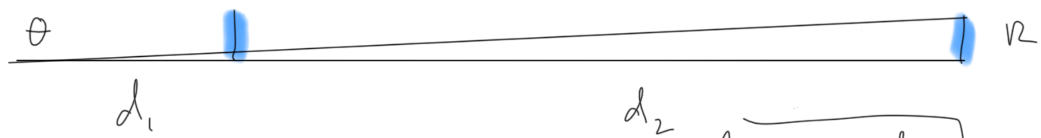
SB → Olber's paradox



$$\tan\theta \approx \theta = \frac{r_1}{d_1} = \frac{R}{d_2} \longrightarrow \boxed{\frac{r_1}{R} = \frac{d_1}{d_2}}$$

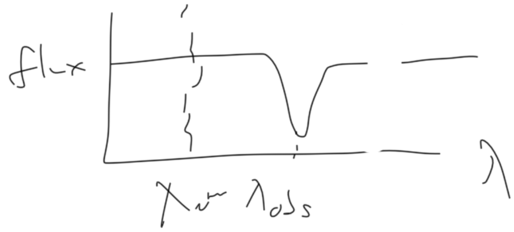
$$F = \frac{L}{4\pi d^2} \qquad \boxed{\frac{A_1}{A_2} \propto \left(\frac{r_1}{R}\right)^2}$$

$$F_2 = \frac{L^{=\int l\, dA_2}}{4\pi d_2^2}, \qquad F_1 = \frac{l\, A_1}{4\pi d_1^2} = \frac{l A_2 \left(\frac{r_1}{R}\right)^2}{4\pi d_1^2}$$

$$\frac{F_2}{F_1} = \frac{l A_2}{l A_2}\,\frac{4\pi}{4\pi}\,\underbrace{\frac{d_1^2}{d_2^2}}_{=\frac{r_1^2}{R^2}}\,\frac{R^2}{r_1^2} = \boxed{1}$$

HW / PDF Syllabus?

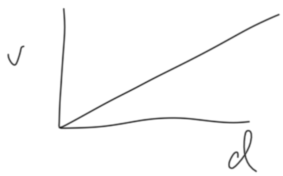
Redshift + Hubble



$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

$$\downarrow$$

$$\boxed{v = cz}$$

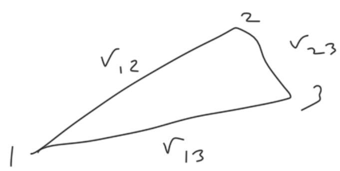


$$y = Ax + \cancel{B}^{0}$$

$$v = Ad = cz, \quad z = \frac{A}{c} d$$

$$A \equiv H_0$$

☆ Ask what means → expanding universe



$$r_{12}(t) = a(t)\, r_{12}(t_0), \text{ etc.}$$

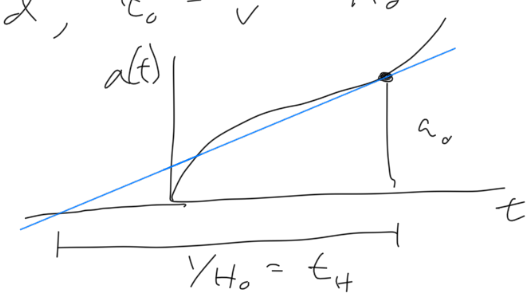
$$v_{12}(t) = \frac{dr_{12}}{dt} = \frac{da}{dt} r_{12}(t_0)$$

$$= \frac{da}{dt} \frac{r_{12}(t)}{a(t)} = \frac{\dot{a}}{a} r_{12}(t)$$

$$\boxed{H = \frac{\dot{a}}{a}}$$

If constant expansion, can turn back → Hubble time

$$x = vt = d, \quad t_0 = \frac{d}{v} = H_0^{-1}$$



$$a(t) = Bt$$

$$\dot{a} = B$$

$$\boxed{H_0 = \frac{\dot{a}}{a}(t_0) = \frac{1}{t_0}}$$

Steady state Universe

→ discuss isotropy & homogeneity

anisotropic
but hom.

isotropic
(center)
inhomog.

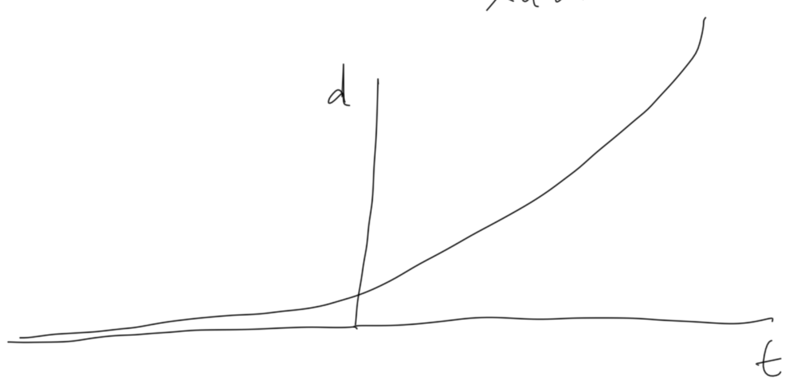
cosmological principle → have both

↑ perfect → H_0 constant always

$$v = H_0 d, \quad \frac{d}{dt}(d) = H_0 d$$

$$\int \frac{1}{d}\, d(d) = \int H_0\, dt$$

$$\ln d = H_0 t \rightarrow d \propto e^{H_0 t}$$



have to create matter → amounts to
1 H per km^3 per year

⟶ problems w/ this?

$\boxed{\text{CMB}}$ Integrate over all freq.

$$\varepsilon_\gamma = \alpha T^4$$

The pressure of a photon gas is $P_\gamma = \varepsilon_\gamma / 3$

If we have a volume V that's expanding

$$V \propto a(t)^3$$

The 1st law of thermo gives

1) conserv. E: $\dfrac{dE}{dt} = \Delta U = Q - W$

heat applied

amount of work

system expands, $W = PdV$

$Q = 0$, $\boxed{\dfrac{dE}{dt} = -P\dfrac{dV}{dt}}$ $\quad E = \varepsilon_\gamma V$

$$\frac{d}{dt}\left(\varepsilon_\gamma V\right) = -\frac{\varepsilon_\gamma}{3}\frac{d}{dt}(V)$$

$$V\alpha\frac{d}{dt}T^4 + \alpha T^4 \frac{d}{dt}V = -\frac{\alpha T^4}{3}\frac{dV}{dt}$$

$$4V\alpha T^3 \frac{dT}{dt} = -\frac{4}{3}\alpha T^4 \frac{dV}{dt}$$

$$\frac{1}{T}\frac{dT}{dt} = -\frac{1}{3}\frac{1}{V}\frac{dV}{dt} = -\frac{1}{3}\frac{1}{a^3}\frac{d}{dt}a^3$$

$$= -\frac{1}{3}\frac{3a^2}{a^3}\frac{da}{dt} = -\frac{1}{a}\frac{da}{dt}$$

$$\frac{d}{dt}(\ln T) = -\frac{d}{dt}(\ln a) = \frac{d}{dt}(\ln a^{-1})$$

$$\Rightarrow T \propto a^{-1}$$

☆ Based on T to shut off nucleosynthesis

$\sim 3000\,K$

$a(t_{decouple}) = \frac{a(t_0)}{1000}$, so $T_0 \sim 3K$