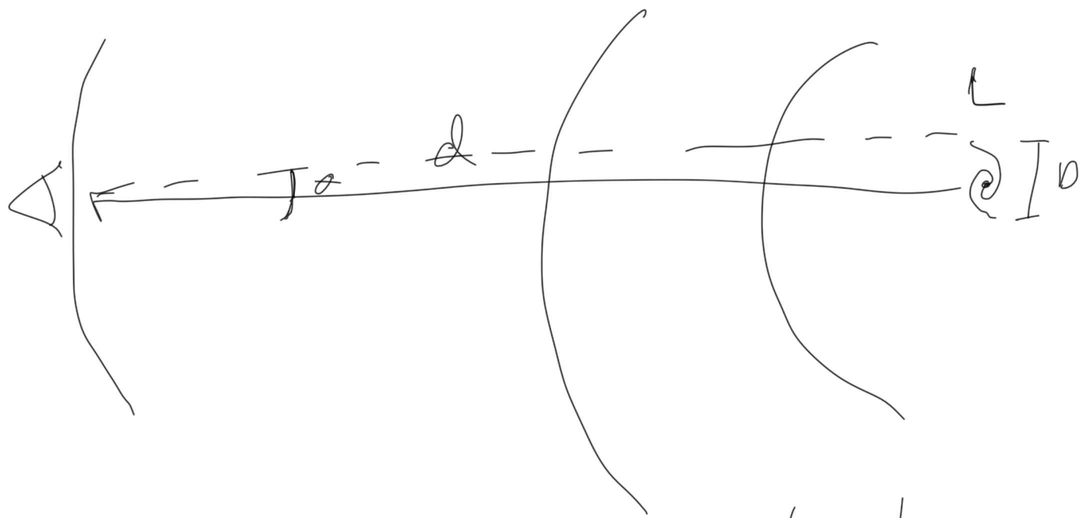
ASTR 4080 - Week 6



Putting out energy @ same rate L

isotropically

so energy per area changes as size of spherical surface changes

$$\text{flux} = \frac{L}{A} = \frac{L}{4\pi d^2}$$ inverse square law

Also, if obj. extended, $\tan\theta = \frac{D}{d} \approx \theta$

$$\boxed{\begin{aligned} d_L &= \sqrt{\frac{L}{4\pi f}} \\ d_A &= \frac{D}{\theta} \end{aligned}}$$

in flat, static universe, $d_L = d_A = d_p$

$d_p \rightarrow ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$

for d_L, the area of the expanding spherical shell depends on the metric

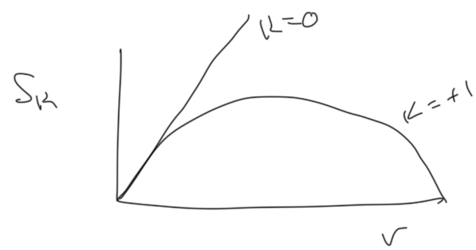
$$ds^2 = -c^2 dt^2 + a^2\left[dr^2 + S_k(r)^2 d\Omega^2\right]$$

$$S_k \begin{cases} R_0 \sin r/R_0 & k=+1 \\ r & k=0 \\ R_0 \sinh r/R_0 & k=-1 \end{cases}$$

Pick t_0 to define proper surface area
(ask what this means)

$$A_p(t_0) = 4\pi S_k(r)^2$$

In $k=+1$ universe, will obj. be brighter or fainter?



A_p is smaller, more light per unit area, so obj. appears <u>brighter</u> than in flat space

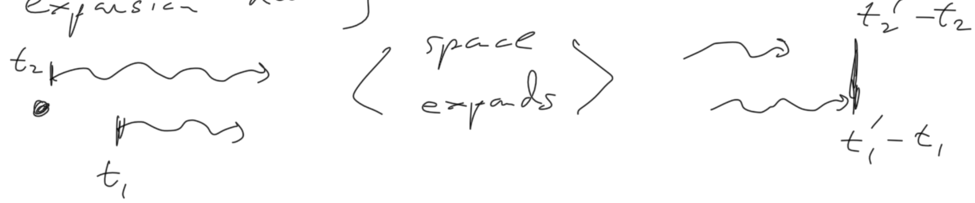
Is that the only effect? <u>Look @ the metric</u>

<u>2 effects</u>: redshifting due to expansion
+ delayed photon arrival times

1) $\lambda_0 = \frac{1}{a(t_e)} \lambda_e = (1+z)\lambda_e$

→ energy per photon drops by $1+z$

2) expansion during travel time



$\delta t_e = t_1 - t_2$

$d_p(t_e) = c\,\delta t_e$

$d_p(t_0) = (1+z)\, d_p(t_e)$

→ so rate of arriving photons lower by $1+z$

$$f = \frac{L}{4\pi S_k(r)^2 (1+z)^2} = \frac{L}{4\pi d_L^2}$$

$$\boxed{d_L = S_k(r)(1+z)}$$

$K = 0, \quad d_L = r(1+z) = d_p(t_0)(1+z)$

For $d_A = \frac{D}{\theta}$, θ is affected by geometry

$$ds = a(t_e)\, S_k(r)\, d\theta = D$$

$$d_A = \frac{D}{d\theta} = a(t_e)\, S_k(r) = \boxed{\frac{S_k(r)}{1+z} = d_A}$$

$$k=0, \quad d_A = \frac{d_p(t_0)}{1+z} = \frac{d_L}{(1+z)^2}$$

★ Show plots of d_L / d_A

(How to measure $a(t)$ nearby [in space & time])

Have a theoretical picture, but who knows if that's really right → want to make measurements that can test models

Assuming $a(t)$ varies smoothly, can do a Taylor expansion & ignore higher terms b/c ↑order derivatives small

$$a(t) \simeq a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0} (t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0} (t-t_0)^2 + \dots$$

$a(t_0) = 1$ by def., so can divide by it

$$\frac{a(t)}{a(t_0)} \simeq 1 + \left.\frac{\dot{a}}{a}\right|_{t=t_0} (t-t_0) + \frac{1}{2}\left.\frac{\ddot{a}}{a}\right|_{t=t_0} (t-t_0)^2$$

$$H_0 \equiv \left.\frac{\dot{a}}{a}\right|_{t=t_0} \rightsquigarrow 1 + H_0(t-t_0) + \frac{1}{2}\frac{\ddot{a}a}{\dot{a}^2} H_0^2 (t-t_0)^2$$

can define

$$q_0 \equiv -\left.\frac{\ddot{a}a}{\dot{a}^2}\right|_{t=t_0} = -\left.\frac{\ddot{a}}{aH^2}\right|_{t=t_0}$$

$$a(t) = 1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2$$

What is q_0? With the accel. eq., have $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} \sum_{i=1}^{N} \varepsilon_i (1+3w_i)$

Can put in terms of q_0 via

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2} \left[\frac{8\pi G}{3c^2 H^2}\right] \sum_{i=1}^{N} \varepsilon_i (1+3w_i)$$

Recall $\varepsilon_{crit} = \frac{3c^2 H^2}{8\pi G}$ & $\Omega_i = \frac{\varepsilon_i}{\varepsilon_{crit}}$

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2} \sum \Omega_i (1+3w_i)$$

$$\boxed{q_0 = -\frac{\ddot{a}}{aH^2}\bigg|_{t=t_0} = \frac{1}{2} \sum \Omega_{i,0} (1+3w_i)}$$

For the general case (rad., matter, Λ), get

$$\boxed{q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}}$$

Benchmark model, have $q_0 \approx -0.53$
($q_0 > 0$ is decel., < 0 is accel.)

To make sense of d_L or d_A, need the proper distance, which then connects to $a(t)$ / underlying cosmological params

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

Similar Taylor expansion for $\frac{1}{a(t)}$

$$\frac{1}{a(t)} \approx 1 - H_0(t-t_0) + \left(1+\frac{q_0}{2}\right)H_0^2(t-t_0)^2$$

(show this?)

Using & integrating, $d_p(t_0) = c(t_0 - t_e) + \frac{cH_0}{2}(t_0-t_e)^2$

But we measure z, so should use $z+1 = \frac{1}{a(t)}$

& solving for t: $t_0 - t_e \approx H_0\left[z - \left(1+\frac{q_0}{2}\right)z^2\right]$

And FINALLY,

$$\boxed{d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right)z^2\right] + \frac{cH_0}{2}\frac{z^2}{H_0} = \frac{cz}{H_0}\left[1 - \frac{1+q_0}{2}z\right]}$$

At low z, $d_L = d_p(1+z) = \frac{cz}{H_0}\left(1 - \frac{1+q_0}{2}z\right)(1+z)$

→ ignoring highest z orders, this becomes

$$\boxed{d_L \cong \frac{cz}{H_0}\left(1 + \frac{1-q_0}{2}z\right)}$$

And $d_A = d_p/(1+z)$

$$\boxed{d_A = \frac{cz}{H_0}\left(1 - \frac{3+q_0}{2}z\right)}$$

The no good, very bad, terrible magnitude system

$$\boxed{\begin{aligned} m &\equiv -2.5\log_{10}(f/f_{ref}) \\ f_{ref} &= 2.53\times10^{-8}\,\mathrm{W\,m^{-2}} \end{aligned}}$$

$$\boxed{\begin{aligned} M &\equiv -2.5\log_{10}(L/L_{ref}) \\ L_{ref} &= 78.7\,L_\odot \end{aligned}} \rightarrow M \Rightarrow \text{value of } m \text{ @ } 10\,\mathrm{pc}$$

so $M_\odot = 4.74$

$$\boxed{\begin{aligned} M &= m - 5\log_{10}\left(\frac{d_L}{10\,\mathrm{pc}}\right) \\ M &= m - 5\log_{10}(d_L/1\,\mathrm{Mpc}) - 25 \end{aligned}} \quad (\text{b/c } L = 4\pi d_L^2 f)$$

Optical people are crazy, so they measure distances in the distance modulus

$$m - M = 5\log_{10}(d_L/1\,\mathrm{Mpc}) + 25$$

Using expression for $d_L(H_0, q_0)$ & defining $h_{68} = \frac{H_0}{68\,\mathrm{km/s/Mpc}}$

$$\boxed{\begin{aligned} m - M \simeq\ & 43.23 - 5\log_{10} h + 5\log_{10} z \\ & + 1.086(1-q_0)z \end{aligned}}$$

From flux & known luminosity, can plot $m-M$ vs. z and measure both H_0 & q_0!