ASTR 4080 - Week 9

Early Universe radiation dominated

→ how does ε_{rad} vary w/ a?

→ how does $a(t)$ vary w/ t?

$a \propto t^{1/2}$

→ how does $T(a)$ behave?

$T \propto a^{-1}$

$a_{rm} \sim 3\times10^{-4}$, $t_{rm} \sim 50\,kyr$

RADIATION

$T \approx 10^{10} K \left(\frac{t}{1s}\right)^{-1/2}$

$kT \approx 1\ MeV$ ( )

$\langle E \rangle = 2.7\,kT(t)$

$= 3\ MeV \left(\frac{t}{1s}\right)^{-1/2}$

✱ LHC → $\sim 7\times10^6$ MeV [7 TeV]

↳ physics well-understood

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nomenclature: ${}^{A}E$ → $p = {}^{1}H$

$A =$ mass #

deuterium $D = {}^{2}H$

$1n + 1p$

nucleons → $Z = \#_p$, $N = \#_n$

$A = Z + N$

$Li \Rightarrow Z = 3$, $Be \Rightarrow Z = 4$

${}^{4}He \rightarrow Z = 2, N = 2$

☆ Go to slide on Nuclear Binding E

@ $t \ll 0.1$s, quarks + anti-q. created + destroyed, in balance

T falls until stray force takes over

@ $t \sim 0.1$s, nucleons have formed ($p + n$)

- however, neutrons outside nuclei are unstable + decay

$$n \to p + e^- + \bar{\nu}_e$$

$$N(t) = N_0 e^{-t/\tau_n}, \quad \tau_N = 898\,s$$

→ neutrons need to be in nucleus to survive

→ radiation $kT \sim 10$ MeV $> m_e c^2$

so $e^- + e^+$ still created/destroyed

$$\gamma + \gamma \rightleftharpoons e^- + e^+$$

ν_e + e^-/e^+ interact as well

$$n + \nu_e \rightleftharpoons p + e^- \quad / \quad n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

B/c of all the interactions, all particles in equilibrium ($kT \sim 3$ MeV)

Can use prev. expression for particle ratios

Next Slide $(m_n - m_p)c^2 = 1.3\,\text{MeV}$

for $kT \gg 1.3$ MeV, $\frac{n_n}{n_p} \sim 1$

but declines as T drops

At some T, the n+p decouple b/c they require a ν_e mediator
↳ ν interact via the weak force

$$\sigma_w \sim 10^{-47} m^2 \left(\frac{kT}{1\,\text{MeV}}\right)^2$$

$\rightarrow$ $T \propto a^{-1} \propto t^{-1/2}$, so $\sigma_w \propto t^{-1}$

interaction rate $\Gamma = n_\nu \sigma_w c \propto t^{-3/2} t^{-1} = t^{-5/2}$
(since $n_\nu \propto a^{-3} \propto t^{-3/2}$)

$H \propto t^{-1}$, so Γ will intersect H at which point ν no longer interact

When this occurs ($\Gamma \approx H$), n/p can't find any ν to interact with, so even though there's enough energy to convert b/t them, this process stops

↳ ratio "freezes out"

$kT_{freeze} \sim 0.8\ MeV\ (9\times10^9 K) : t_{freeze} \sim 1s$

$$\frac{n_n}{n_p} \approx e^{-1.29/0.8} = 0.2$$

$\tau_n = 880s$, so ratio holds for a few minutes

★ Explains why universe mostly H → not enough neutrons to combine

p+n → just σ_{np} ‖ p+p → repel, must overcome Coulomb barrier (eff. reduced σ_{pp})

$p + p \rightleftharpoons {}^2He$ (diproton or He-2)

↳ $\tau \sim 10^{-23}$ s back to $p+p$

↳ $\tau_{weak} \geq 10^{-2}$ s : $\rightarrow D + e^+ + \nu_e$
(actual value unknown)

→ this can produce D in the sun,
but not enough time (< 1 hr) in BB

left w/ 2n for every 10p → max ^4He
that can be made (relative to H)
is 1 ^4He for every 8p

✡ Start from here

$$Y_{max} = \frac{\rho(^4He)}{\rho_{tot}} = \frac{4}{12} = \frac{1}{3}$$

Y_p (measured) $= 0.24 < Y_{max}$ ✓

Why? — some n might decay if they don't find a p quickly enough
- may not end up in He : stuck in D
- may get fused in ↑ element (Li, Be)

Deuterium ++

$t = 2s$ $\quad \frac{n_n}{n_p} = 0.2 \quad Y_{max} = \frac{1}{3}$

ν's have decoupled, so can't be used, but γs still interacting

$$p + n \rightleftharpoons D + \gamma$$

$$B_D = (m_n + m_p - m_D)c^2 = 2.22 MeV$$

$E_\gamma > 2.22$ MeV can destroy the D

★ same process as w/ atoms + recombination

2. What equation determines their relative #s? Saha Eq.!

Not identical, but same form:

$$\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi \hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right)$$

↑ T : free n + p $\left(\frac{n_n}{n_p n_n} \to 0\right)$

↓ T : D preferred exponentially

When does nucleosynthesis happen?

↳ isn't a single instant in time

Define as when $\frac{1}{2}$ n are in D, or

$$\frac{n_D}{n_n} = 1$$

→ move n_p to other side, just need an expression for n_p

$$n_p \approx 0.8\, n_{baryons} = 0.8\,\eta\, n_\gamma$$

(known well)

$n_\gamma = 0.2436 \left(\frac{kT}{\hbar c}\right)^3$ → integral of BB equation

$$\Rightarrow \quad \frac{n_D}{n_n} \approx 6.5\,\eta \left(\frac{kT_{nucl}}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT_{nucl}}\right) \simeq 1$$

$$T_{nucl} \approx 7.6\times10^8 K \;/\; \frac{66\,keV}{k} \;/\; B_D/34$$

↳ occurs @ $t = 200\,s$

200s is not $\ll \tau_n = 880s$

Let's ignore the gradual creation of p + assume all forms @ 200s

↳ $\#_n$ will have decreased

@ ~1s, have $\frac{n_n}{n_p} = \frac{1}{5}$

ratio changes as $n \to p + e^- + \nu_e$

$$f = \frac{n_n + \frac{\partial n}{\partial t}}{n_p + \frac{\partial p}{\partial t}}, \quad \frac{\partial n}{\partial t} = -\left(n_n - n_n e^{-t/\tau_n}\right).$$

$$\frac{\partial p}{\partial t} = -\frac{\partial n}{\partial t}$$

$$f = \frac{n_n e^{-t/\tau_n}}{n_p + n_n - n_n e^{-t/\tau_n}} = \frac{e^{-t/\tau_n}}{n_p/n_n + 1 - e^{-t/\tau_n}}$$

$t = 200, \tau_n = 880, \frac{n_p}{n_n} = 5$

↳ ≈ 0.15

Est. Y_{max} was too high, should be

$$Y_{max} = \frac{0.15}{0.2}\,\frac{1}{3} \approx \frac{1}{4}$$

~~*~~ to slides