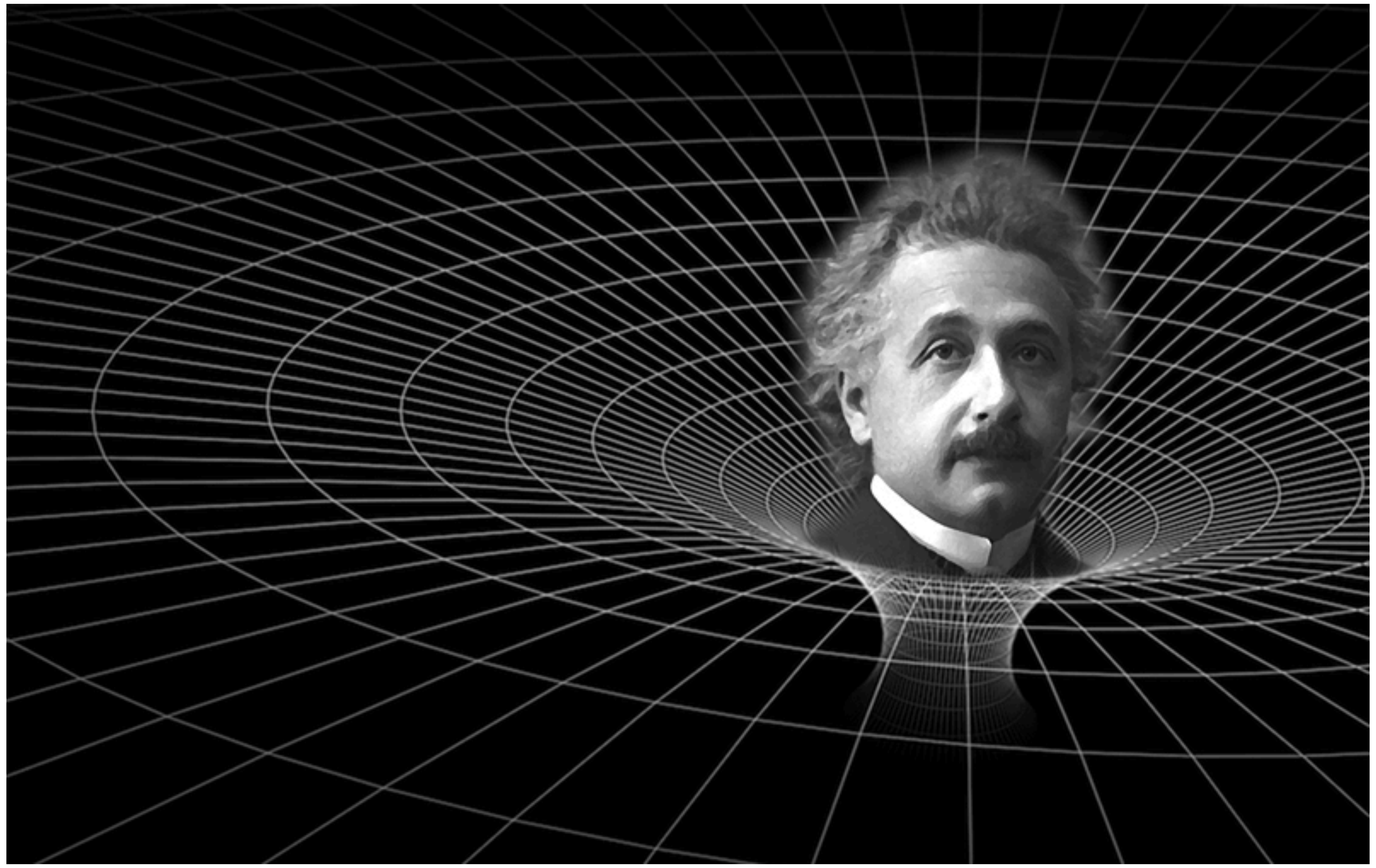
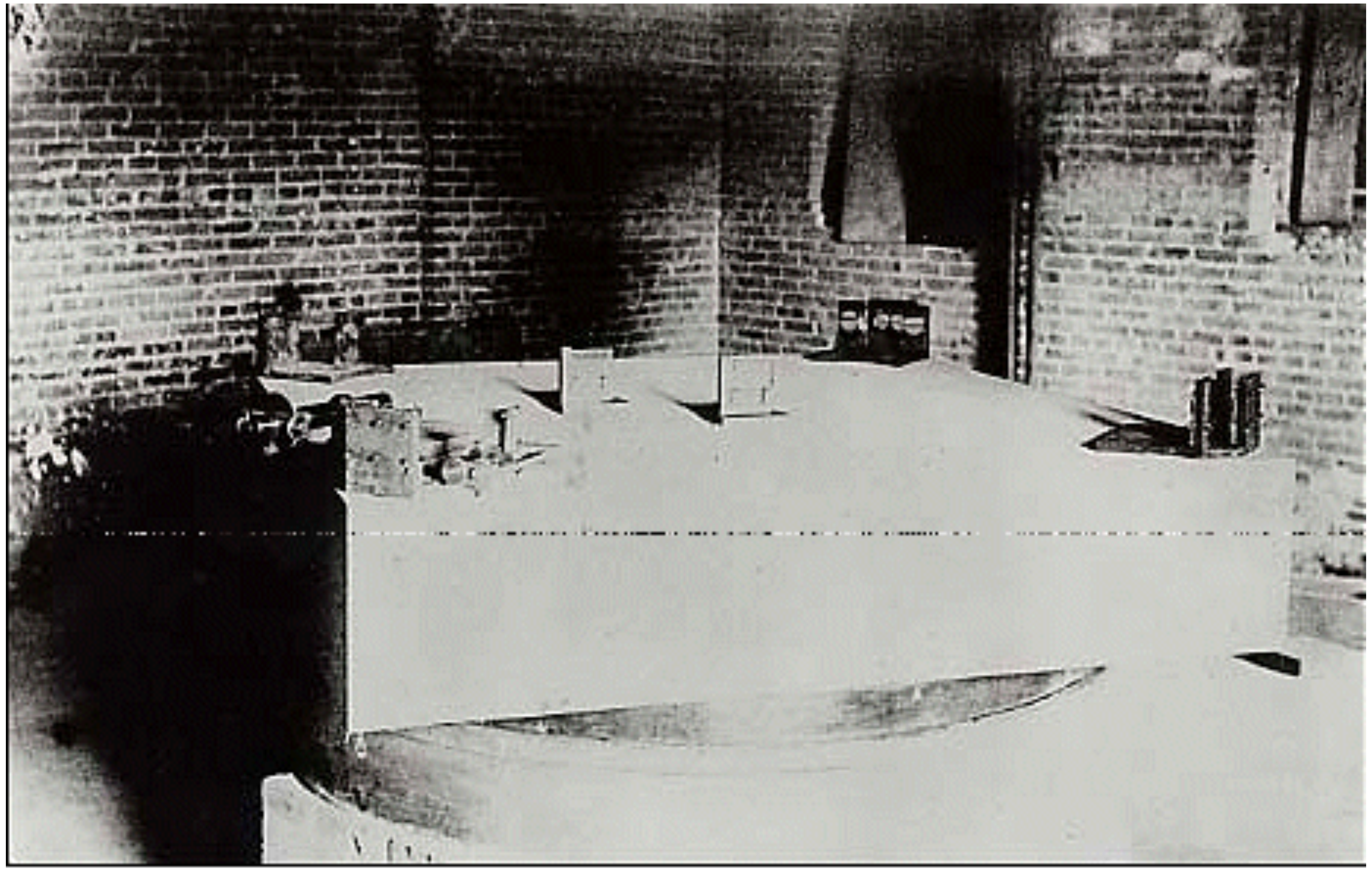
Special & General Relativity

ASTR/PHYS 4080: Intro to Cosmology

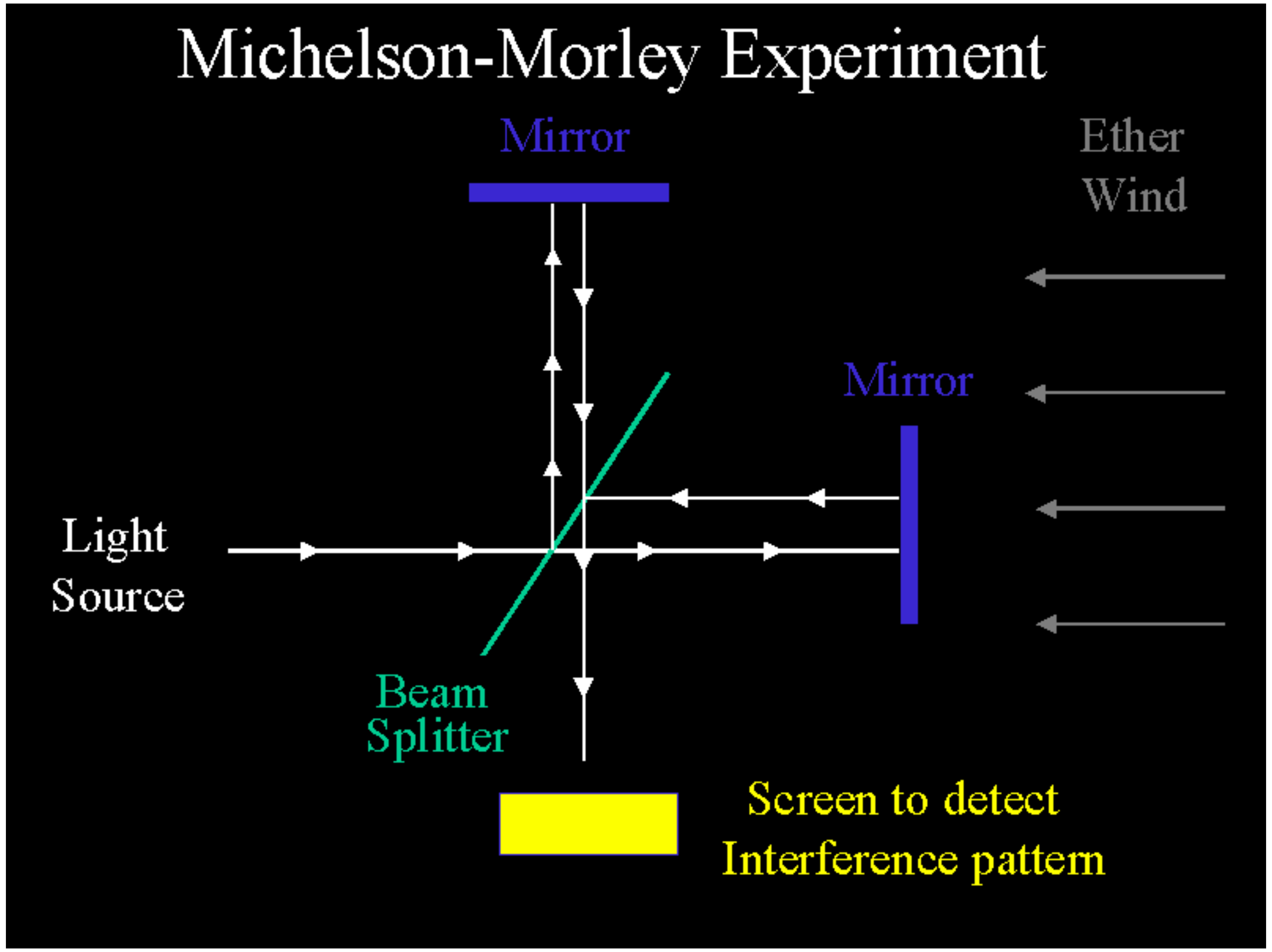
Week 2



Special Relativity: no "ether"



Michelson & Morley's 1887 interferometer
built in the basement of Western Reserve
Photo: Case Western Reserve Archive



Presumes absolute space and time, light is a vibration of some medium: the ether

Equivalence Principle(s)

$$\mathbf{F} = m_I \mathbf{a}$$

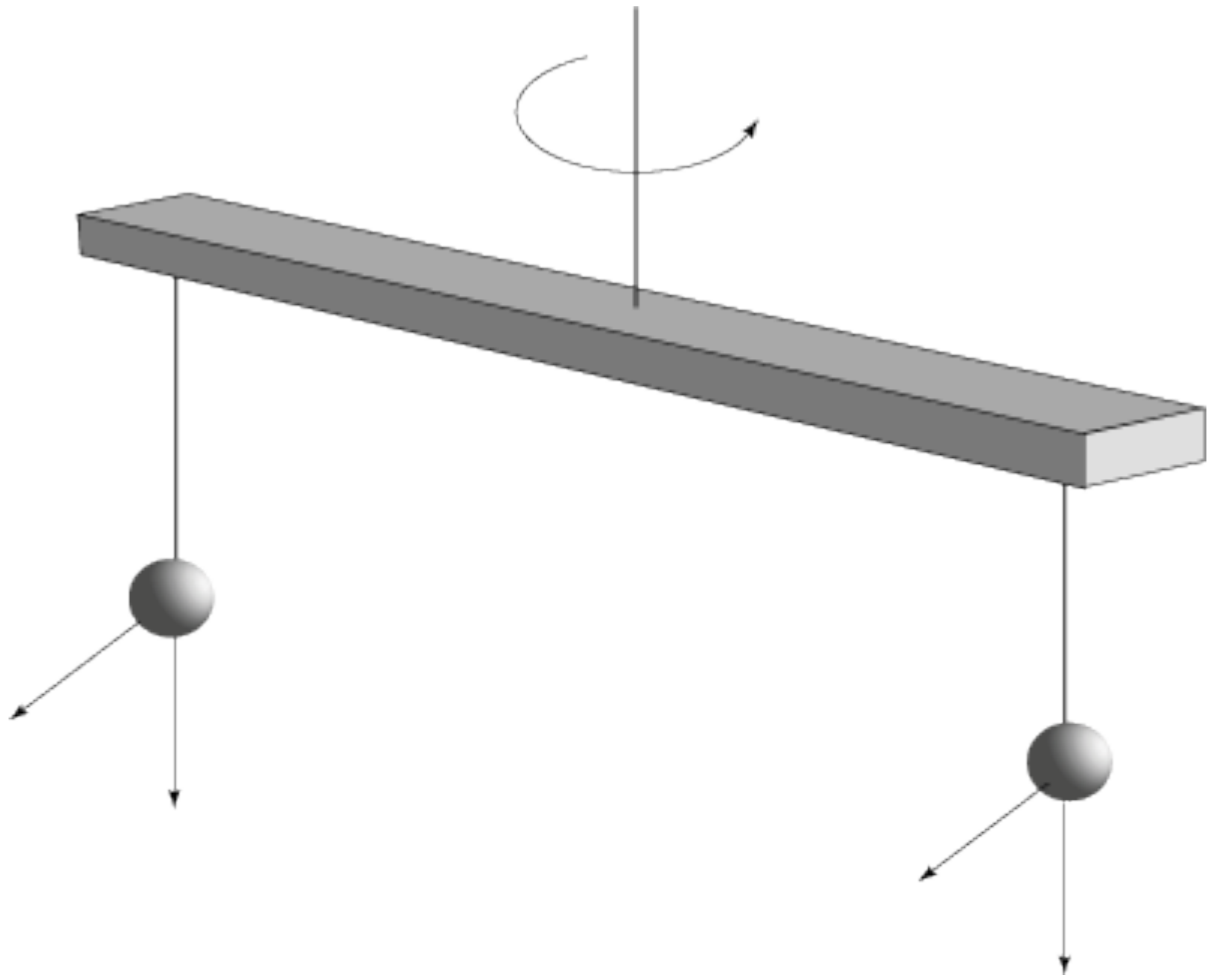
reflect an object's inertia
(how hard to make it move)

$$\mathbf{F} = -\frac{GM_G m_G}{r^2}\hat{r} = m_G \mathbf{g}$$

reflect the strength of the grav. interaction;
nothing to do with inertia at all;
may just call it "gravity charge" (like electric charge)

Galileo, and later Eötvös, experimentally demonstrated that:

$$m_I = m_G$$



suspicious…

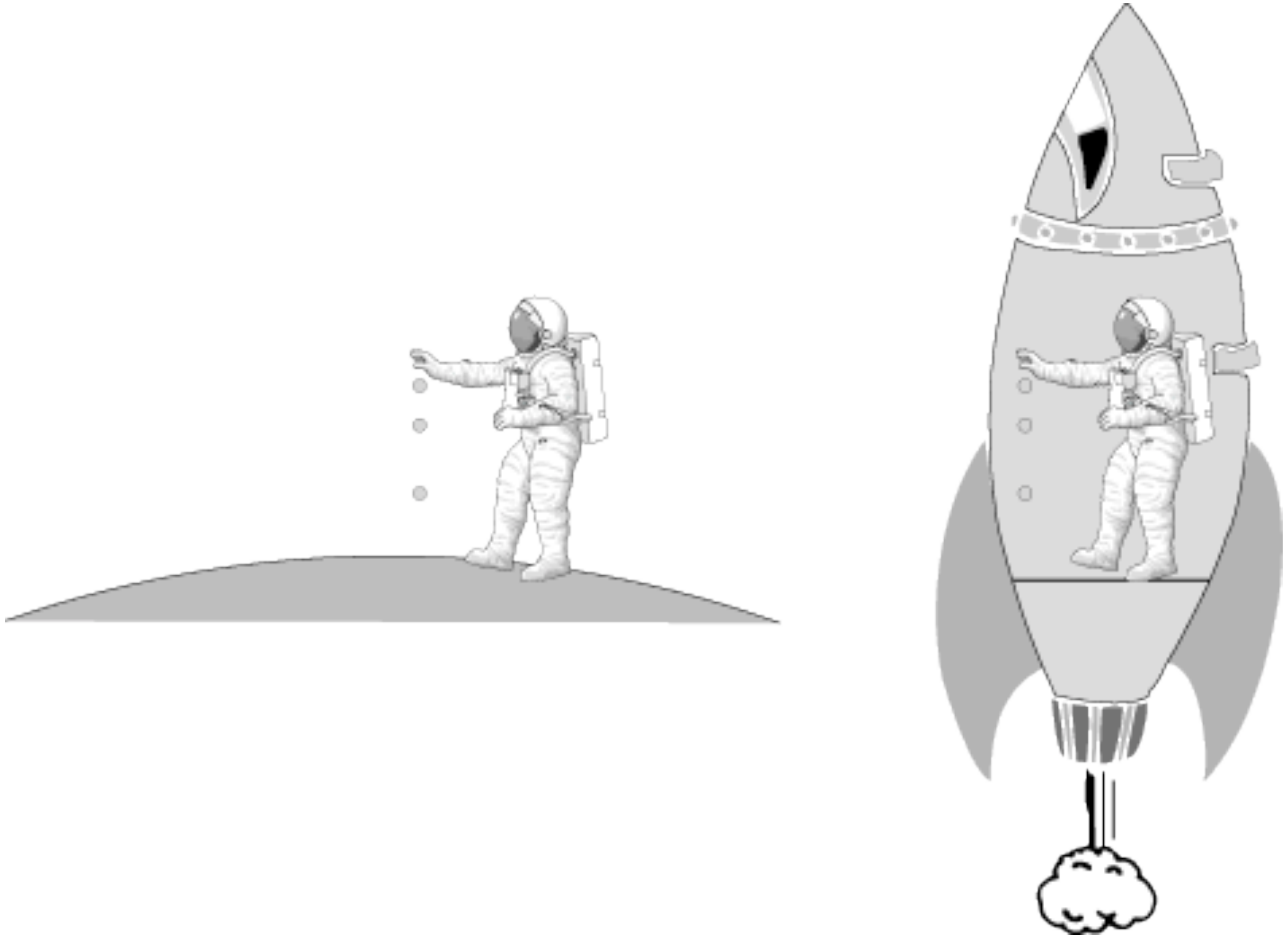
Equivalence Principle: Newton

"Gravitational mass" and "inertial mass" are equivalent

You cannot distinguish gravity from any other acceleration

Gravity even affects massless particles like light

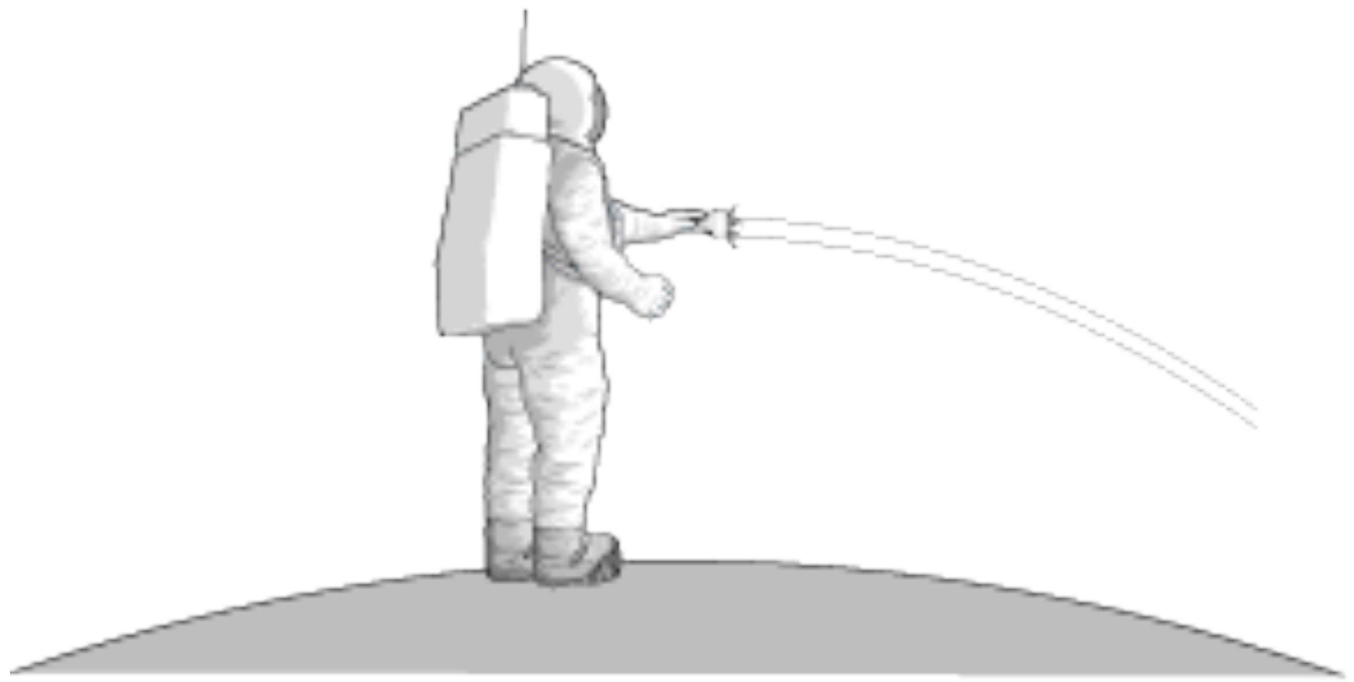
Only applies to mechanics: E&M not included until special relativity

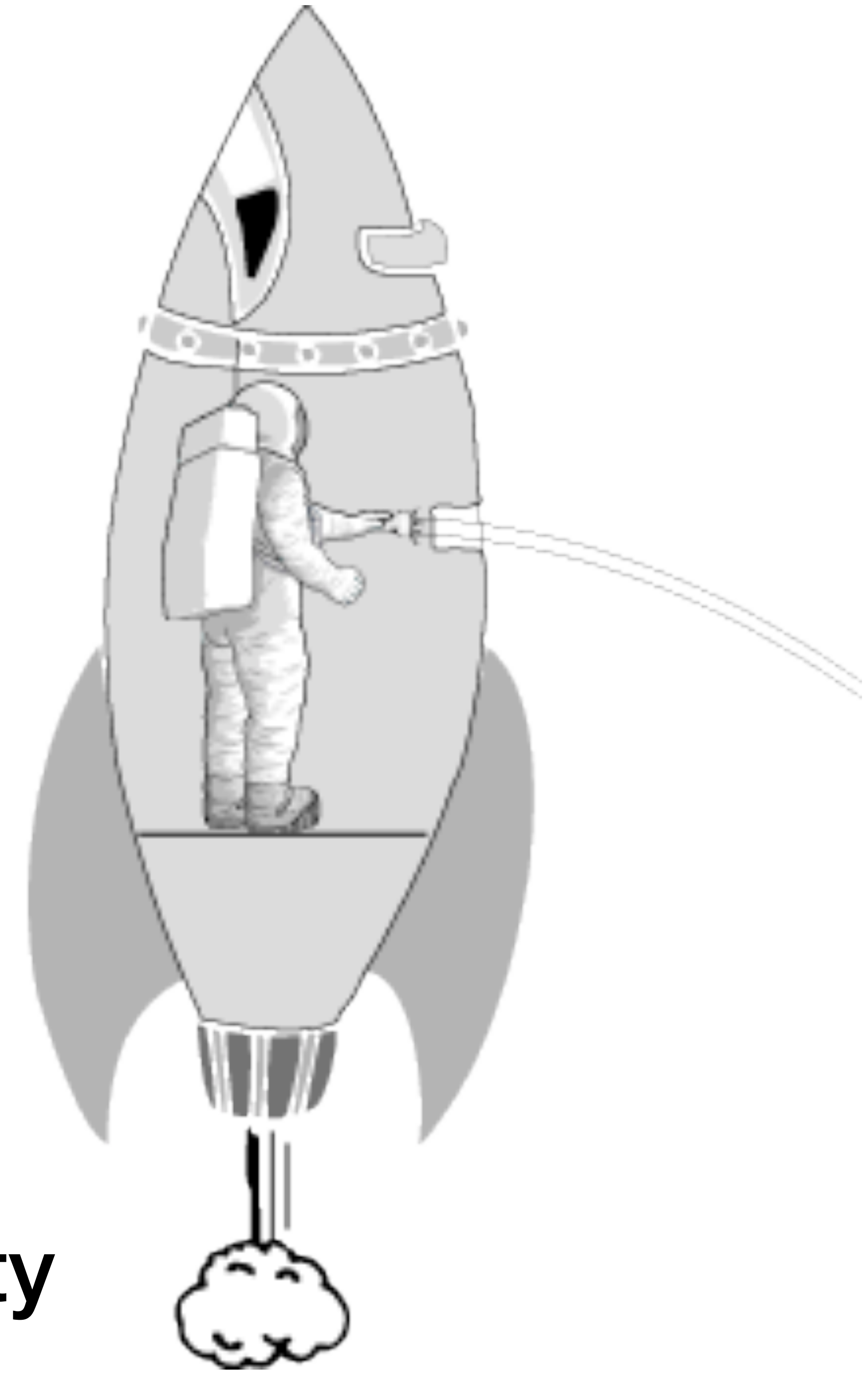


Equivalence Principle: Einstein

No experiment can distinguish between an accelerated frame and a gravitational field – they are completely equivalent

Also, implies gravitational redshifting





“Special” relativity applies in the absence of gravity
“General” relativity generalizes the postulates of SR to include gravity

Mach’s Principle: inertial frames aren’t absolute, but determined by the distribution of matter — can’t have motion without something else a thing is moving relative to

Implication of Stricter Equivalence for Light

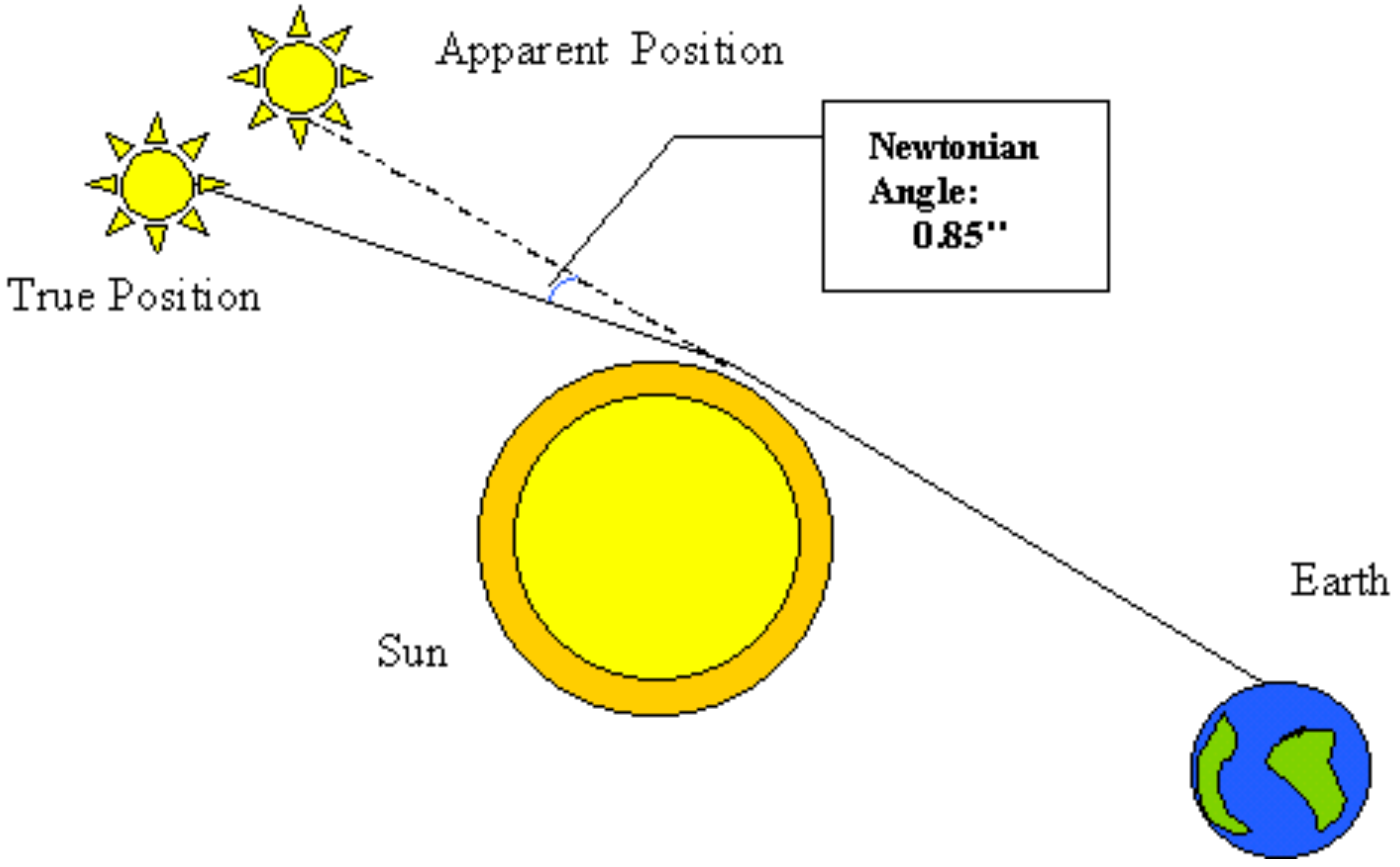
Fermat's Principle in optics states that light travels the minimum distance between two points

If light takes a curved path, space cannot be Euclidean (flat) because the shortest path in Euclidean geometry is a straight line

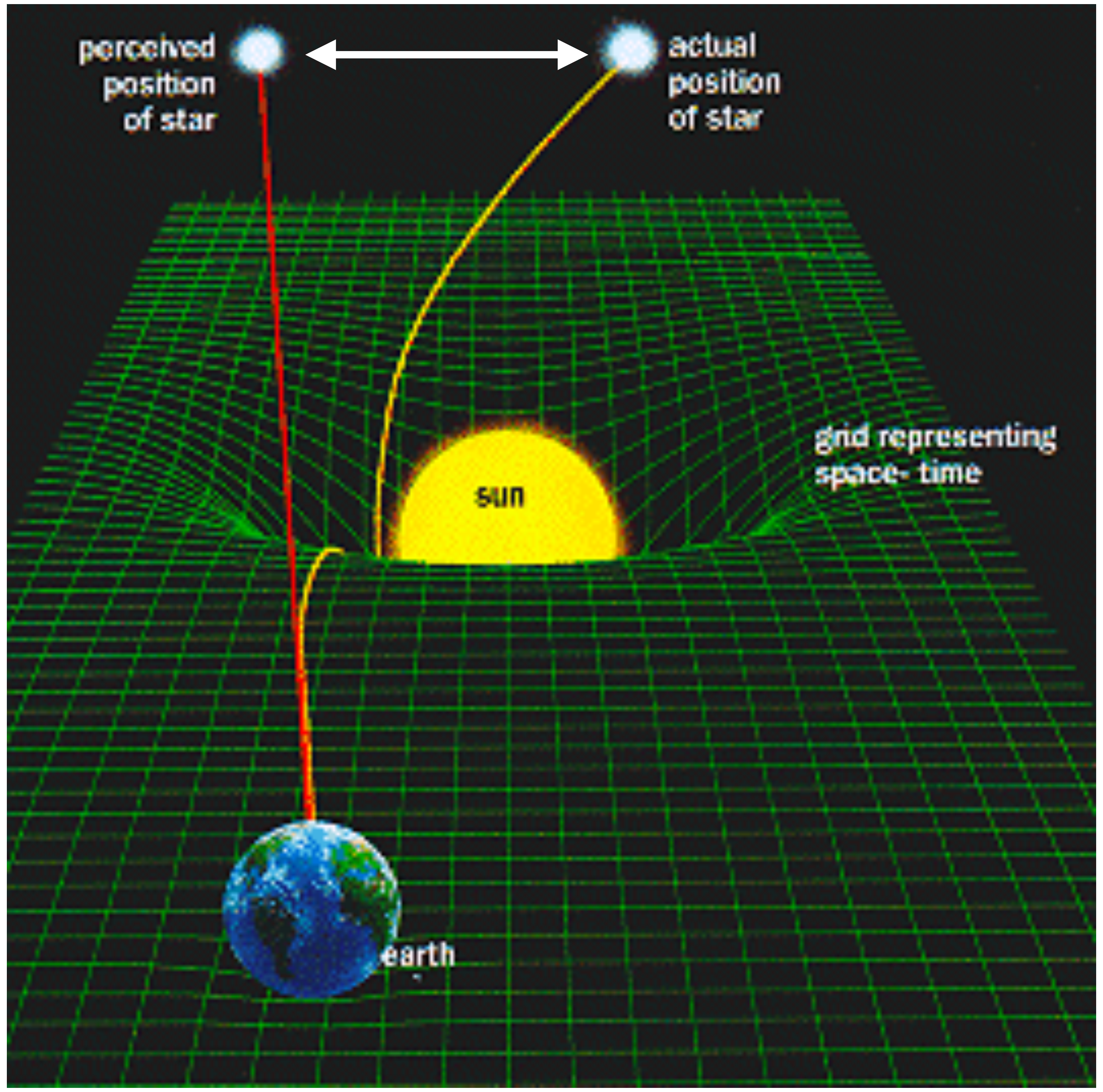
If space is curved (like surface of a sphere), then Fermat's Principle may still hold

—> Matter (and Energy, b/c $E=mc^2$) tells spacetime how to curve, and curved spacetime tells matter (and energy) how to move

Experimental Confirmation of GR



Angle in GR is ~1.75”:
additional deflection due to curved space-time

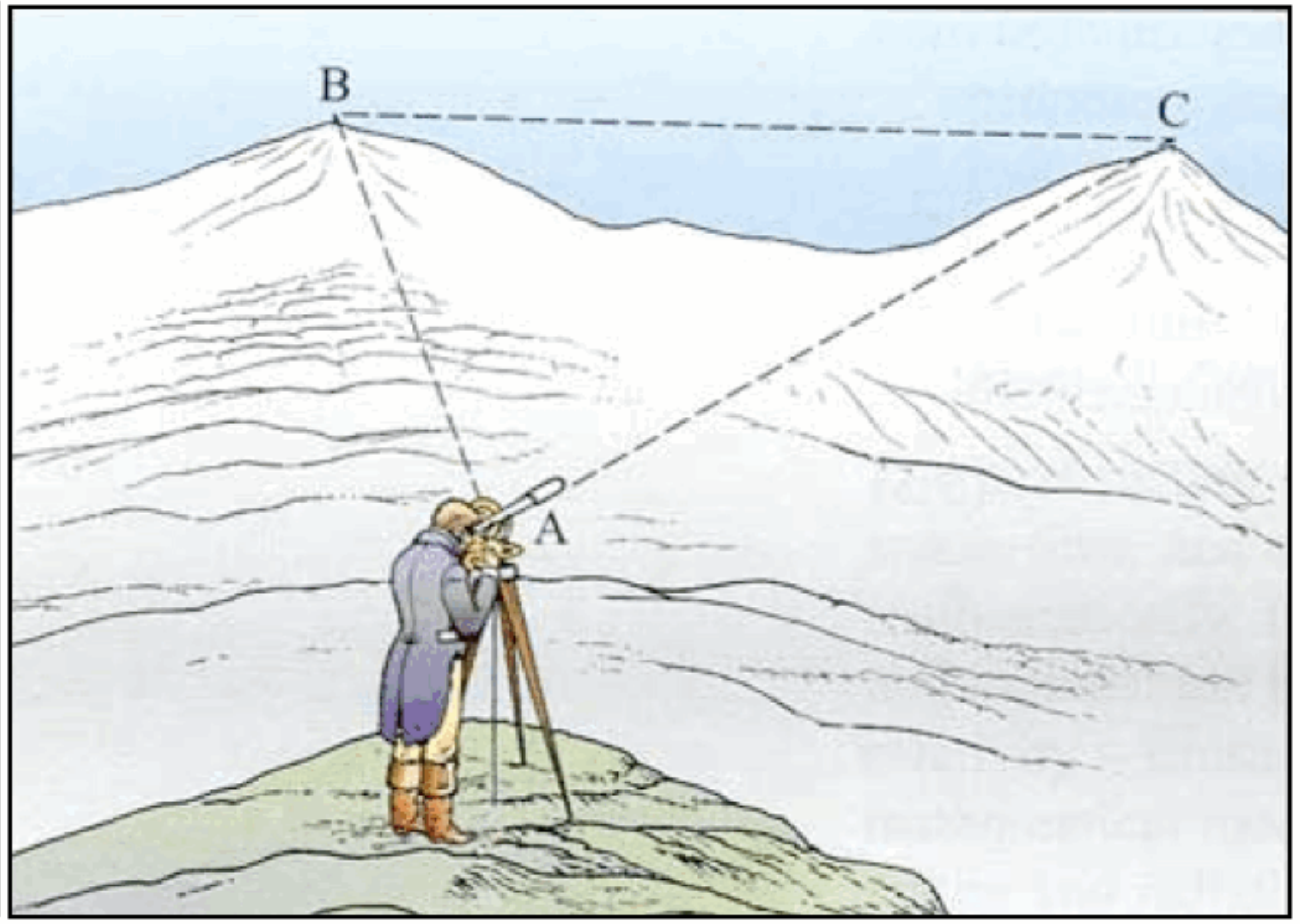


“Confirmed” by Arthur Eddington during the 1919 solar eclipse
—> reason Einstein became famous

Curvature

How can we measure the curvature of spacetime?

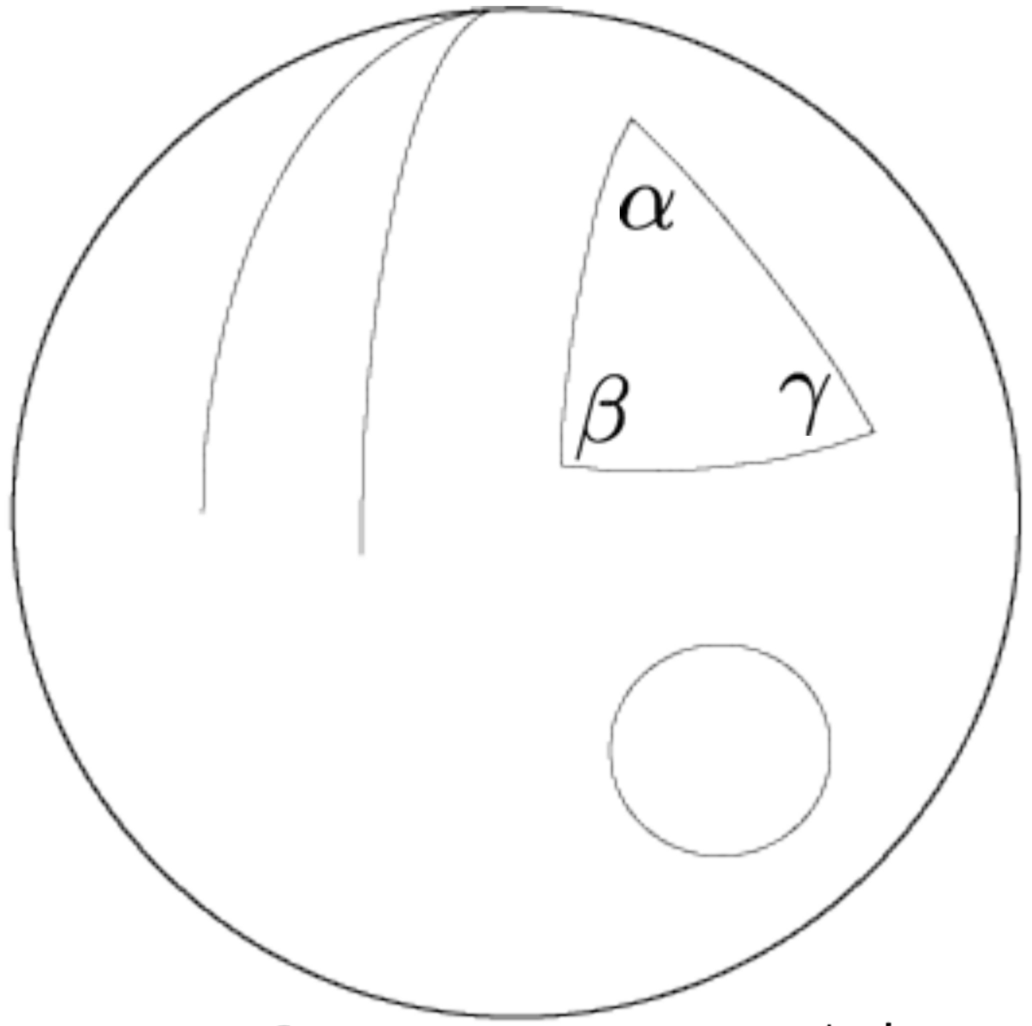
Carl Friedrich Gauss
1777 - 1855



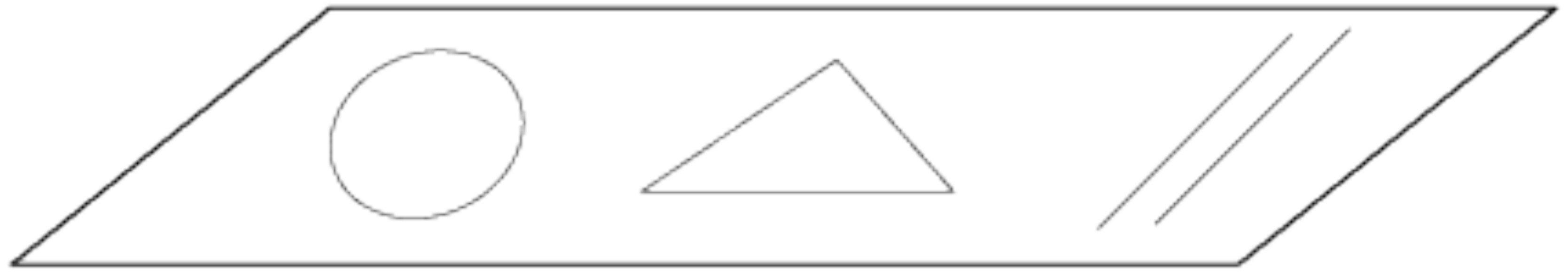
Gauss finds 180 degrees in large survey triangles:
Space is not (grossly) non-Euclidean

A = area of triangle R = Radius of Curvature

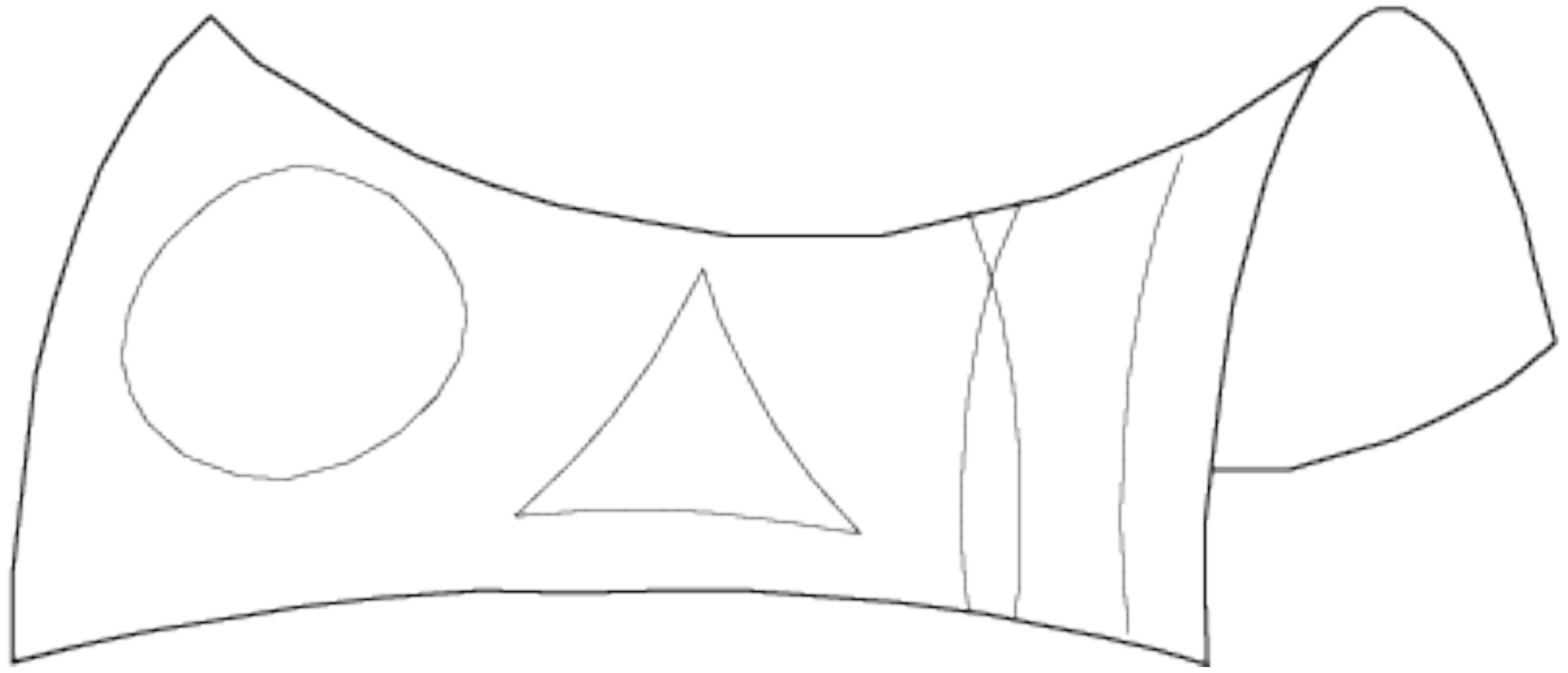
Only possible geometries that are homogeneous/isotropic



$$\alpha + \beta + \gamma = \pi + A/R$$



$$\alpha + \beta + \gamma = \pi$$



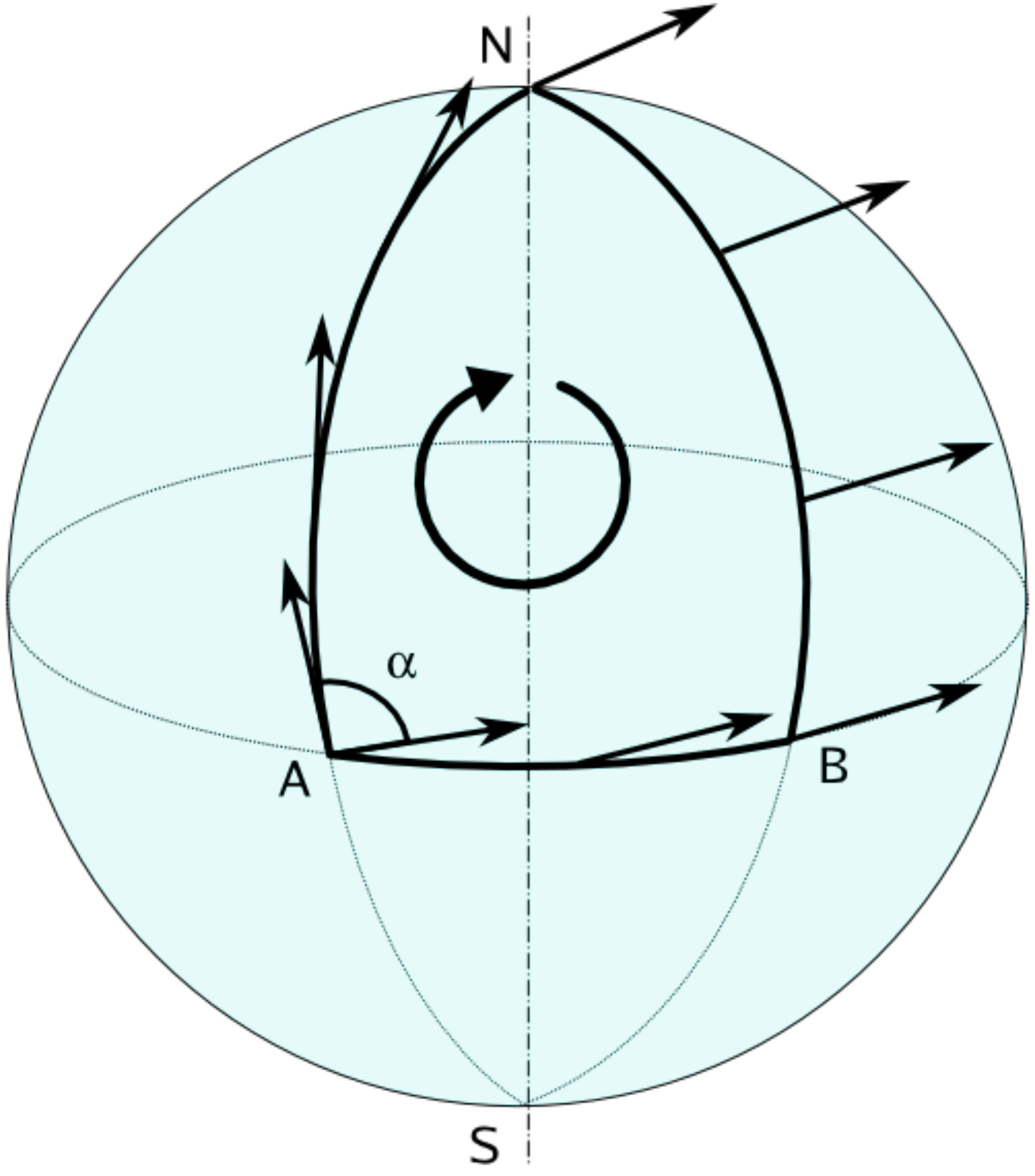
$$\alpha + \beta + \gamma = \pi - A/R$$

Characterizing Curvature

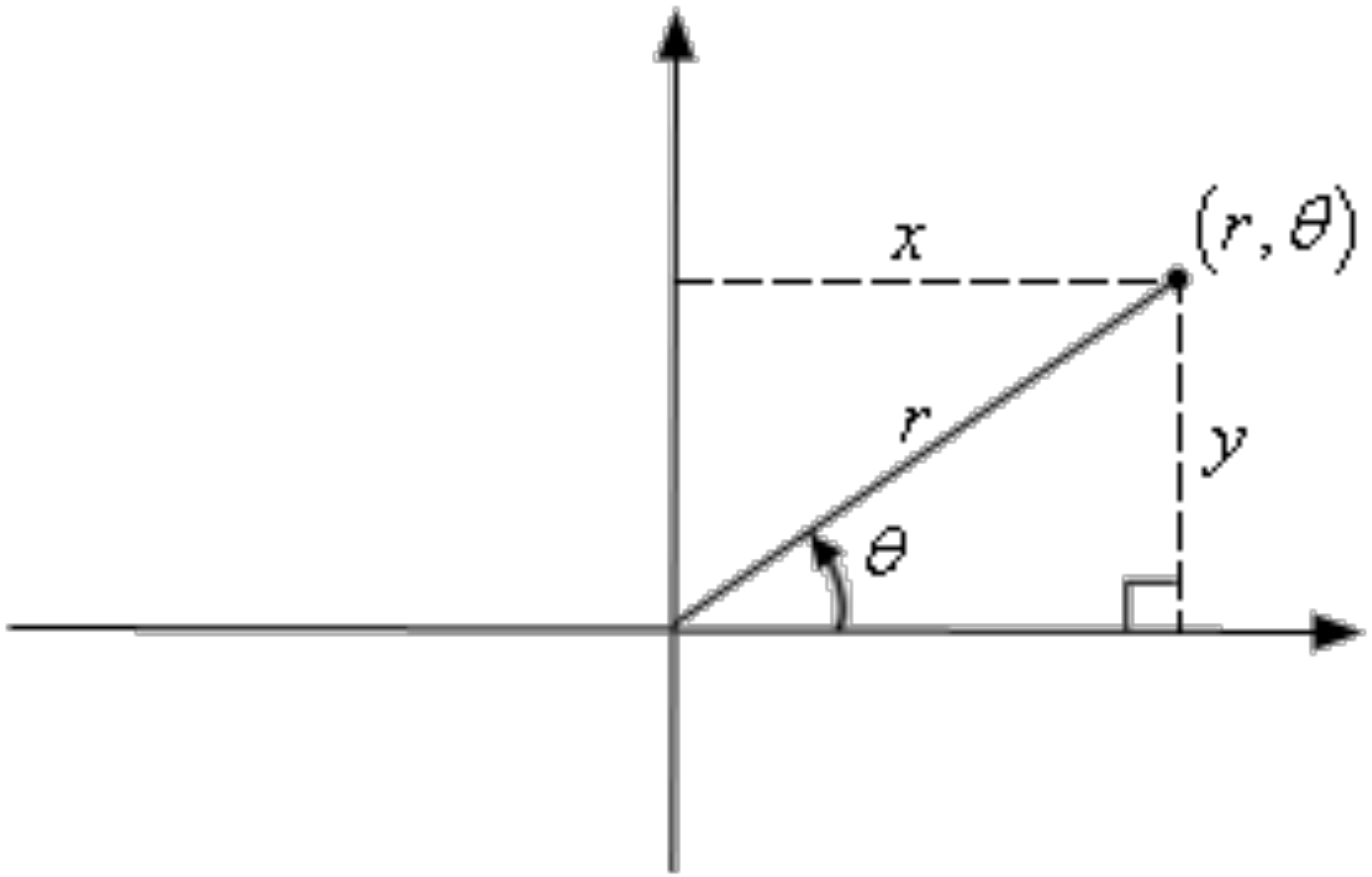
<u>Parallel Transport</u>

transport a vector around a triangle, keeping the vector at the same angle wrt your path at all times

change in vector when you arrive back at your starting position ⟶ curved space



Length of a (Euclidean) Line



$$(x, y) \rightarrow (x + dx, y + dy)$$

$$d\ell^2 = dx^2 + dy^2$$

$$(r, \theta) \rightarrow (r + dr, \theta + d\theta)$$

$$d\ell^2 = dr^2 + r^2 d\theta^2$$

$$x = r\cos\theta, y = r\sin\theta$$

Lengths of Geodesics (3D, polar coords)

↳ straight lines in a given geometry

<OR>

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

flat or Euclidean space:

$$d\ell^2 = dr^2 + r^2 d\Omega^2$$

elliptical or spherical space:

$$d\ell^2 = dr^2 + R^2 \sin^2 \frac{r}{R} d\Omega^2$$

hyperbolic space:

$$d\ell^2 = dr^2 + R^2 \sinh^2 \frac{r}{R} d\Omega^2$$

$$d\ell^2 = dr^2 + S_\kappa(r)^2 d\Omega^2$$

$$S_\kappa(r) = \begin{cases} R\sin\frac{r}{R} & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh\frac{r}{R} & (\kappa = -1) \end{cases}$$

Minkowski & Robertson-Walker Metrics

metrics define the distance between events in spacetime

Minkowski (no gravity: metric in SR)

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

Robertson-Walker (with gravity, if spacetime is homogeneous & isotropic)

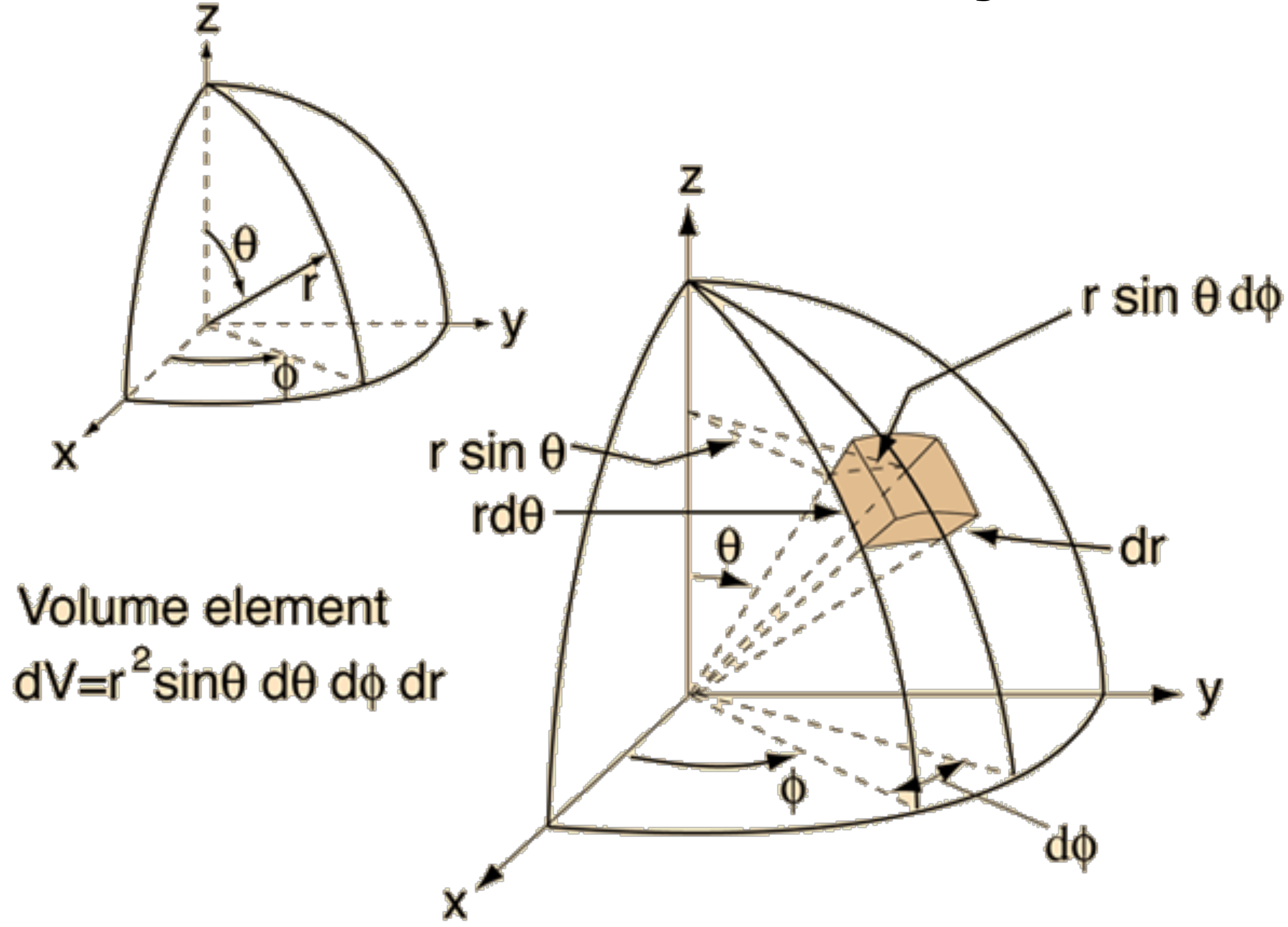
$$ds^2 = -c^2 dt^2 + a(t)[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

light travels along null geodesics, i.e.: $ds^2 = 0$

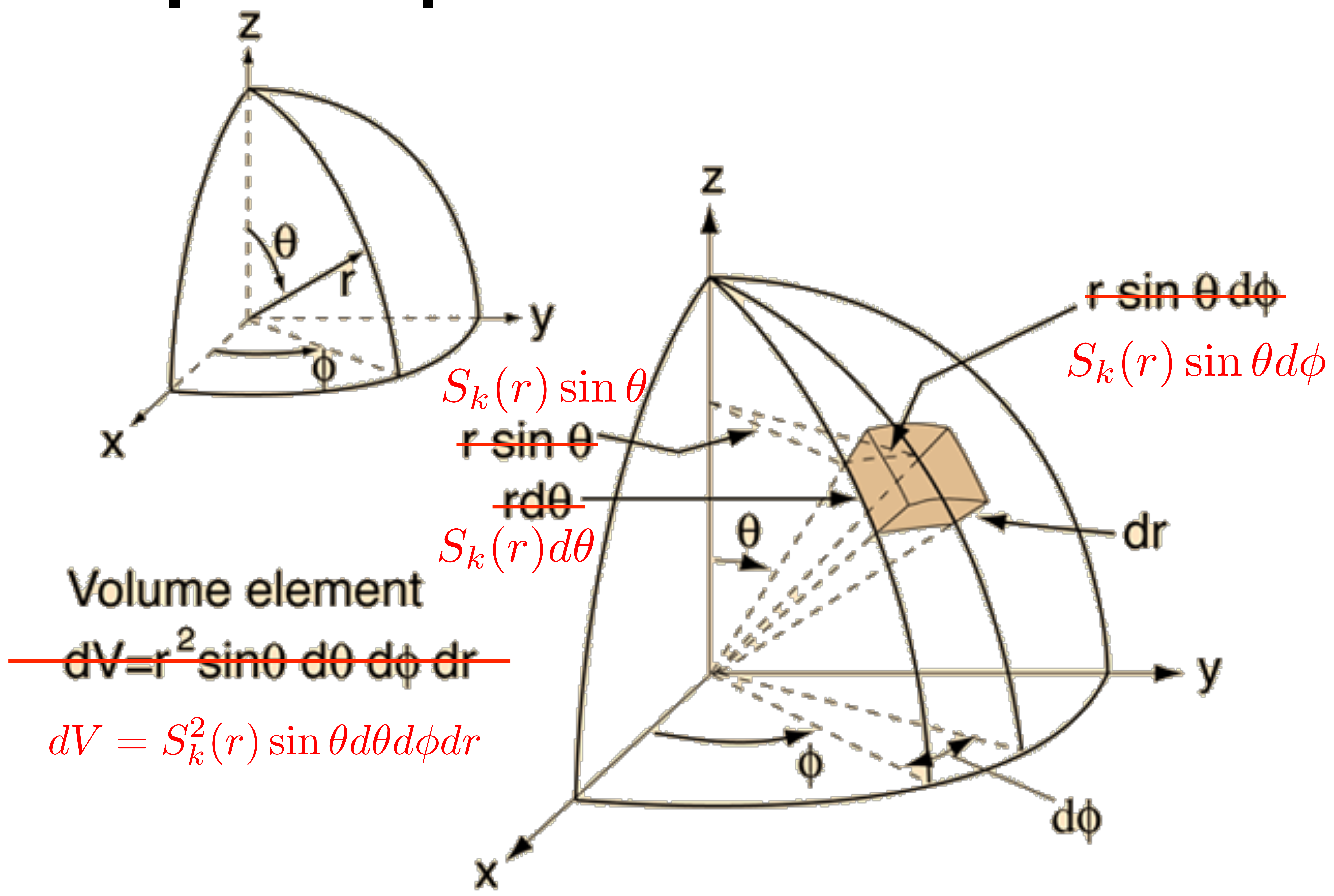
cosmological proper time or cosmic time

(r, θ, ϕ) comoving coordinates

Spherical Coordinate System



Spatial part of RW metric



At time t, $adr, \;\; aS_k(r)d\theta, \;\; aS_k(r)\sin\theta d\phi$

Proper Distance

In an expanding universe, how do we define the distance to something at a cosmological distance?

The distance between 2 objects at the same instant of time is given by the RW metric:
called the "proper distance"

$$ds = a(t)dr$$

$$d_p(t) = a(t)\int_0^r dr = a(t)r$$

$$\dot{d}_p = \dot{a}r = \frac{\dot{a}}{a}d_p$$

$$v_p(t_0) \equiv H_0 d_p(t_0) \rightarrow d_H(t_0) \equiv c/H_0$$

Redshift and Scale Factor

Proper distance is not usually a practical distance measure.
For example, you might rather want to know the distance light has traveled from a distant object so you know the "lookback time" or how far you're looking into the past.

Relatedly, we measure redshift, but would like to know how redshift is related to the change in scale factor between emission and observation, which is:

$$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}$$