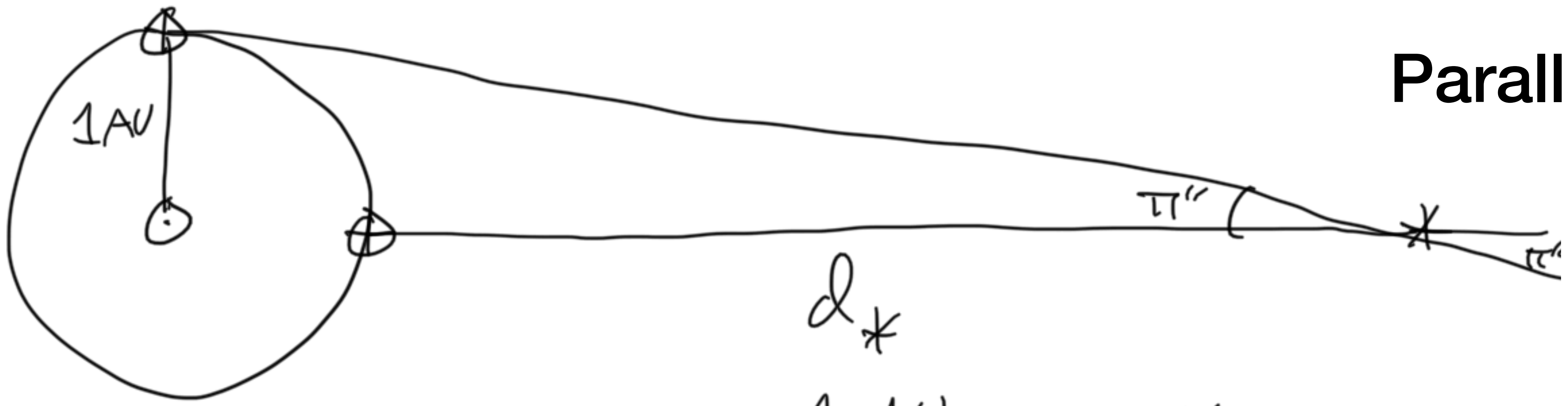
- Read Chapter 5, 6.1, 6.4-7
- HW 3 due September 17th at 11:59pm via Canvas upload

- HEAP talk at 4pm in INSCC auditorium
  - Former Utah REU student (now at NASA Ames) talks about haze in planetary atmospheres

https://utah.zoom.us/j/94983299819
Meeting ID: 949 8329 9819
Passcode: 115S1400E

HW2

Parallax



$$d = \frac{1\ \text{pc}}{(\pi''/\text{arcsec})}$$

$$\tan \pi'' = \frac{1\,\text{AU}}{d_*} \approx \pi''$$

$$\pi'' = \frac{1\ \text{pc}}{d}\ \text{arcsec} = \frac{1\ \text{pc}}{4000\ \text{pc}}\ \text{arcsec} = 2.5 \times 10^{-4}\ \text{arcsec}$$

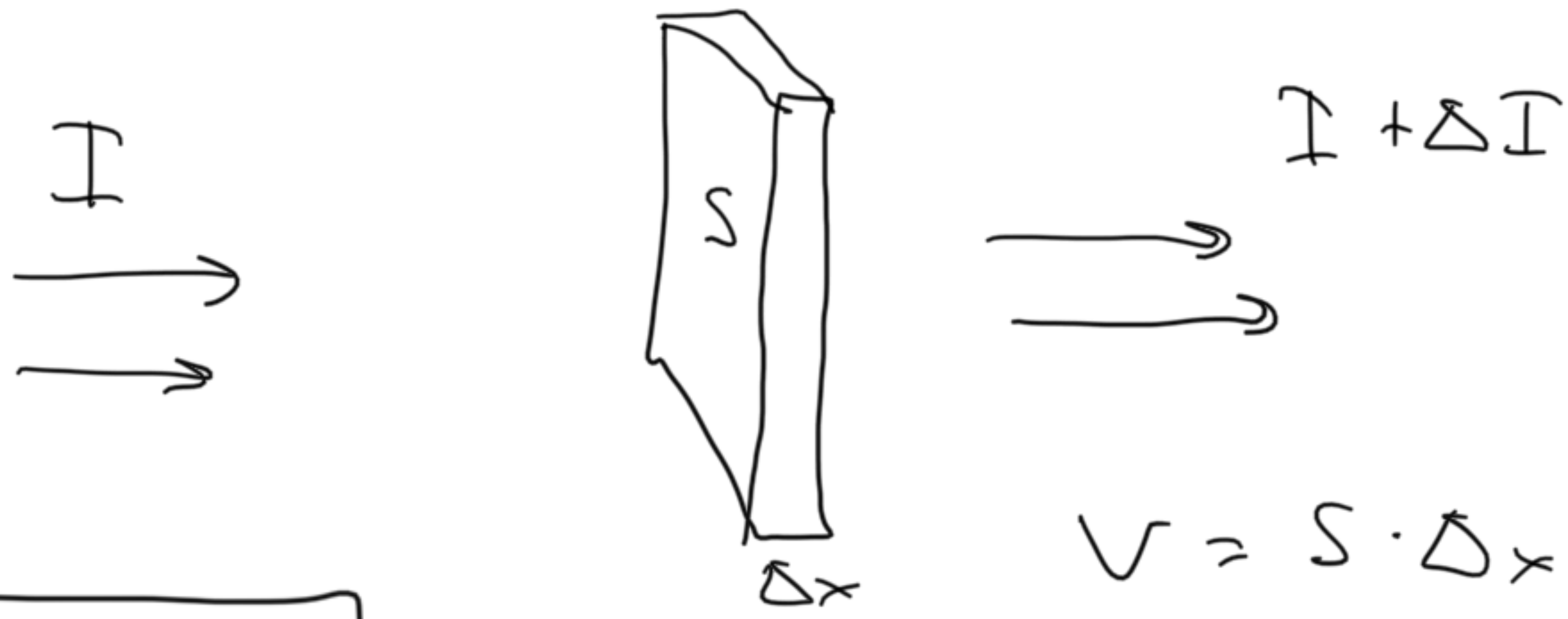
$$d_* = \frac{1\,\text{AU}}{\pi''(\text{rad})} = \frac{206265}{(\pi''/\text{arcsec})}\ \text{AU}$$

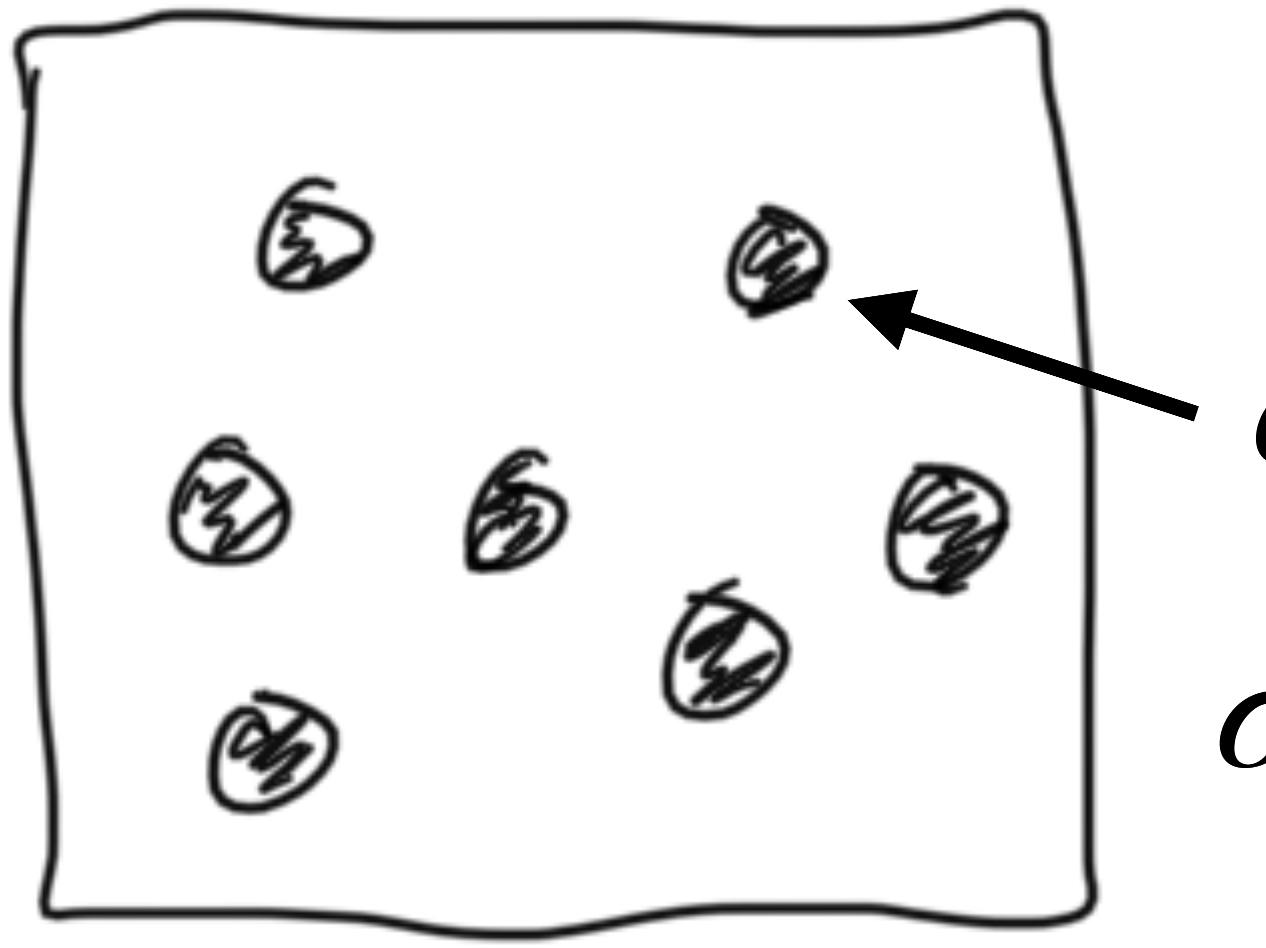
Kepler's 3rd Law

$$P^2 = \frac{4\pi^2}{G(M+m)} a^3 \approx \frac{4\pi^2}{GM} a^3 \text{ if } M \gg m$$

$$\frac{4\pi^2}{GM_\odot} = \frac{4 * 3.14159^2}{6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2} \cdot 1.989 \times 10^{30}\ \text{kg}} \cdot \frac{(1.496 \times 10^{11}\ \text{m/AU})^3}{(3.1 \times 10^7\ \text{s/yr})^2} = \frac{39.48}{1.33 \times 10^{20}} \cdot \frac{3.35 \times 10^{33}}{9.61 \times 10^{14}} \frac{\text{yr}^2}{\text{AU}^3} = 1.0\ \frac{\text{yr}^2}{\text{AU}^3}$$

Radiative Transfer





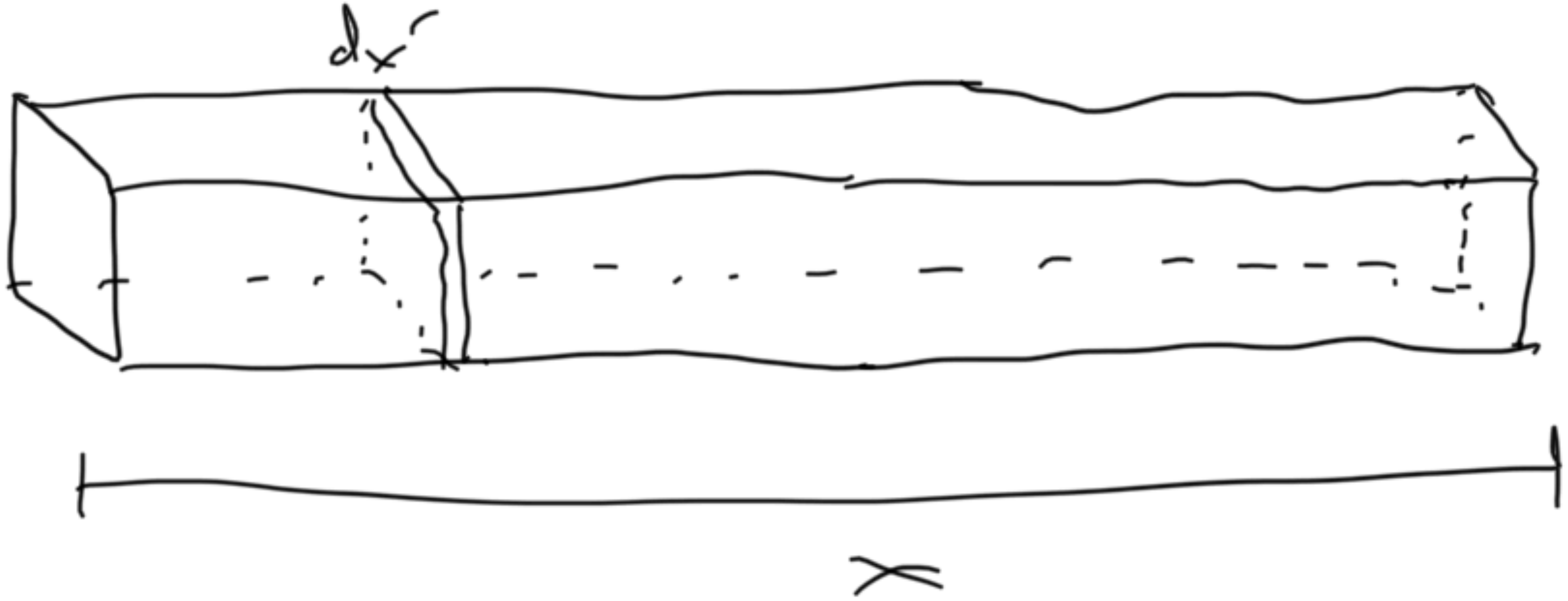
$$\sigma_p = \pi r^2$$

$$\sigma_{\rm tot} = \sigma_p N$$

$$\frac{\Delta I}{I} = -\frac{\sigma_{\rm tot}}{S} = -\frac{N\sigma_p}{S}$$

Column Density

$$N(x) \equiv \int_0^x n(x')\,dx' = nx$$

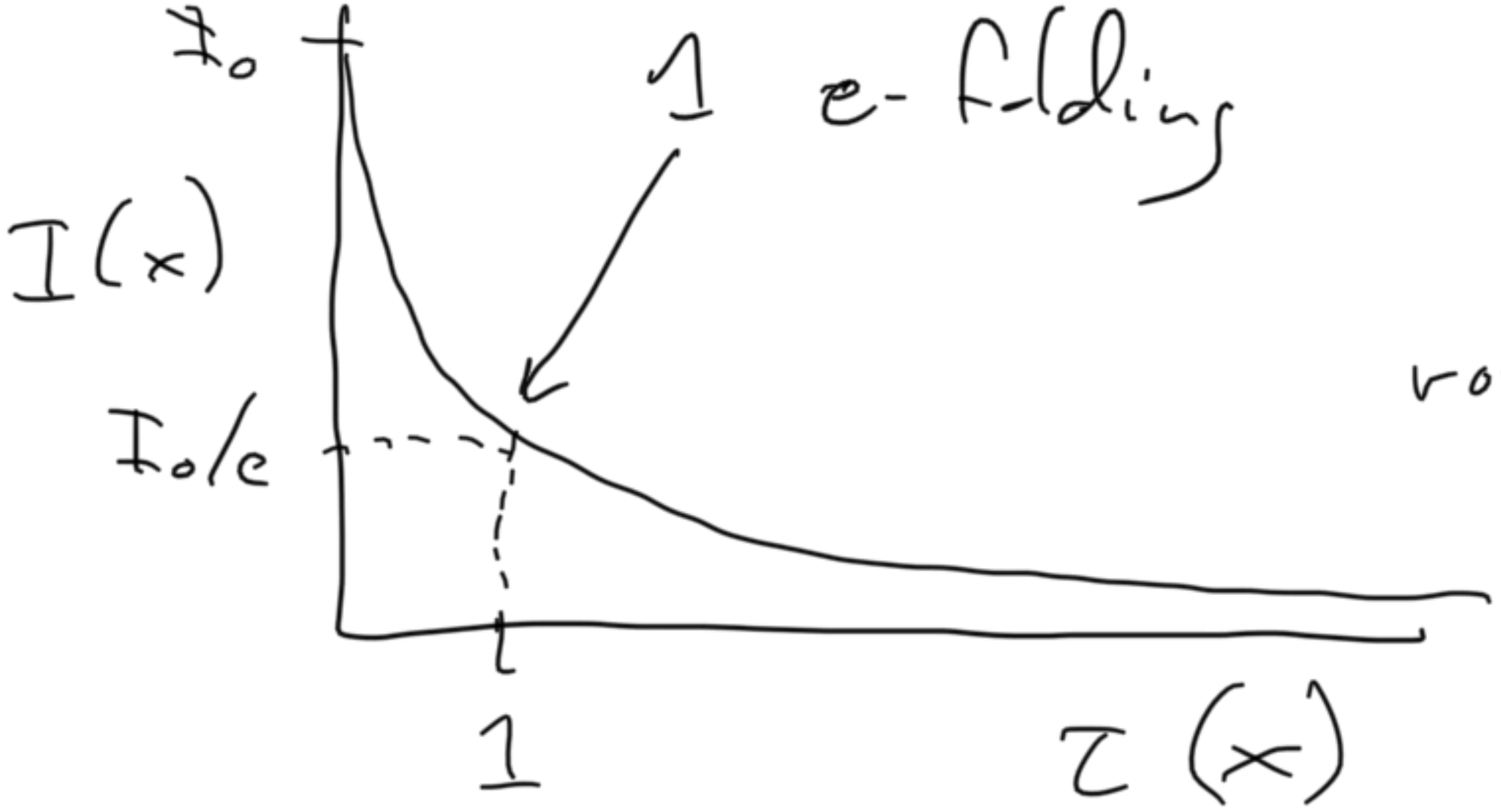


Optical Depth

$$I(x) = I_0 e^{-n\sigma_p x}$$
$$= I_0 e^{-\tau(x)}$$

$$\tau = n\sigma_p x = \sigma_p N(x)$$

- τ: optical depth
- n: # density
- σ_p: cross-section
- $N(x)$: column density



$e \approx 2.72$

roughly $\frac{1}{3}$ of photons left when $\tau = 1$

Mean Free Path

Average distance a photon travels before being absorbed or scattered

$$\lambda_{\mathrm{mfp}} = \langle x \rangle = \frac{1}{n\sigma_p}$$

(Set optical depth = 1, solve for x)

Radiative transfer depends on frequency

$$I_\nu = I_{\nu,0} e^{-\tau_\nu(x)}$$

Group Problems

A gas cloud has a column density of 10^{26} atoms per square meter and is 10 pc ($= 3.26 \times 10^{17}$ m) across on the sky.

What is the number density of atoms in the cloud?
What assumptions do you have to make to arrive at that estimate?
How does this compare to the number density of molecules of air in this room?

If the cross-section of each atom is $\sigma_p = 10^{-28}$ m^2, what fraction of light from a star on the far side of the cloud makes it through to us?

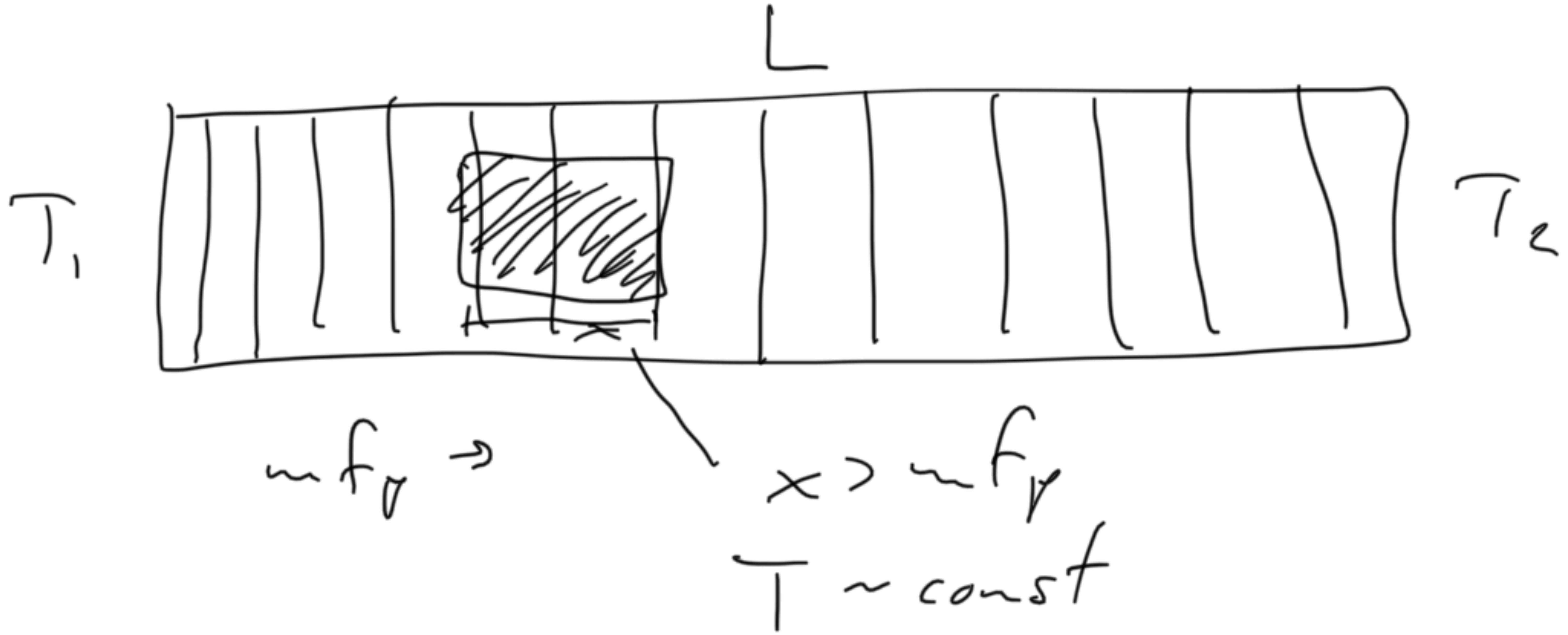
Local Thermodynamic Equilibrium (LTE)

$x \gg \lambda_{\rm mfp}$

$\Delta T \ll T$

Follows avg. statistics
Max.-Bolt. dist.

No time
dependence



LTE —> Blackbody/Planck function

1. Photons & massive particles have a high number density
2. System is optically thick

$$I_\nu(T)d\nu = \frac{2h\nu^3}{c^2}\frac{d\nu}{e^{h\nu/kT} - 1}$$

$$I_\nu d\nu(\nu \to \nu + d\nu) =$$

$$I_\lambda d\lambda(\lambda \to \lambda + d\lambda)$$

$$I_\lambda(T)d\lambda = \frac{2hc^2}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

$$\nu = \frac{c}{\lambda} \;\; \longrightarrow \;\; d\nu = -\frac{c}{\lambda^2}d\lambda$$

Integrate BB over all freq. & angles

<u>Flux</u>
(energy per area per time)

$$F = \frac{2\pi^5}{15}\frac{k^4}{c^2 h^3}T^4 = \sigma_{\mathrm{SB}} T^4$$

Stephan-Boltzmann constant

$$\sigma_{\mathrm{SB}} = 5.67 \times 10^{-8}\ \mathrm{J\ s^{-1}\ m^{-2}\ K^{-4}}$$

What is the total energy emitted?

Assume star is spherical

$$L = F \cdot A_* = F \cdot 4\pi R_*^2$$

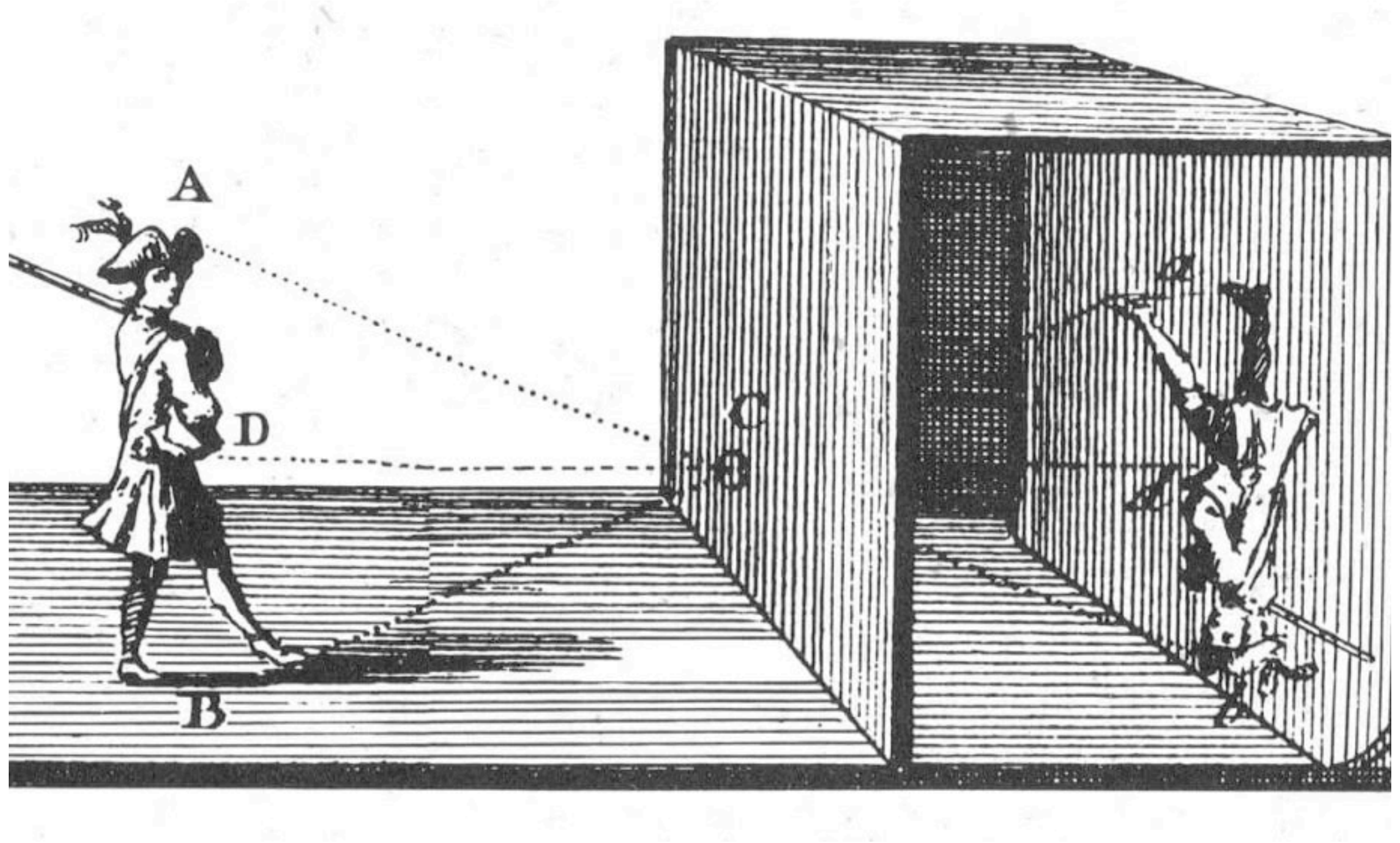
$$\boxed{L = 4\pi R_*^2 \sigma_{\rm SB} T^4}$$

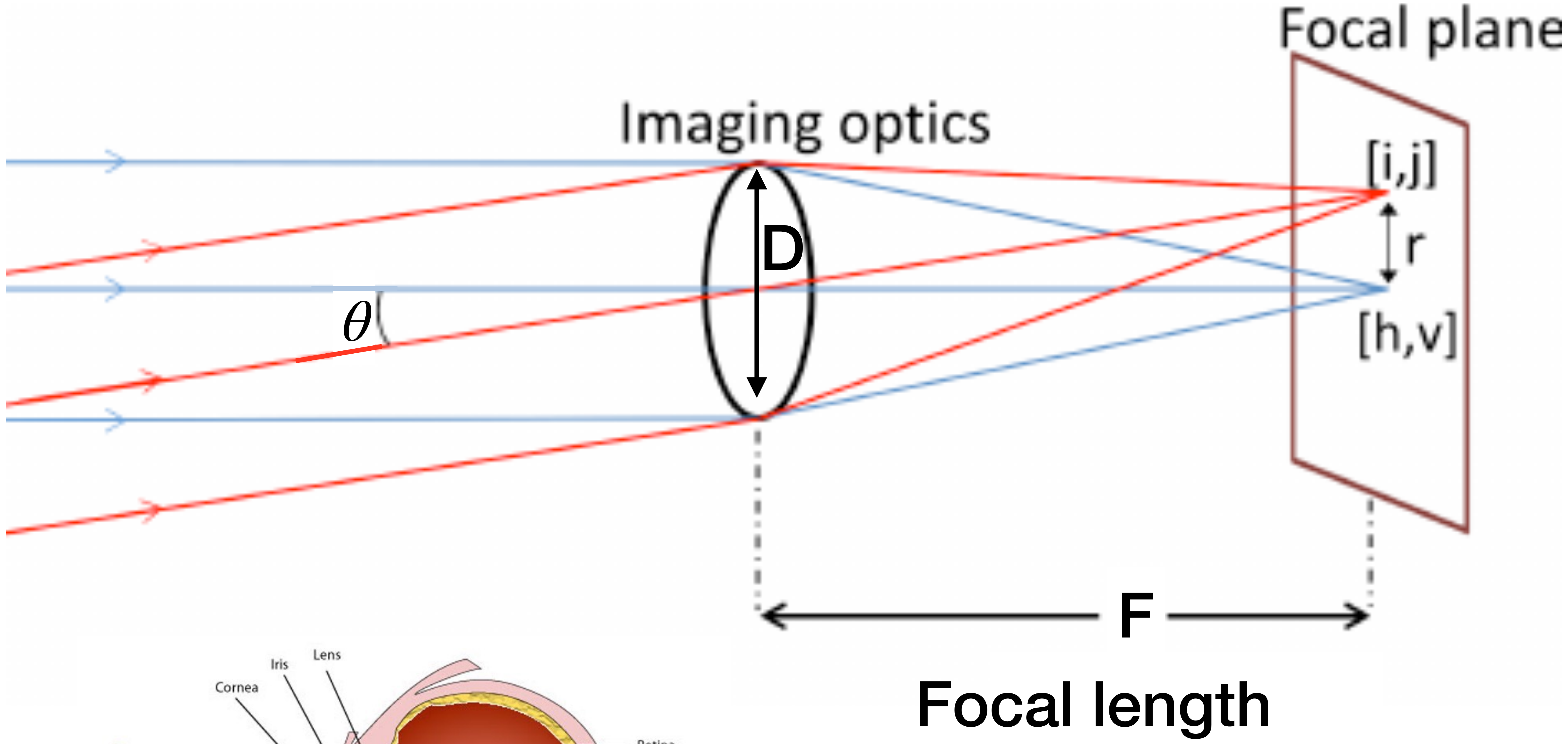
Compare to the Sun:

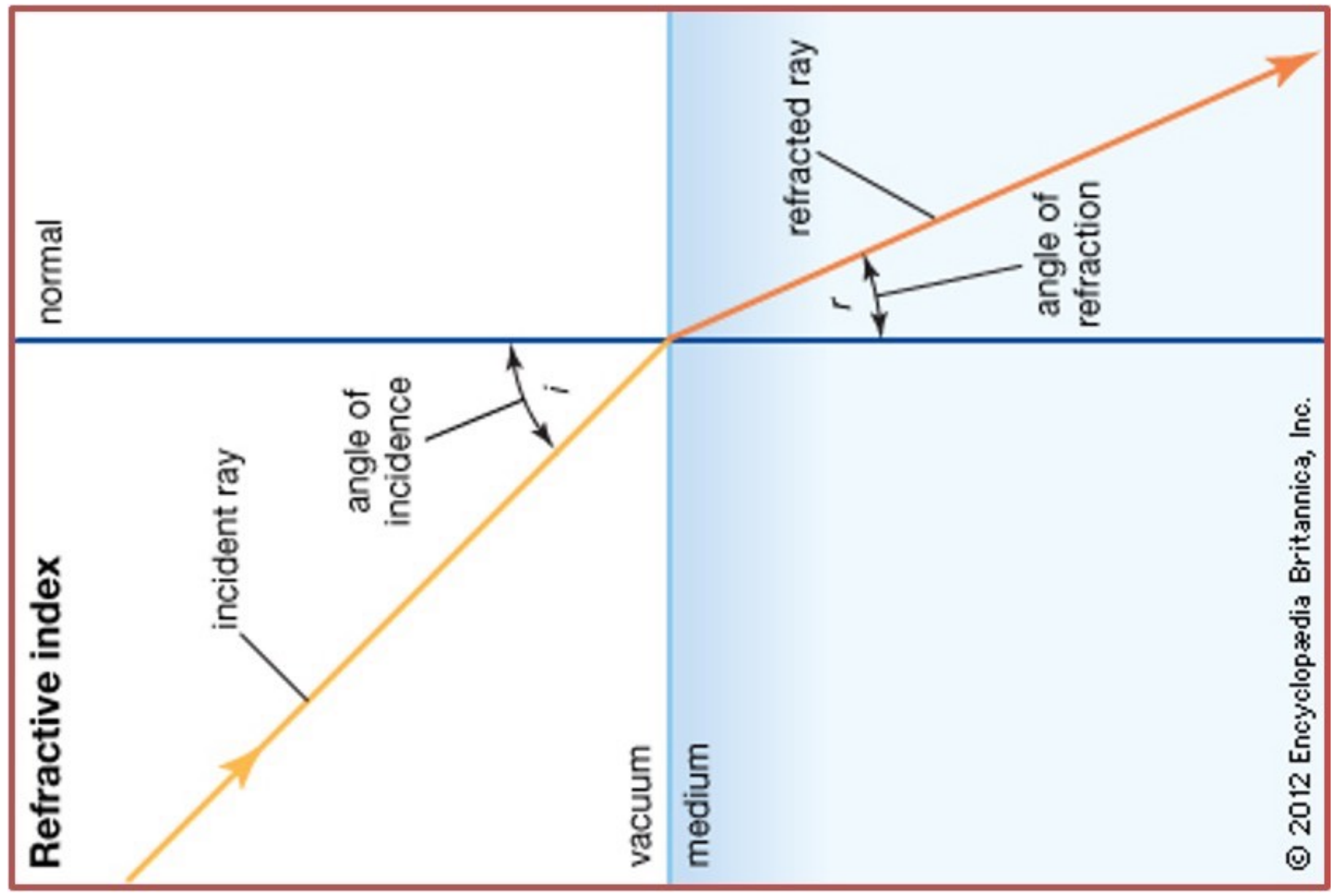
$$\left.\begin{aligned} R_\odot &= 6.96 \times 10^8 \text{ m} \\ T_\odot &= 5780 \text{ K} \\ L_\odot &= 3.8 \times 10^{26} \text{ W} \end{aligned}\right\} \quad L = 1\ L_\odot \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

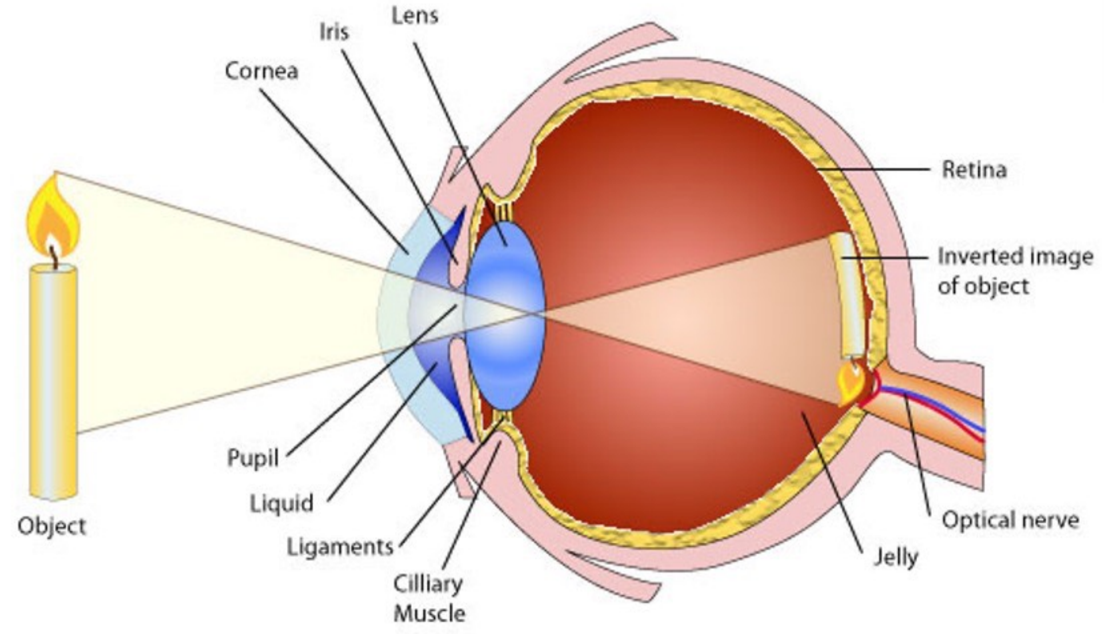
Collecting Light

Telescopes collect (often by focusing) light







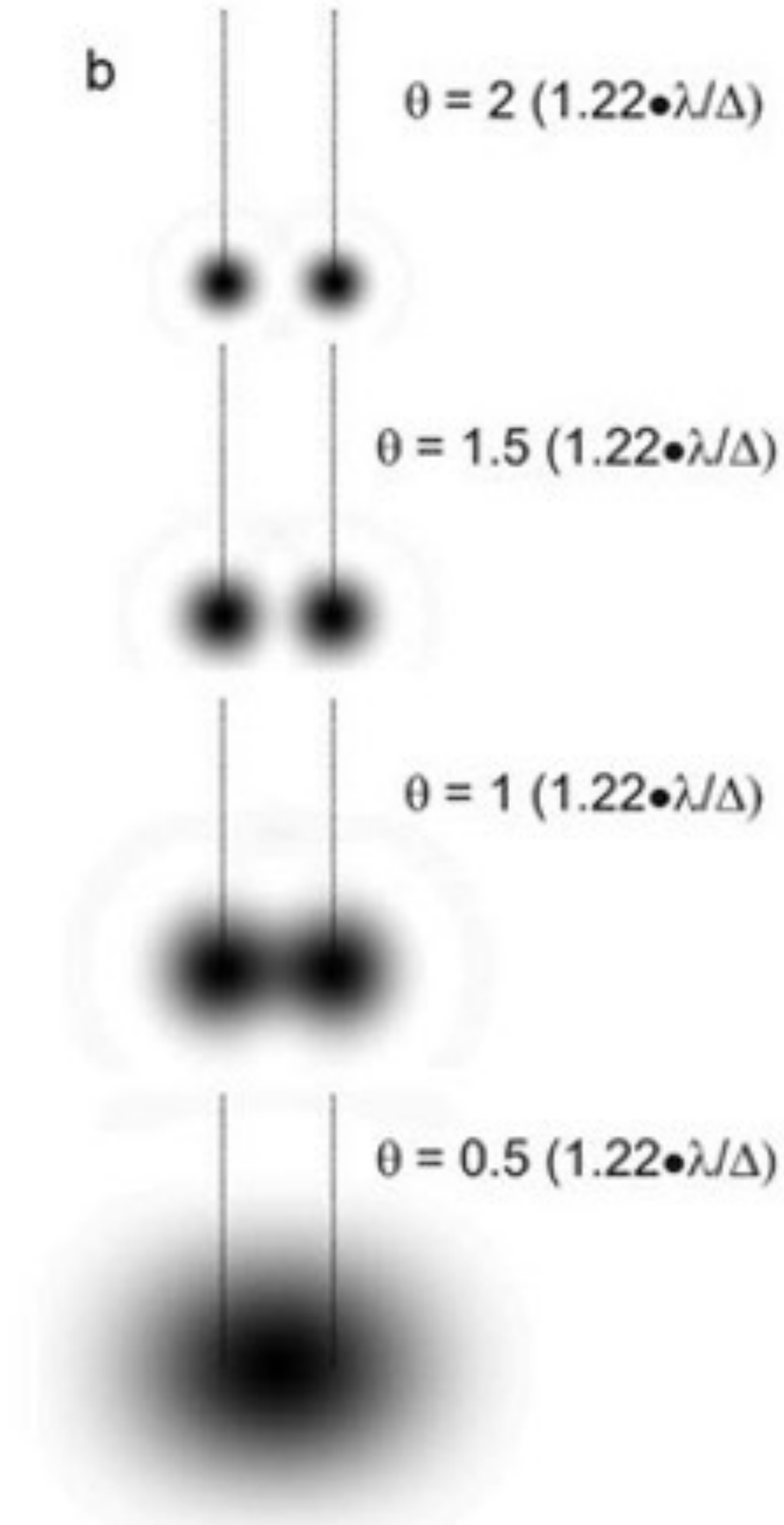


Our eyes are telescopes!

Plate scale = $\frac{\theta}{r}$ (arcsec/mm)

$$\theta_{\min} = 1.22\frac{\lambda}{D}$$

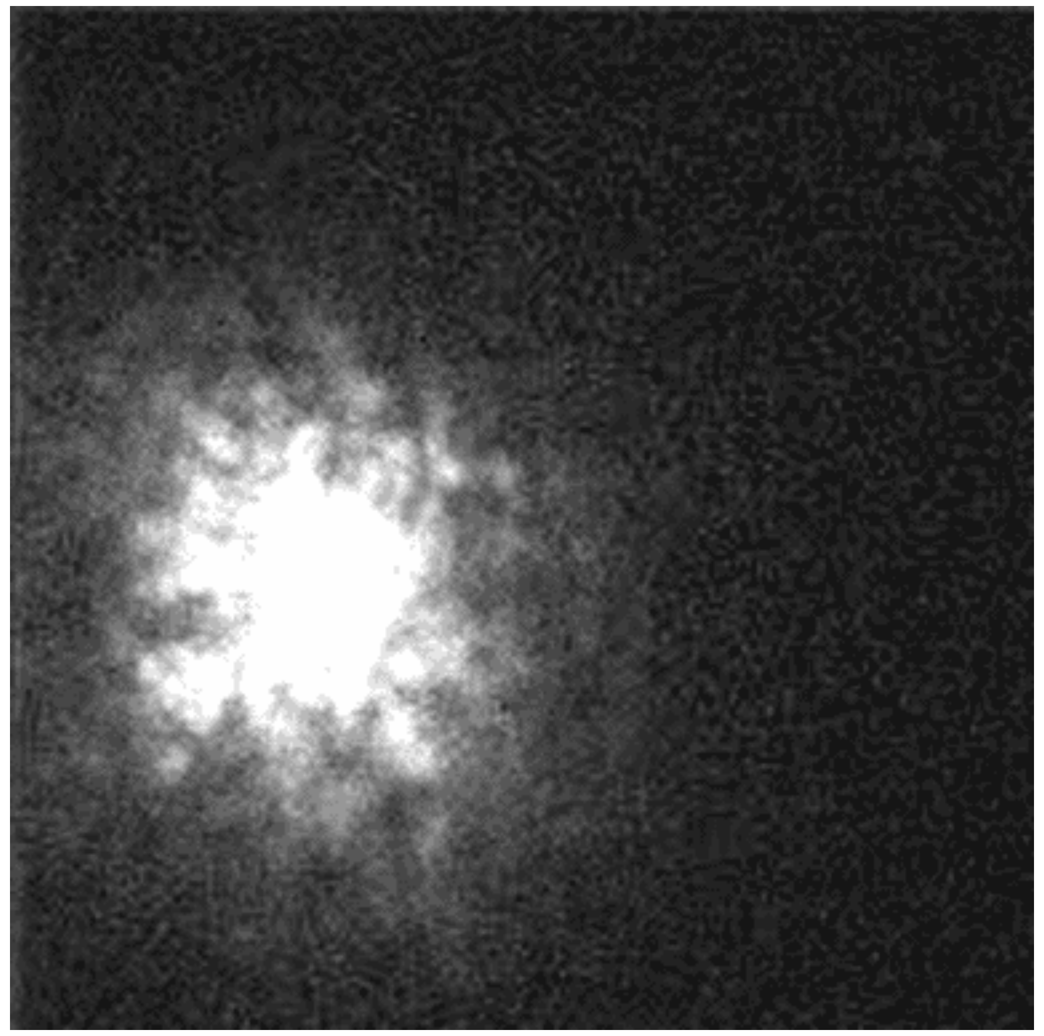
Image Resolution



$$\theta_{\min} = 1.22\frac{\lambda}{D}$$

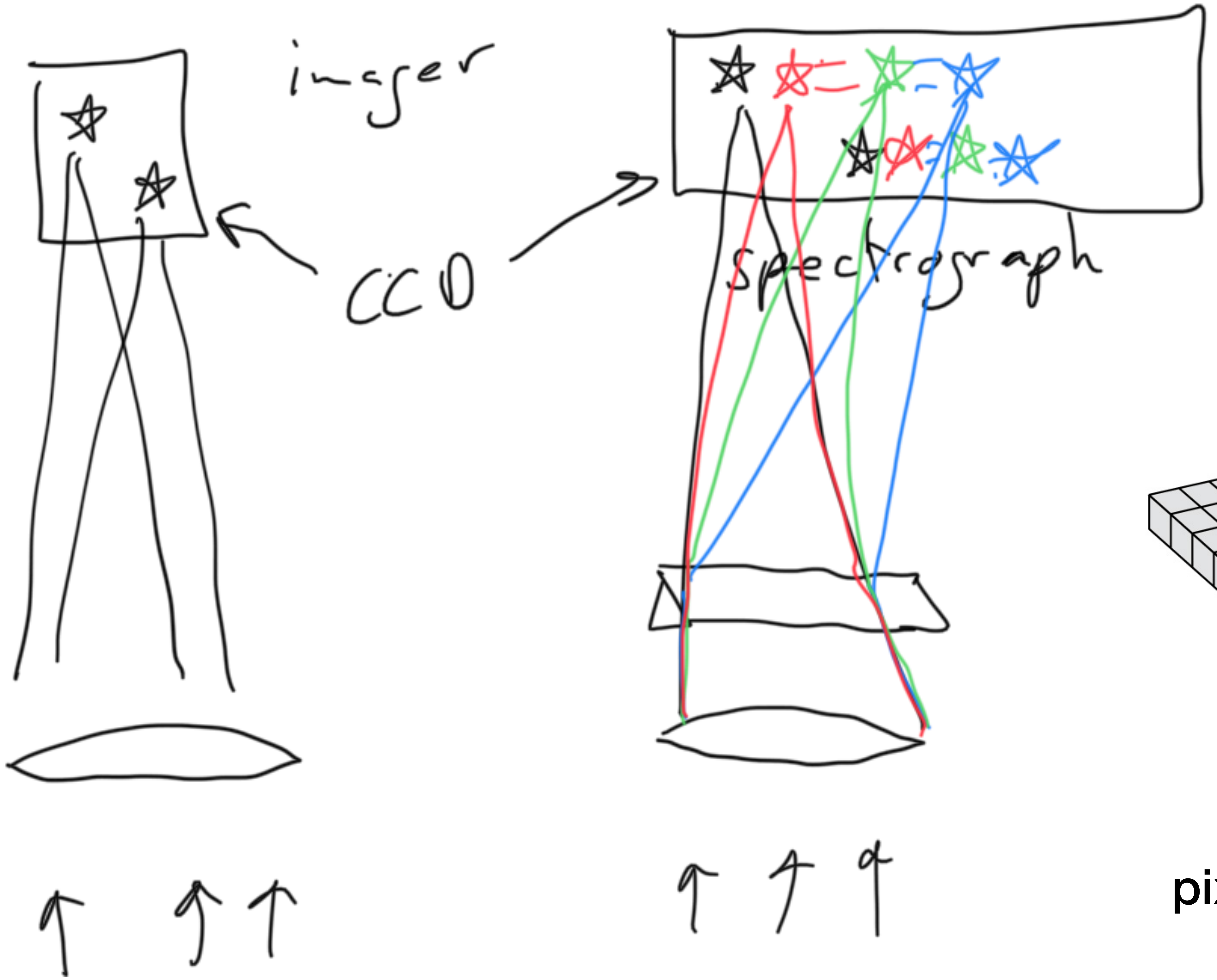
In ideal case, resolution determined by size of mirror

Often, mirror imperfections (misalignments, roughness) or atmospheric effects make the actual resolution worse

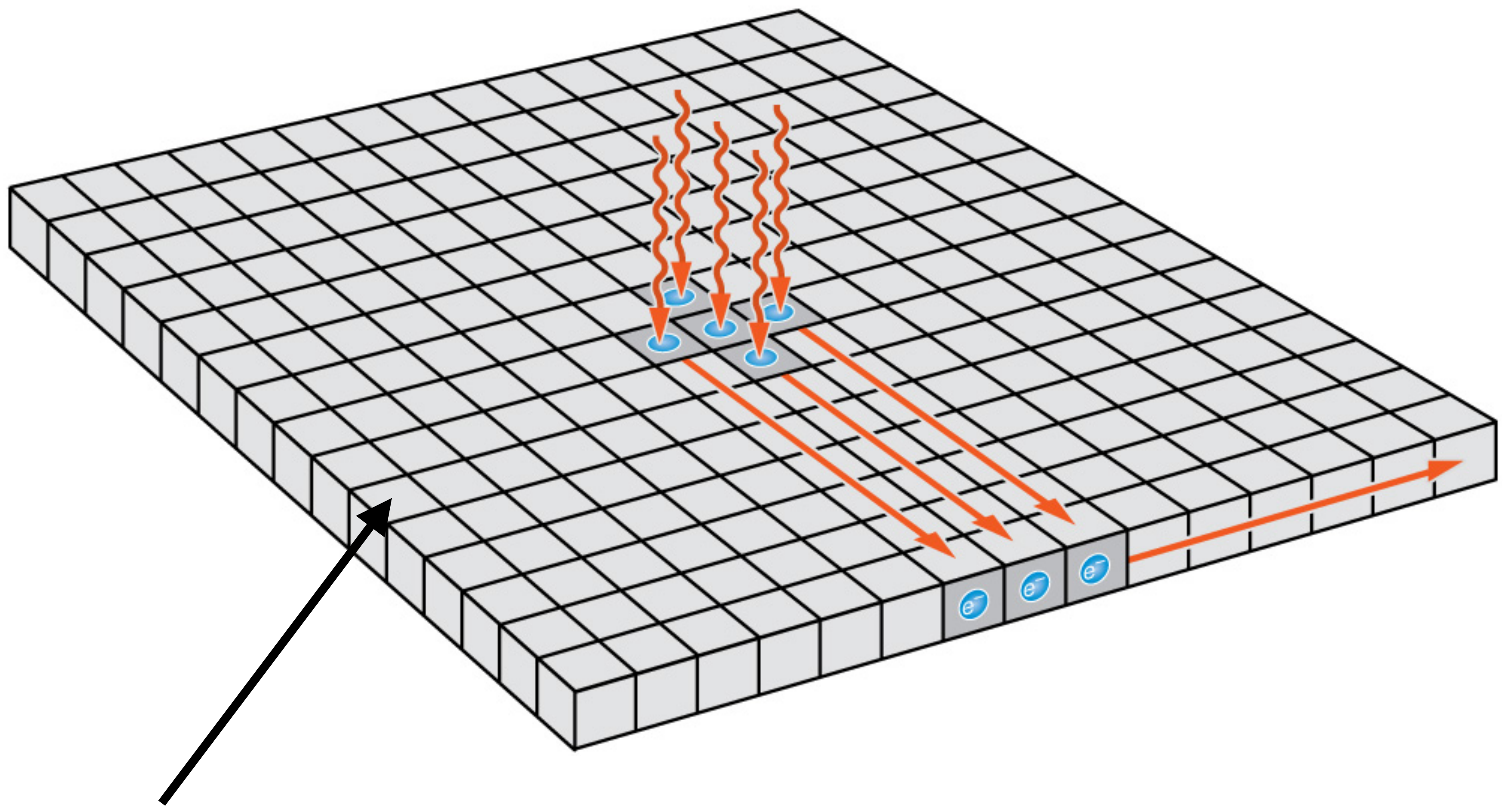


Why stars twinkle

Imaging versus Spectroscopy



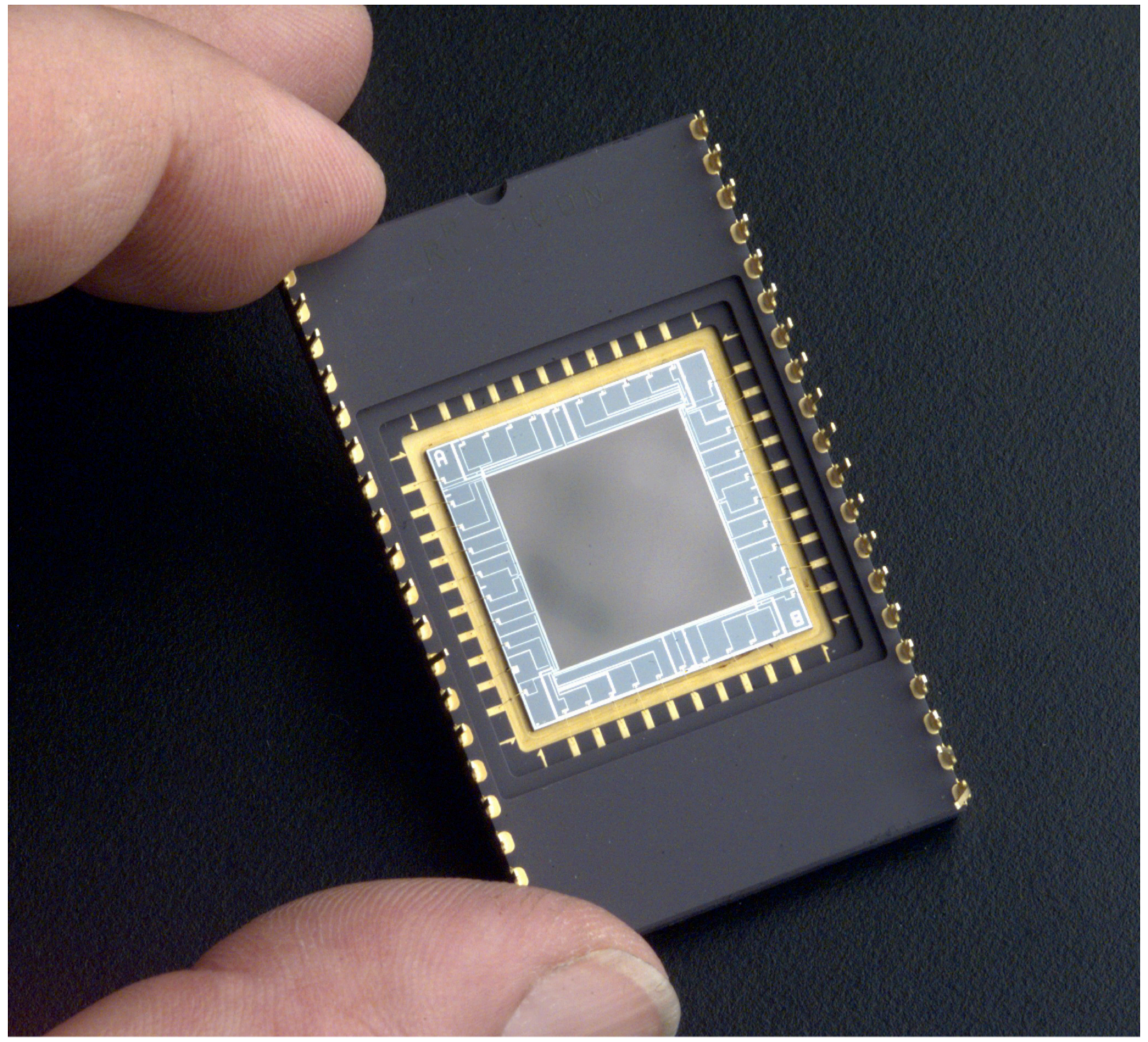
CCD
Charged Coupled Device



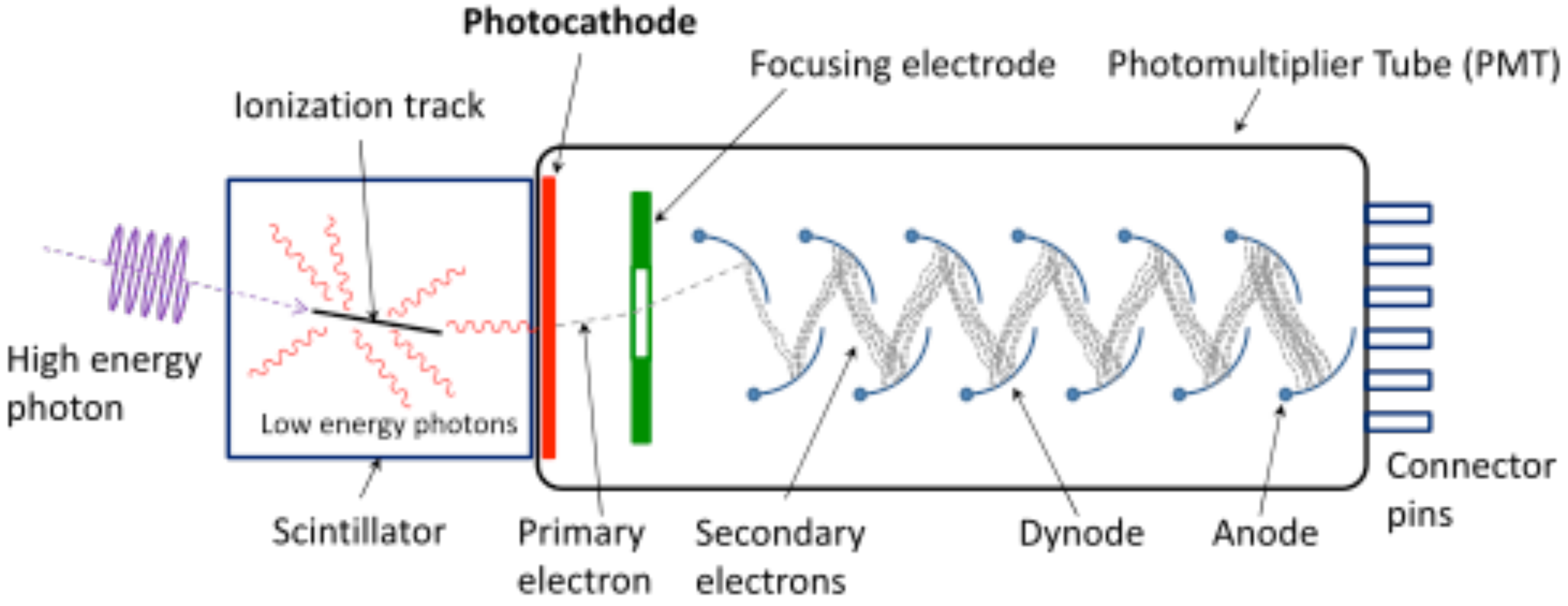
pixel (<u>pic</u>ture <u>el</u>ement)

Detectors

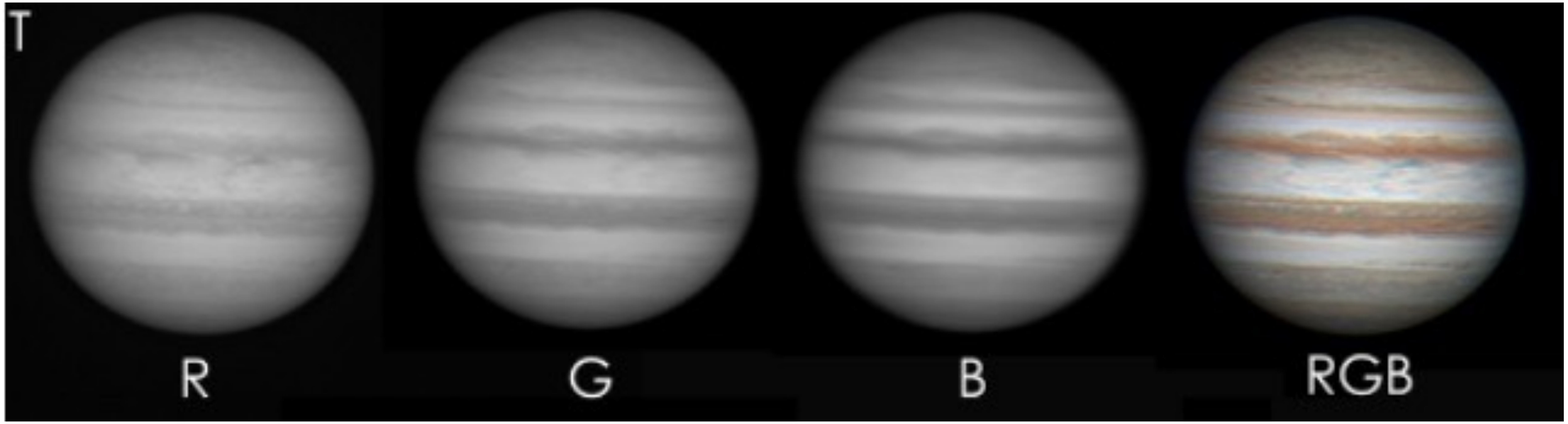
CCD

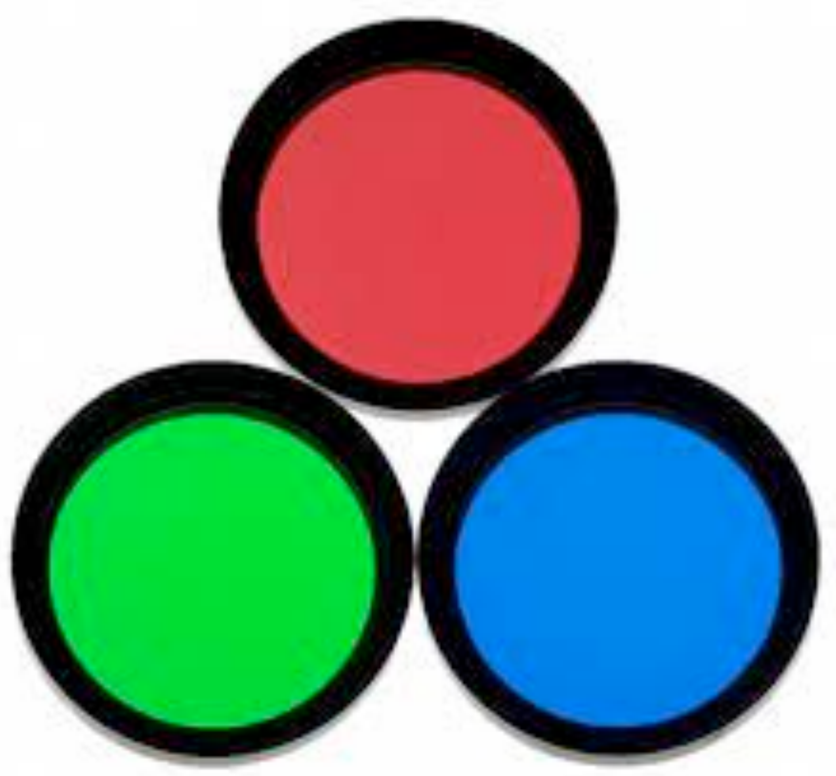


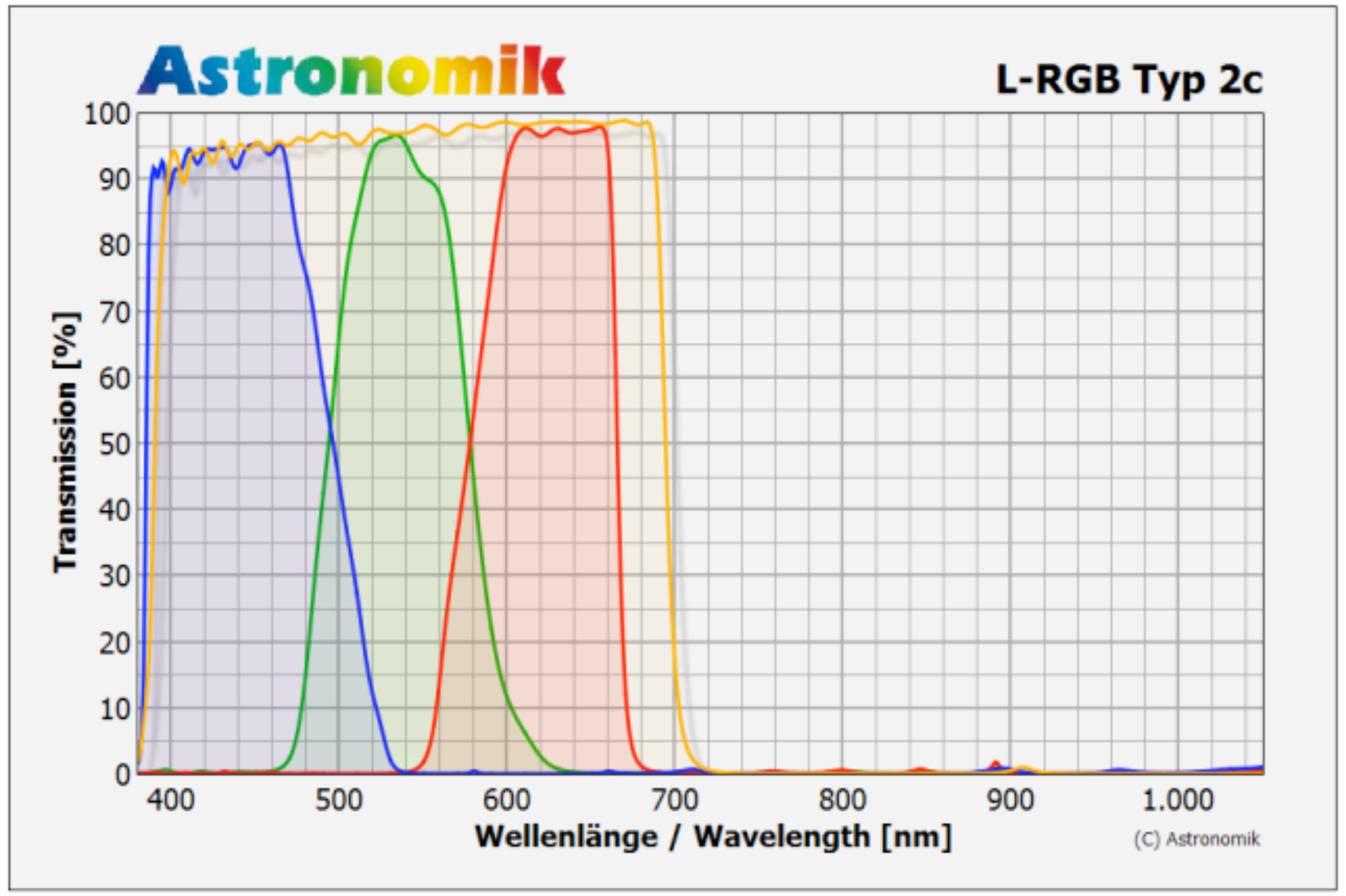
Photomultiplier tube

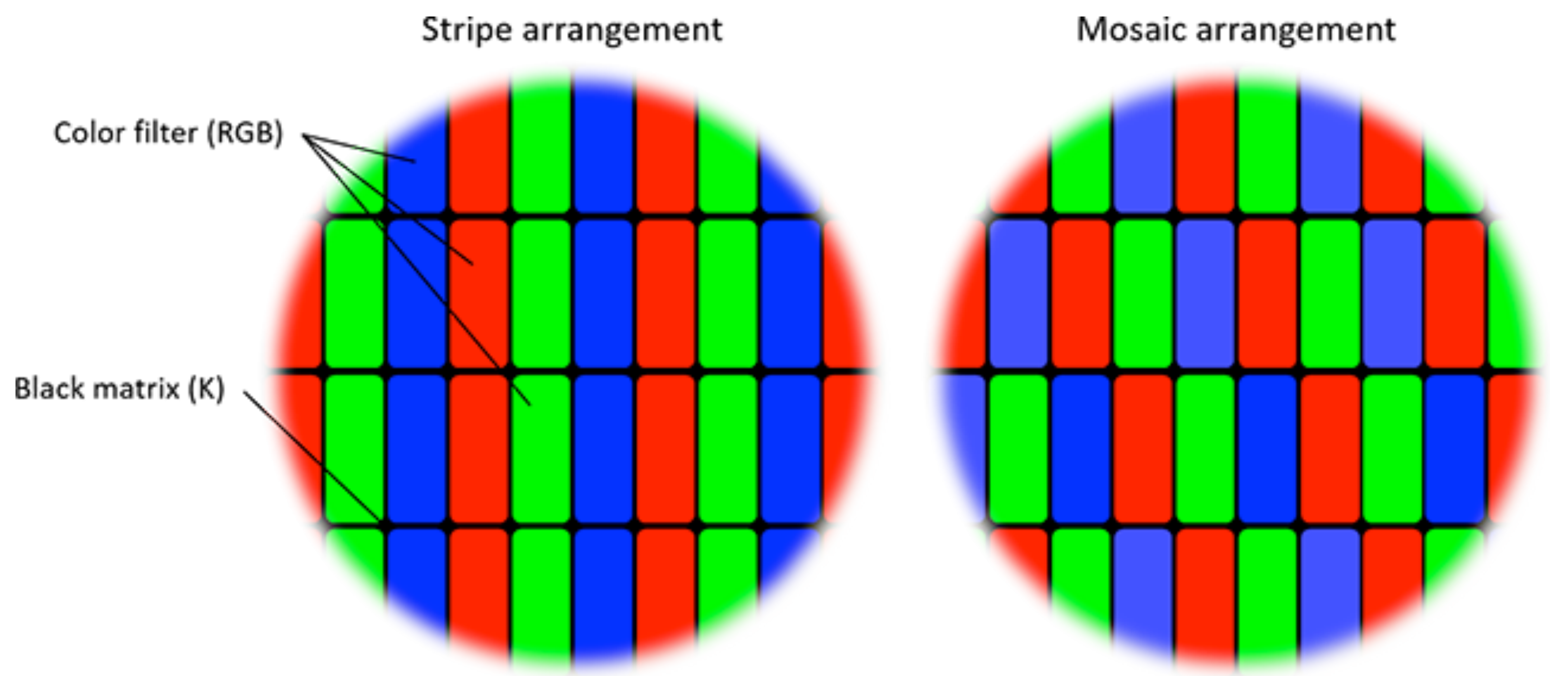


“Color” Imaging



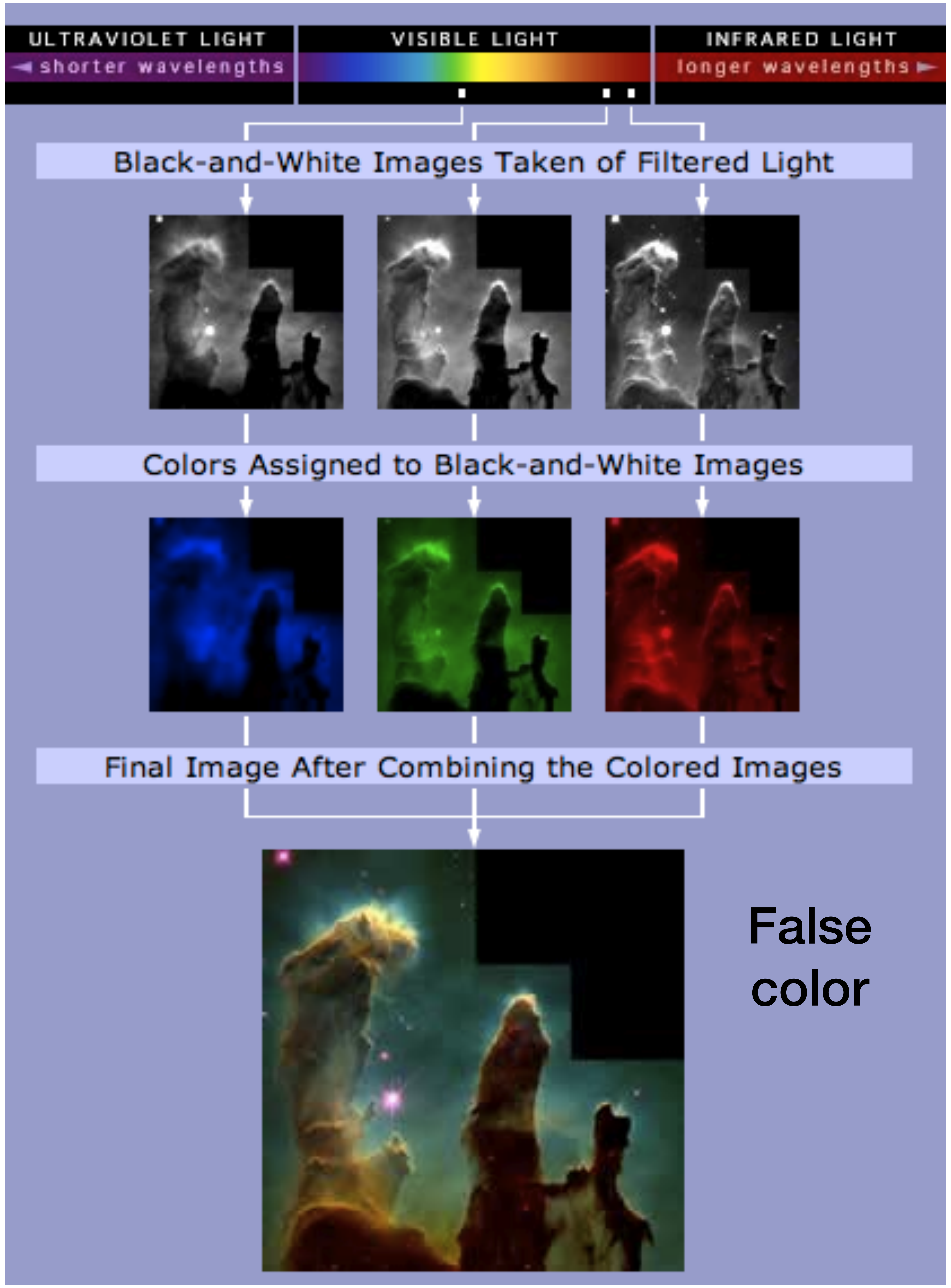






Phone Camera

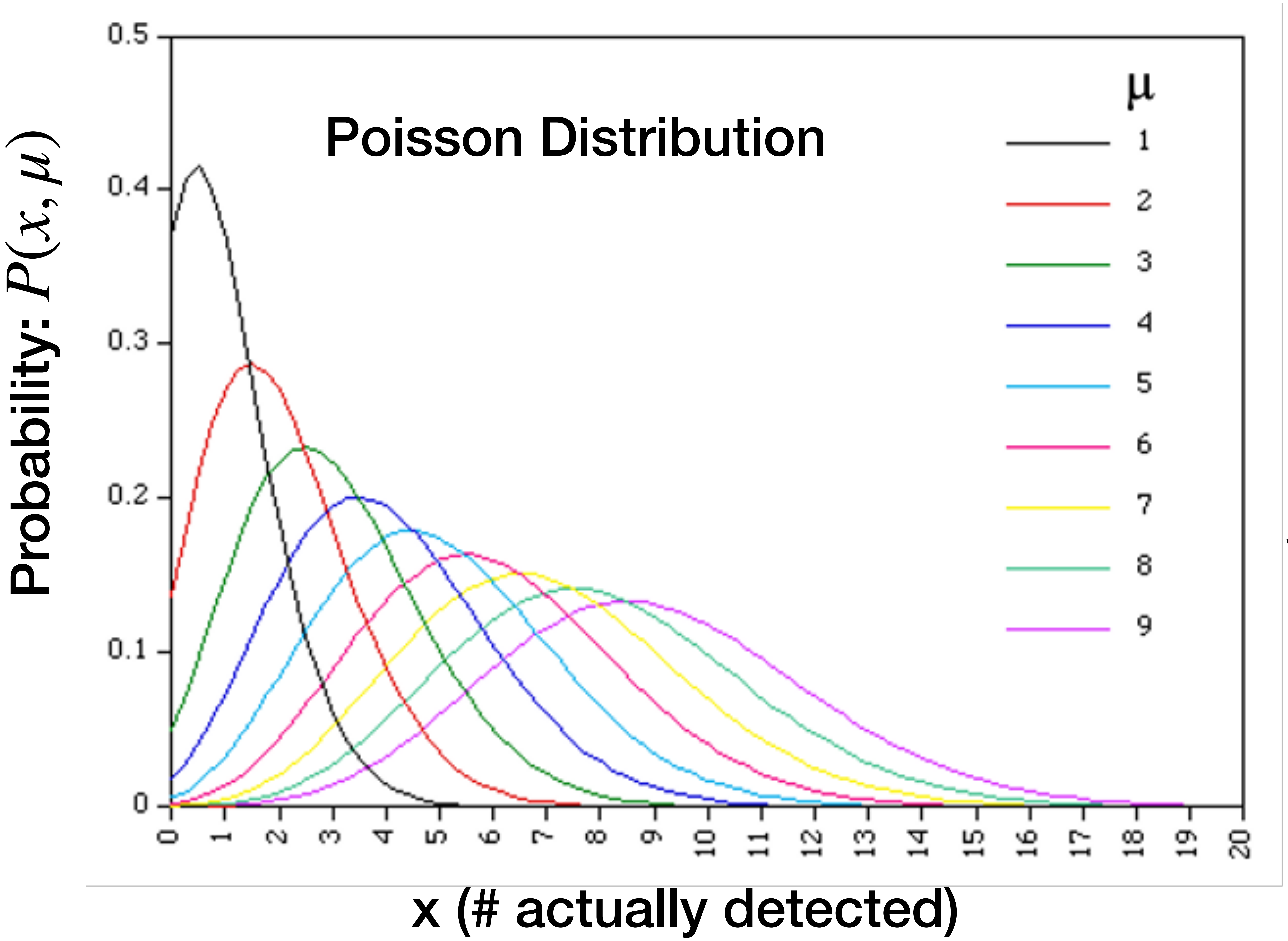
True color



False color

Making Measurements

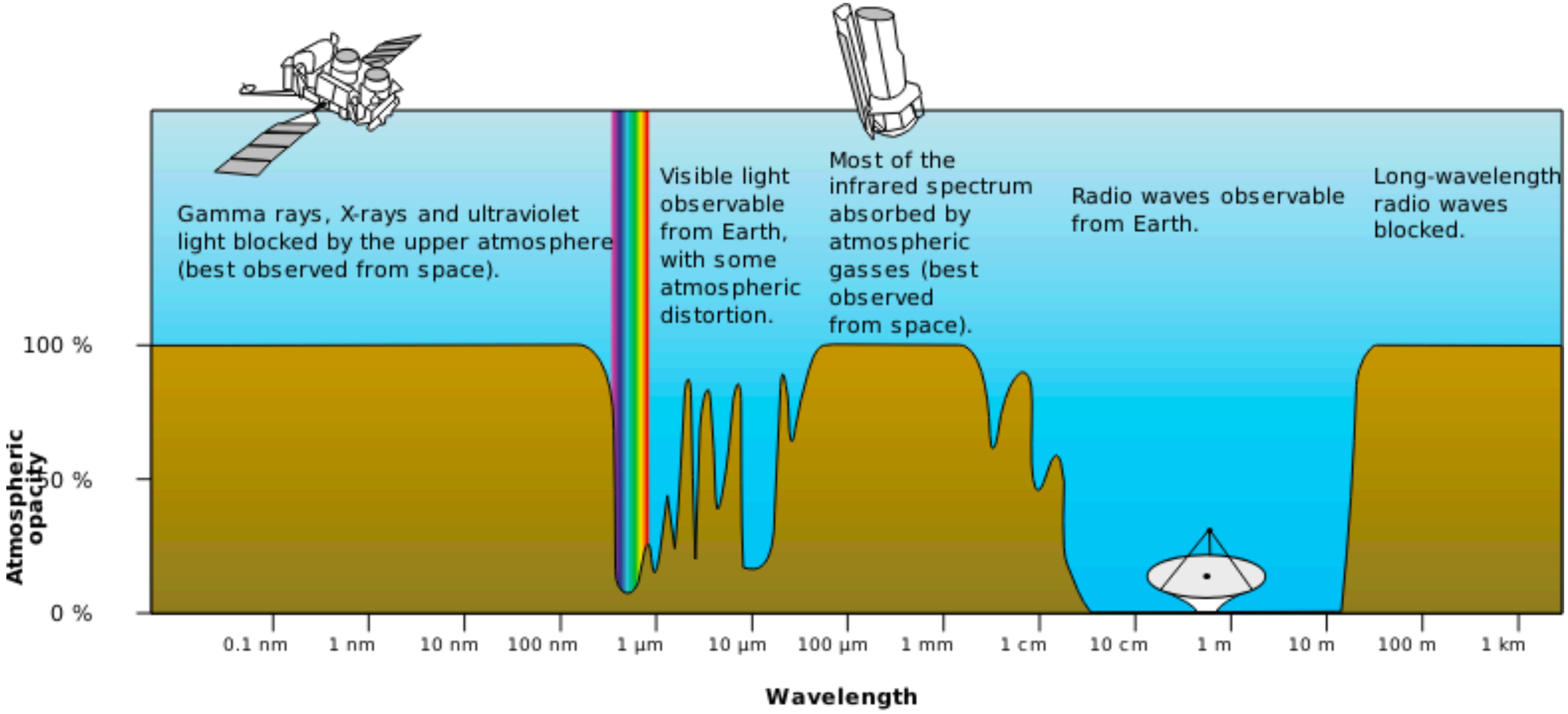
Photons arrive randomly — # detected not necessarily # “should” detect



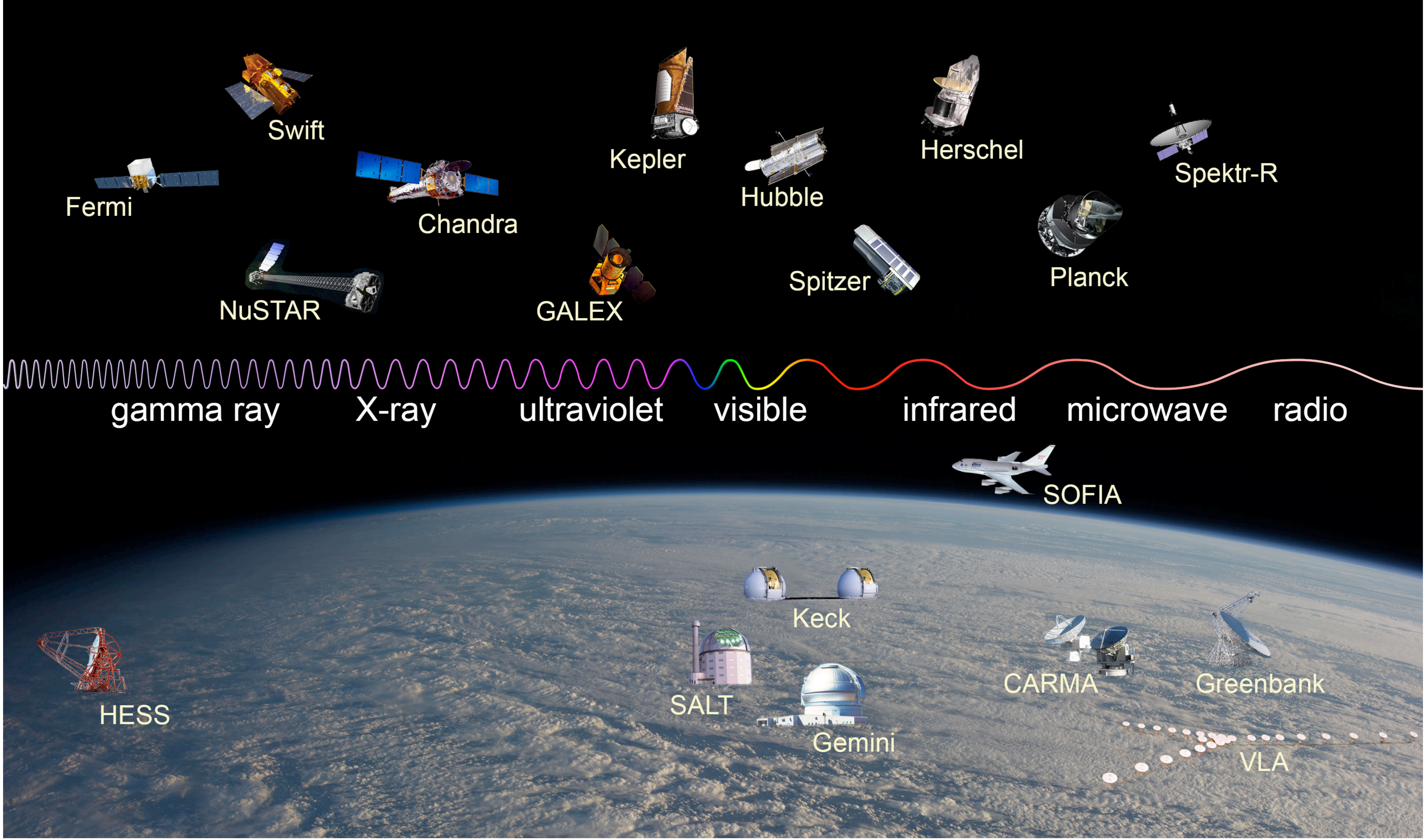
μ —> # “should” detect

$$P(x, \mu) = \frac{\mu^x}{x!} e^{-\mu}$$

Width of the distribution, which gives the uncertainty (or error) of the measurement, is $\sigma = \sqrt{\mu}$

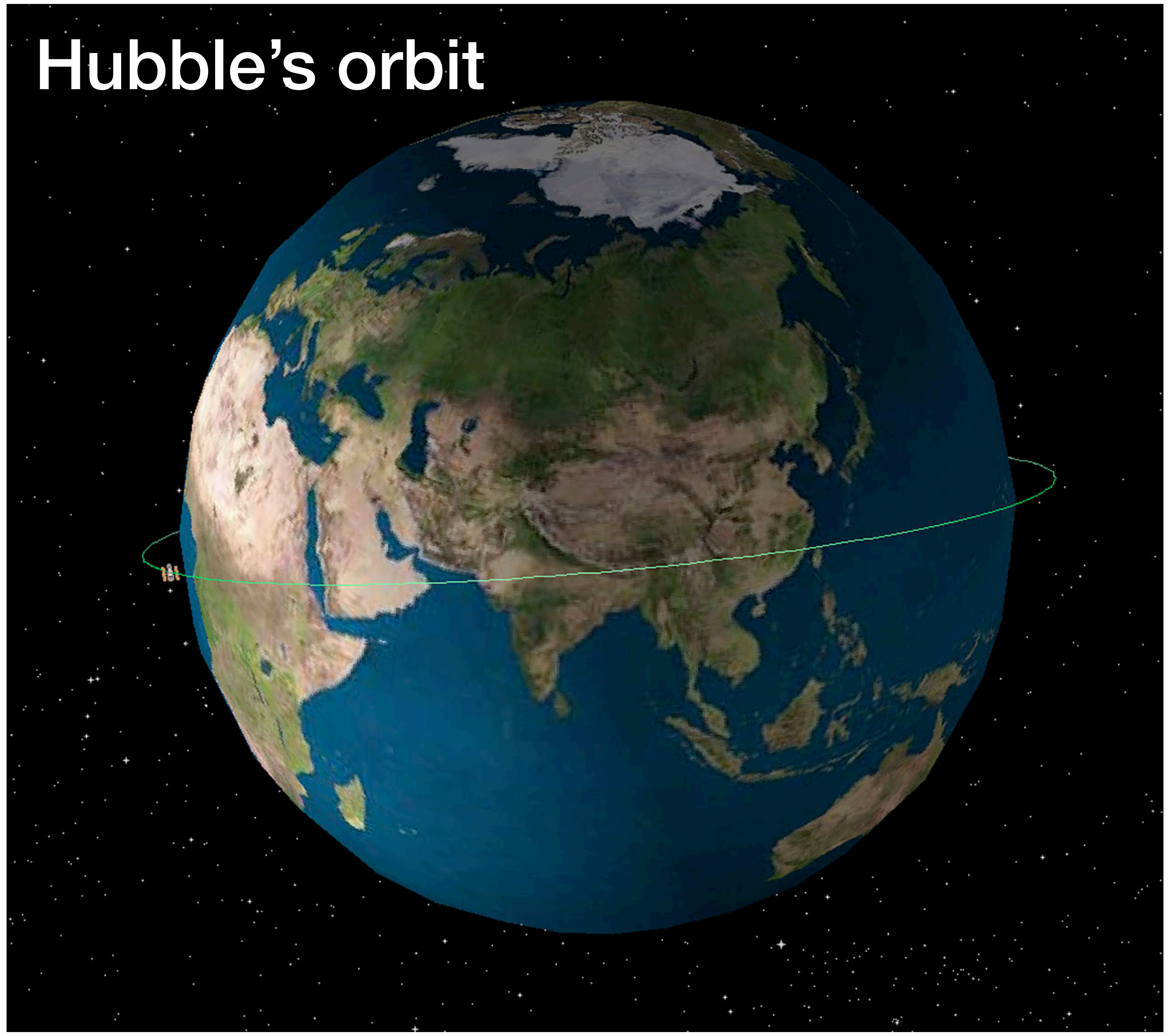

Gamma rays, X-rays and ultraviolet light blocked by the upper atmosphere (best observed from space).
Visible light observable from Earth, with some atmospheric distortion.
Most of the infrared spectrum absorbed by atmospheric gasses (best observed from space).
Radio waves observable from Earth.
Long-wavelength radio waves blocked.
100 %
50 %
0 %
Atmospheric opacity
0.1 nm
1 nm
10 nm
100 nm
1 µm
10 µm
100 µm
1 mm
1 cm
10 cm
1 m
10 m
100 m
1 km
Wavelength

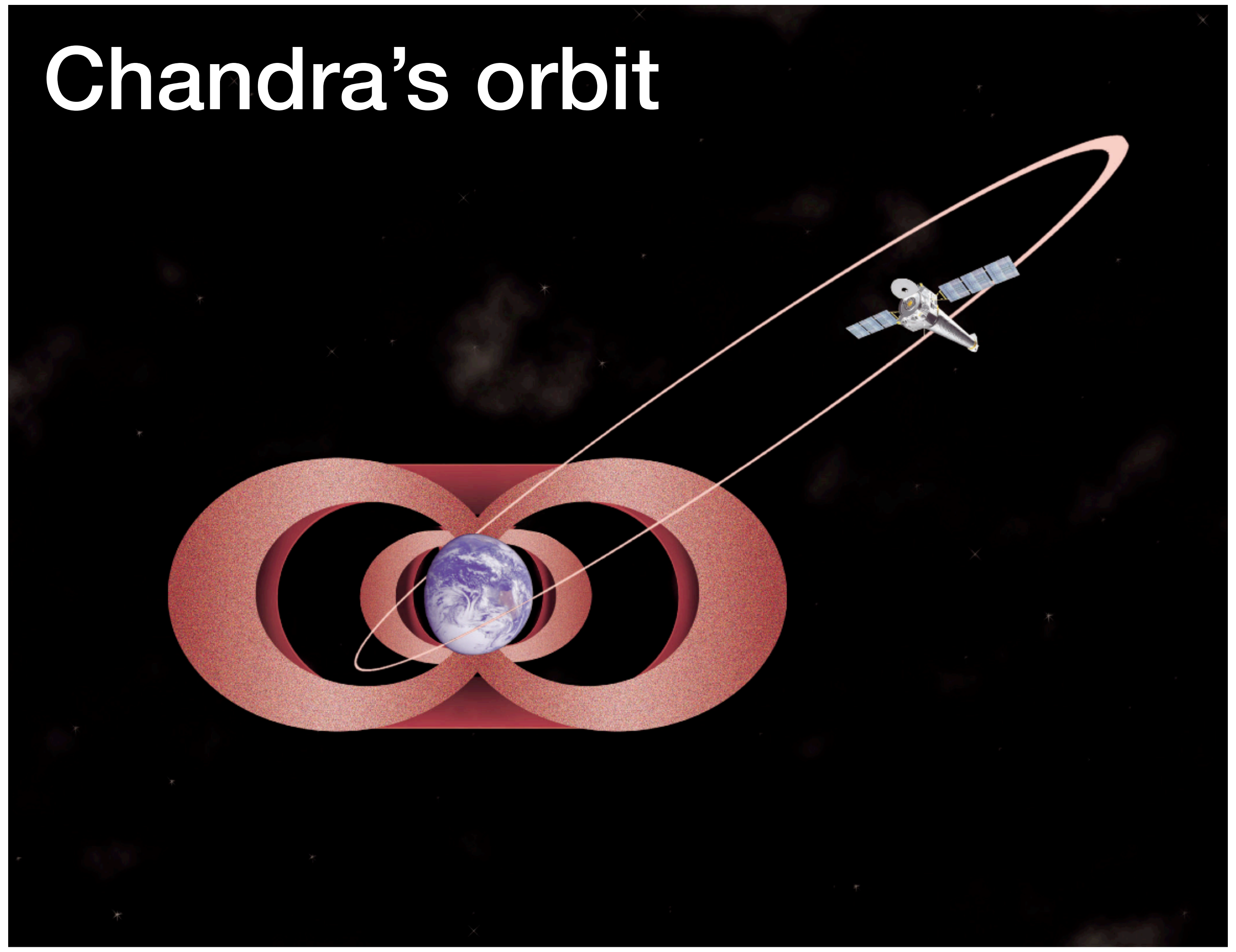

Transparency (%)
100
50
0
Atmosphere is opaque
Atmosphere is opaque
Radio window
Atmosphere is opaque
10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^{1} 10^{2} 10^{3} (m)
0.001 nm
0.01 nm
0.1 nm
1 nm
10 nm
100 nm
1 µm
10 µm
100 µm
1 mm
1 cm
10 cm
1 m
10 m
100 m
1 km
Wavelength


Swift
Fermi
Chandra
NuSTAR
Kepler
GALEX
Hubble
Spitzer
Herschel
Planck
Spektr-R
gamma ray
X-ray
ultraviolet
visible
infrared
microwave
radio
SOFIA
Keck
HESS
SALT
Gemini
CARMA
Greenbank
VLA

3 Misconceptions about Telescopes in Space

- From space, objects can be observed continuously, even during the day
- The sky is much darker in space than on the Earth
- Observations from space are not affected by weather





JWST's orbit