ASTR 3070 Week 9 (Ch.13)

STELLAR MASSES

Masses are harder, but as we've seen, they can be estimated from star-planet systems using Kepler's 3rd law

$$M_A + M_B = \frac{4\pi^2}{G} \frac{a^3}{P^2}$$

Lucky for us, MOST stars are in binary systems & we can use the same techniques for stars as planets and it's generally easier since stars are brighter/more massive/bigger than planets
→ how we know so much about the properties of stars

3 types (just like planets)

Visual: can see both (usually ↑ a)

Spectroscopic : see 1 star, but 1 or 2 sets of lines shifting over time

Eclipsing : see 1 star, but flux regularly dips as they pass in front of each other

Visual, can directly measure a + P, which gives

$$\frac{M_A + M_B}{M_\odot} = \frac{a^3}{P^2} \quad (a = a_A + a_B)$$

+ individual masses come from CoM

$$\frac{M_A}{M_B} = \frac{a_B}{a_A}$$

Spectroscopic, measure velocities, but includes an unknown inclination i

$$\frac{M_B}{(1 + M_A/M_B)^2} \sin^3 i = \frac{P}{2\pi G} (v_A \sin i)^3$$

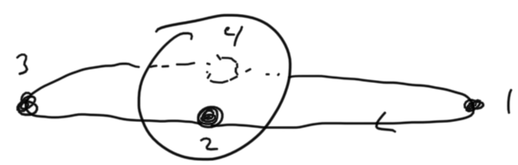
unmeasurable measured quantities

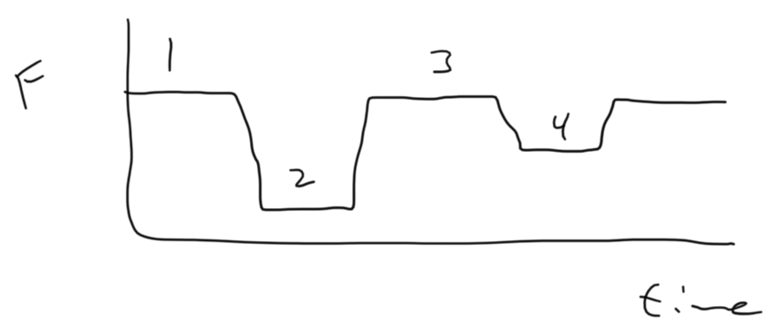
Left side constrains masses given a definite value on right side, call this the "mass function"

$$\boxed{f(M_A, M_B) = \frac{M_B^3 \sin^3 i}{(M_A + M_B)^2}}$$

This reduces to equation we found for exoplanets if $M_A \gg M_B$

Eclipsing





Assuming bigger star is hotter, 2 will be the bigger dip b/c

$$L = 4\pi R^2 \sigma_{SB} T^4$$

more important factor

How does R, T, & lifetime depend on mass?

$$\frac{R}{R_\odot} = \begin{cases} 1.06\,(M/M_\odot)^{0.945} & M < 1.66\,M_\odot \\ 1.33\,(M/M_\odot)^{0.555} & M > 1.66\,M_\odot \end{cases}$$

$$\frac{L}{L_\odot} = \begin{cases} 0.35\,(M/M_\odot)^{2.62} & M < 0.7M_\odot \\ 1.02\,(M/M_\odot)^{3.92} & M > 0.7M_\odot \end{cases}$$

lifetime of a star is determined by its mass & L $\propto$ rate fuel burned

↳ $\propto$ fuel

$$\tau \propto M/L \propto \begin{cases} M^{-1.62} & M < 0.7M_\odot \\ M^{-2.92} & M > 0.7M_\odot \end{cases}$$

STELLAR BRIGHTNESS

Already know some measurements

distance $d = \dfrac{1\,\text{pc}}{\pi''}$

flux & luminosity $F = \dfrac{L}{4\pi d^2}$

$L \rightarrow$ intrinsic brightness
(absolute)

$F \rightarrow$ apparent brightness

Still use the magnitude system, recorded by Hipparchus 2200 years ago

Visible stars were put into 6 brightness bins, brightest mag = 1, faintest mag =

Turns out our eyes (also ears) work on a logarithmic scale, not linear scale (so something that looks 2x as bright is actually emitting >2x as many photons)

In mid-1800s, system was quantified (when photography made measurements more precise)

$\rightarrow$ difference of 5 mag $\rightarrow$ 100x ratio of flux

$m_2 - m_1 = 5$ equivalent

$$\frac{F_1}{F_2} = 100$$

if $\Delta m = 1$, $\quad \frac{F_1}{F_2} = (100)^{\frac{1}{5}}$

$\Delta m = 2$, $\quad \frac{F_1}{F_2} = (100)^{\frac{2}{5}}$ etc.

in general, $\quad \frac{F_1}{F_2} = 100^{(m_2 - m_1)/5}$

$$= 10^{2(m_2 - m_1)/5}$$

$$\log\left(F_1/F_2\right) = \frac{2}{5}\left(m_2 - m_1\right)$$

$$\boxed{m_2 - m_1 = 2.5 \log\left(F_1/F_2\right)}$$

apparent magnitudes

relative system, need a reference mag

$\uparrow m$ means fainter

$$m = m_1 - 2.5 \log F + 2.5 \log F_1$$

reference value

Use the star Vega (could use anythi[ng]
not even a real star)

Vega : $m_1 = 0$, $C = 2.5 \log F_{Vega}$

$$\boxed{m = C - 2.5 \log F}$$

It's a flux, so depends on distance

$$m = C - 2.5 \log\left(\frac{L}{4\pi d^2}\right)$$

$$= C - 2.5\left(\log L - \log 4\pi d^2\right)$$

$$= C - 2.5\left(\log L - 2\log d - \log 4\pi\right.$$

The magnitude equivalent of the luminosity needs a distance ref., so we use 10_{pc} b/c the system is stupid anyway why not

Absolute magnitude

$$M = C - 2.5\left(\log L - 2\log(10) - \log\right.$$

$$= C - 2.5\left(\log L - 2\log d - \log 4\pi\right.$$

Subracting these,

$$m - M = 5 \log d - 5 \log 10_{pc}$$

$$\boxed{m - M = 5 \log \left(\frac{d}{10_{pc}} \right)}$$

<u>distance modulus</u>

Convenient refence is the Sun

$$m_\odot = -26.75 , \quad M_\odot = 4.83$$

In practice, flux is a function of λ and measurements are w/in some range $\lambda_1 \to \lambda_2$

i.e., our eyes are sensitive to light $4000\text{Å} < \lambda < 7000\text{Å}$

Can define the total flux as the integrated light over all λ, called

bolometric flux $F_{bol} = \int_0^\infty F_\lambda \, d\lambda$

bolometric mag $m_{bol} = C_{bol} - 2.5 \log F_{b.}$

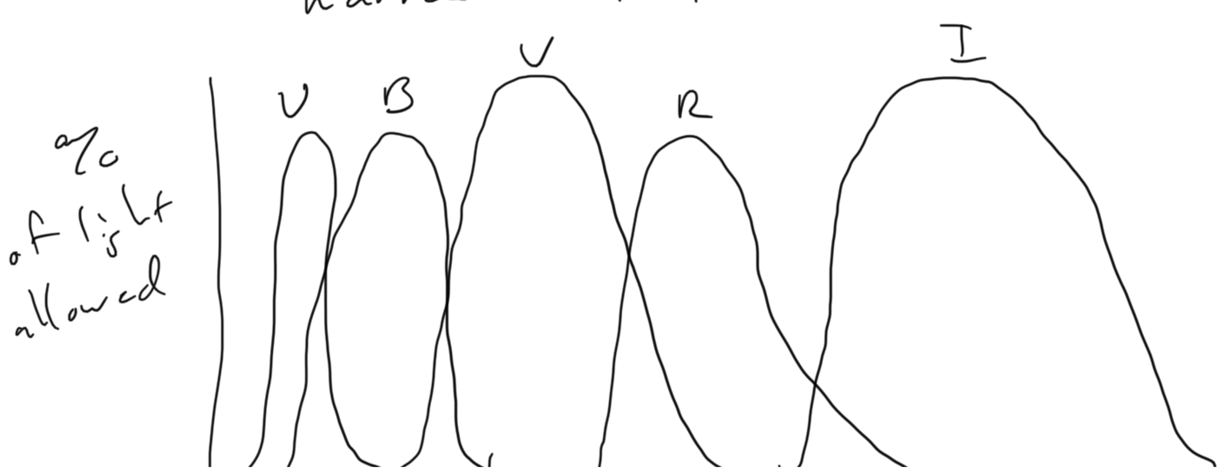
define $M_{\odot,bol} = 4.74$

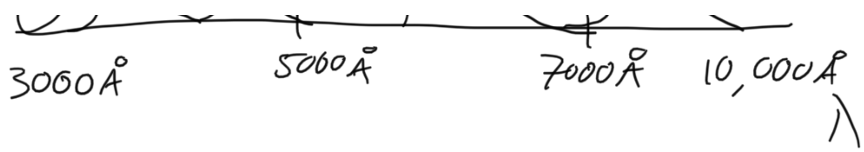
$$\boxed{M_{bol} = 4.74 - 2.5 \log (L/L_\odot)}$$

In practice, no physical detector is sensitive to all λ's; need diff. technology @ diff. λ's

Also, lose info if you just count photons & don't record their exact λ, but hard to do that & make an image, so must choose

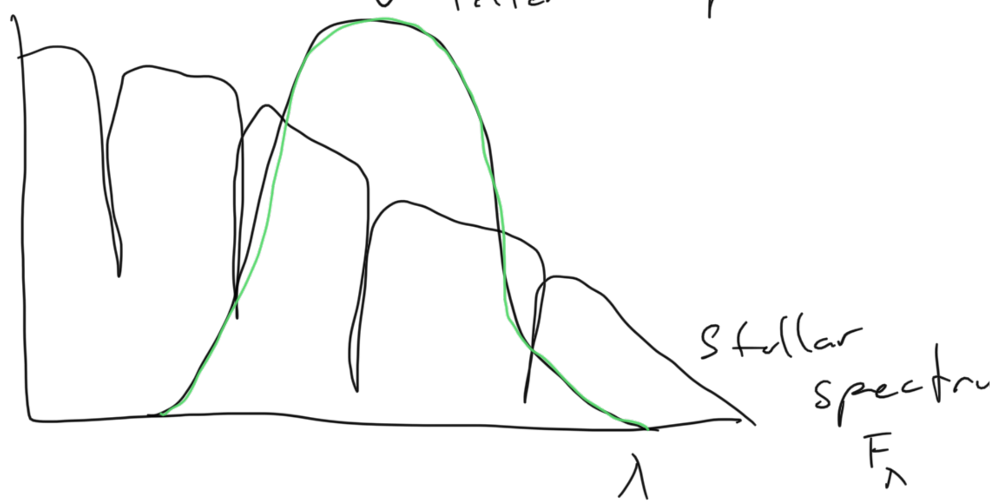
Images → "filter" the light so λ's are restricted to a more narrow band





Johnson-Cousins system

V filter : response $S_V(\lambda$

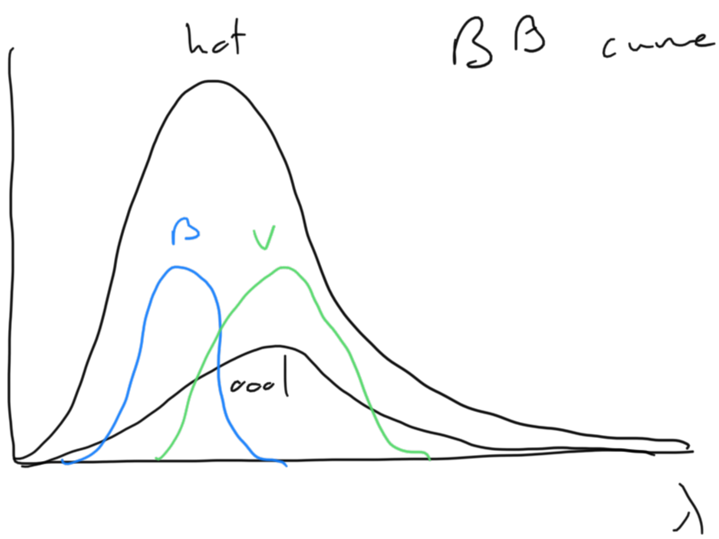


$$F_V = \int_0^\infty F_\lambda S_V(\lambda)\, d\lambda$$

$$m_V = C_V - 2.5 \log F_V$$

similar expressions for other filters

BB curves for stars



$B-V \rightarrow$ color, ratio of fluxes

hot stars have more blue light rel. to green light than cooler star

$$B-V = m_B - m_V$$

Define Vega to have $B-V=0$

$T_{Vega} \approx 10{,}000\,K$, so

$B-V < 0$ (brighter in B rel. to V, bluer, hotter

$\hookrightarrow T > 10{,}000\,K$

$B-V > 0$, $T < 10{,}000\,K$

In practice, stars aren't perfect BBs, but we observe that

$$T \approx \frac{9000\,K}{(B-V)+0.93}$$

Can convert filter mags to bol. mags using a bolometric correction

$$BC = m_{bol} - m_V = M_{bol} - M_V$$

[illegible] to measure

Stellar radii are hard to measure,
but when we can, can estimate its T

$$L = 4\pi R^2 \sigma_{SB} T^4$$

BB relation, which won't apply perfectly to stars, but can use it to define an "effective temperature"

$$T_{eff} = \left(\frac{L}{4\pi R^2 \sigma_{SB}} \right)^{1/4}$$