ASTR 3070 Week 07a (Solar System & Exoplanets)

$$F = \frac{L}{A} = \frac{L}{4\pi d^2} \quad ; \quad \text{flux}$$

$$L_{BB} = A\,\sigma_{SB} T^4 = 4\pi R^2 \sigma_{SB} T^4 \quad ; \quad \text{BB Luminos.}$$

Sun produces $L_\odot$ energy / time

Maximum energy a planet can absorb/reflect is $F(d) \cdot$ cross-section (σ_p)

$$\frac{L_\odot}{4\pi d_p^2} \pi R_p^2 = \frac{1}{4} L_\odot \left(\frac{R_p}{d_p}\right)^2$$

Define albedo as fraction of light reflected: $A = \frac{E_{reflected}}{E_{incident}}$

Rate of energy absorption by planet is

$$W_p = F(d) \cdot \sigma_p (1-A)$$

$$= \frac{1}{4} L_\odot \left(\frac{R_p}{d}\right)^2 (1-A)$$

$$4 \quad (d)$$

If planet emits absorbed radiation like a BB, $W_p = L_p = 4\pi R_p^2 \sigma_{SB} T_p^4$

$$= \frac{1}{4} L_\odot \left(\frac{R_p}{d_p}\right)^2 (1-A)$$

$$\boxed{T_p = \left(\frac{R_\odot}{d}\right)^{1/2} \left(\frac{1-A}{4}\right)^{1/4} T_\odot}$$

Earth: $T_p \approx 250\,K = -23°C = \underline{-9°F}$

★ Why isn't Earth this cold???

★ Why don't astronauts on the Moon freeze on the surface?

To do calculation, need the distance

- use radar: send pulse & wait for it to reflect & return
- use Kepler's 3rd law

$$\left(P/_{yr}\right)^2 = \left(a/_{AU}\right)^3$$

Other fundamental properties: mass, dens.
(infer what they're made of)

$$p^2 = \frac{4\pi^2}{G(M+m)} a^3$$

planet ↗ ↖ satellite $m \ll M$

$$M_p \approx \frac{4\pi^2 a^3}{G p^2}$$

What about radius? Use angular size

$R_p \approx \theta d$; d is distance from us, but can figure out

The density is then $\rho = \frac{M}{V}$

$$\rho = \frac{M_p}{\frac{4}{3}\pi R_p^3}$$

Rocky: $\rho \approx 3000-5500 \text{ kg m}^{-3}$

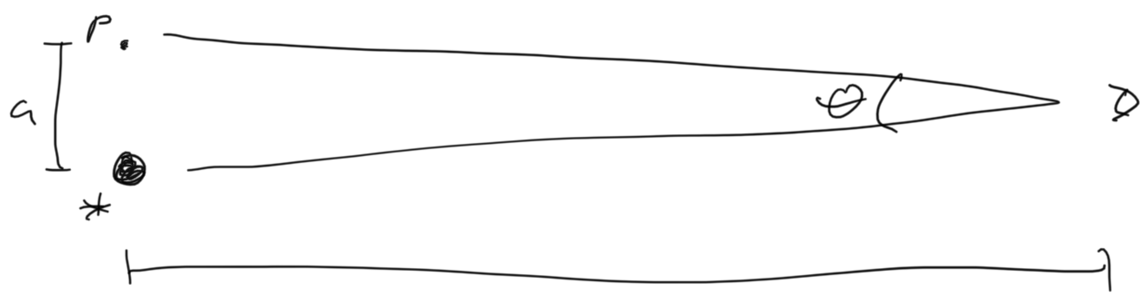
Gas: $\rho = 700-2000 \text{ kg m}^{-3}$

H_2O: $\rho = 1000 \text{ kg m}^{-3}$

Detecting Exoplanets

★ Why is it so hard to detect planets around other stars?

- stars are bright, planets reflect a small fraction of that light, & stars are far away, so the light is hard to distinguish



$d = a/\theta$ ← radians

angle star shifts due to planet : $\theta_* = 0.004'' \left(\frac{m_p/M_*}{0.001}\right)\left(\frac{a}{5.2\,AU}\right)\left(\frac{d}{1.3\,pc}\right.$

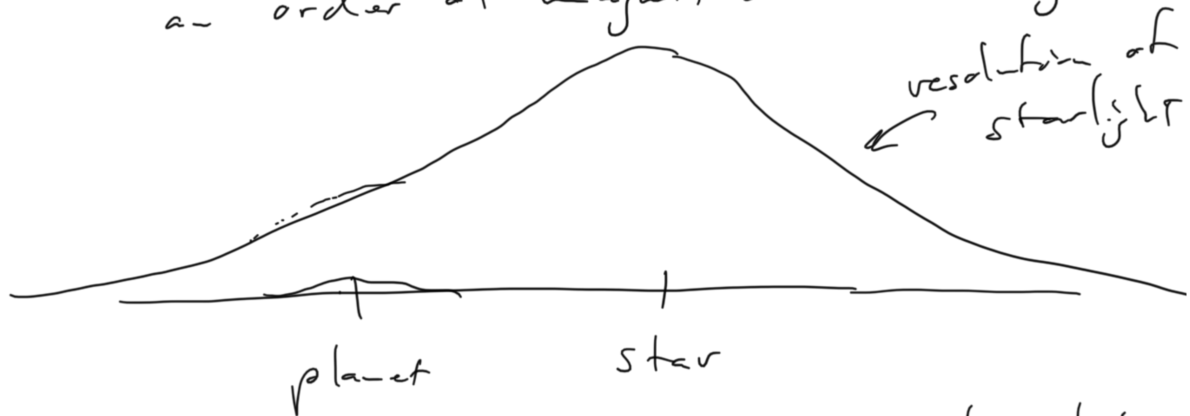
Jupiter's separation from Proxima Centauri is ~4'' (max)

- On the Earth's surface, atmosphere blurs images (just like on a hot day in the desert), limiting $\theta \gtrsim 1''$

- In space, $\theta \gtrsim \frac{\lambda}{D}$; Hubble $D = 2.4m$

so $\theta \gtrsim \dfrac{400\times10^{-9}\text{m}}{2.4\text{m}} = 0.03''$,

an order of magnitude too large



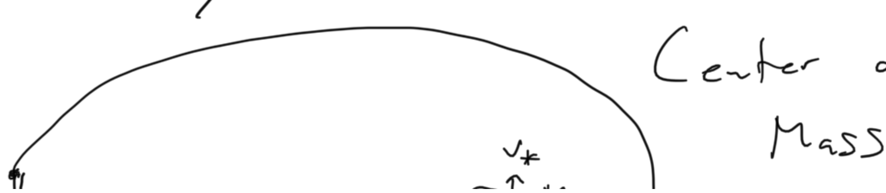
↳ in reality, starlight much brighter

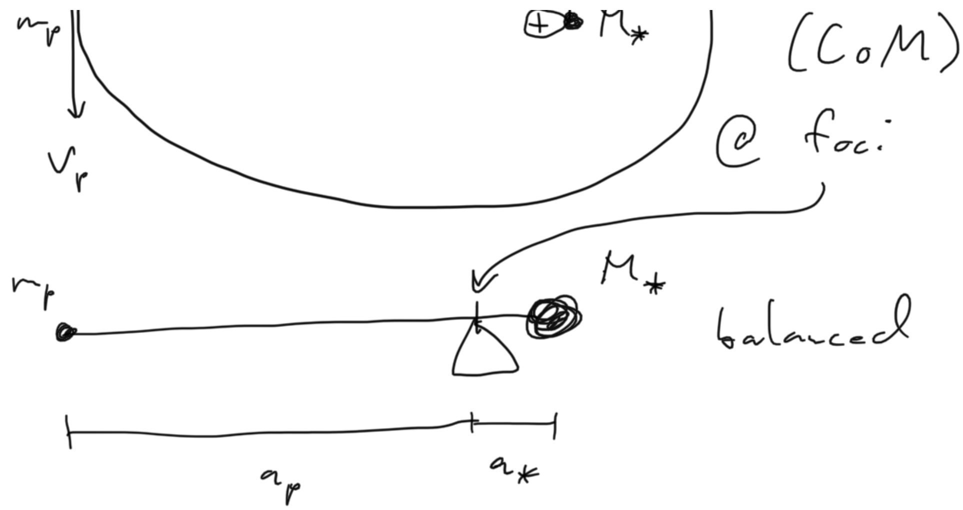
<u>Direct imaging</u> very challenging

<u>Radial velocities</u> also hard, but comparatively easier

↳ look @ how spectral lines shift over time due to the induced motion caused by gravity of a planet on its star

☆ Stars are <u>NOT</u> @ the foci of elliptical orbits → they also orbit





$$\frac{m_p}{M_*} = \frac{a_*}{a_p}$$

Always keep the focus/CoM b/t them so their periods are the same

$$P = \frac{2\pi a_p}{v_p} = \frac{2\pi a_*}{v_*}$$ (circular vel. or avg.)

$$\frac{v_p}{v_*} = \frac{a_p}{a_*} = \frac{M_*}{m_p}$$

Radial velocities come from Doppler shifts:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

measure this

infer this

Find the velocity of the star, can infer the velocity of a planet

↳ estimate M_* somehow, use to get m_p!

Velocities are small: Jupiter is $1000\times \downarrow$ mass than $\odot$, $v_J = 13\ \mathrm{km/s}$

so $v_\odot = 13\ \mathrm{m/s}$

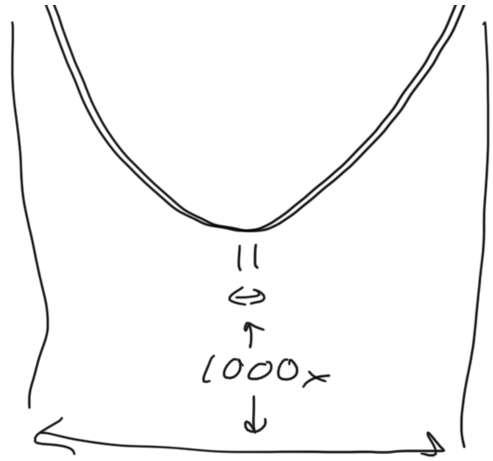
$$\frac{\Delta\lambda}{\lambda} = \frac{13\ \mathrm{m/s}}{3\times10^8\ \mathrm{m/s}} \sim 4\times10^{-8}$$

Measure λ to about ~~than~~ 1 part in 100 million

But the <u>width</u> of a line due to thermal motions is $\frac{\Delta\lambda}{\lambda} \simeq 3\times10^{-7}\left(\frac{T}{1\mathrm{K}}\right)^{1/2}$ A

For H in a solar-like star, $T = 5780\mathrm{K}$,

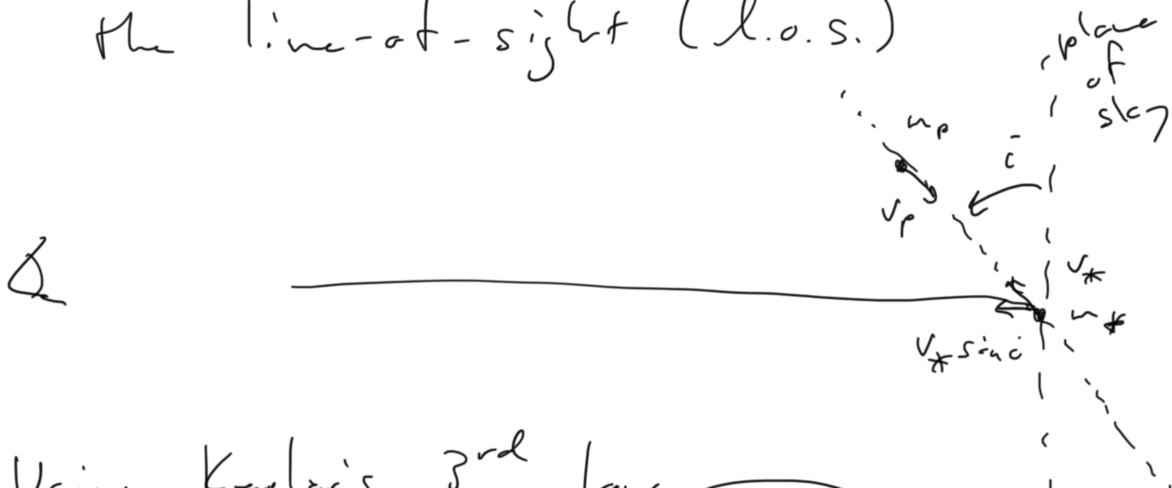
$\frac{\Delta\lambda}{\lambda} \sim 2\times10^{-5}$, $1000\times$ wider than the shift



Very hard

But easier if planet mor[e] massive & close.

But, only measure velocity along the line-of-sight (l.o.s.)



Using Kepler's 3rd law

$$\underbrace{m_p \sin i}_{\text{calc. this}} \approx \left(\frac{M_*^2 P}{2\pi G} \right)^{1/3} \underbrace{v_* \sin i}_{\text{measure}}$$

infer

So, only get limit on mass of planet w/o some other way to constrain i

$$m_p \geq m_p \sin i$$

Be able to dectect motions for

But, harder to detect
orientations w/small i

Selection Effects

- what you observe does not necessarily correspond to the true underlying population, but can be <u>biased</u> by the detection method

Radial velocity method favors massive planets near the star (can detect in a shorter time plus velocity larger) & orbital planes aligned w/our l.o.s.

Transit Method (3rd way)

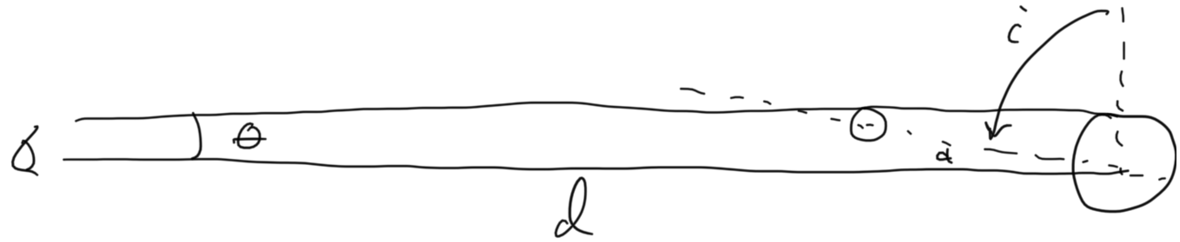
- see the flux from a star dip as a planet passes in front of it

☆ What are some selection effects of this method?

Dips are small $\quad \dfrac{\delta F}{F} = \dfrac{\pi R_p^2}{\pi R_*^2}$

$R_J \sim 0.1 R_\odot$, $\quad \dfrac{\delta F}{F} \sim 0.01$

Even MORE sensitive to i



To see a transit, $\cos i \lesssim \dfrac{R_* + R_p}{a}$

BACK to SLIDES