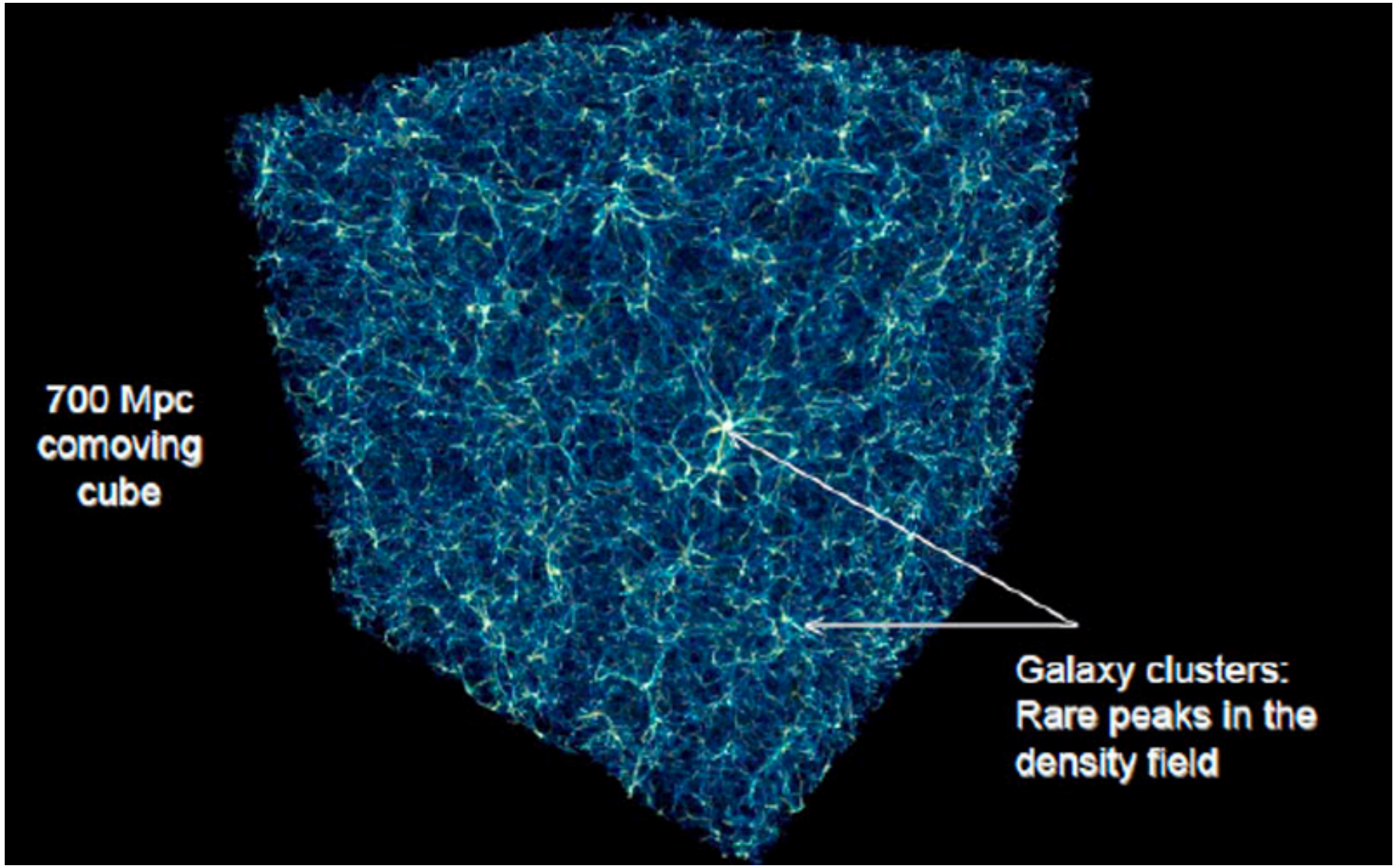
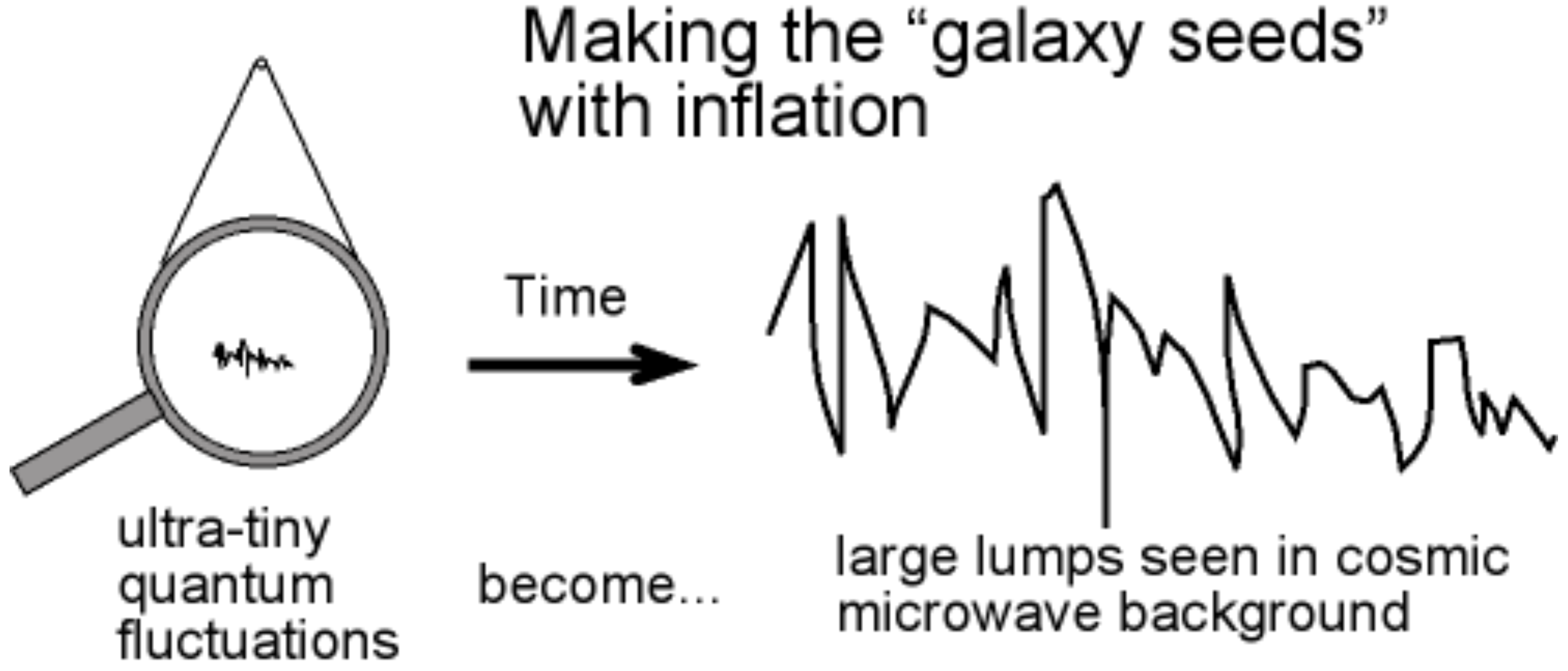
# Structure Formation: Gravitational Instability

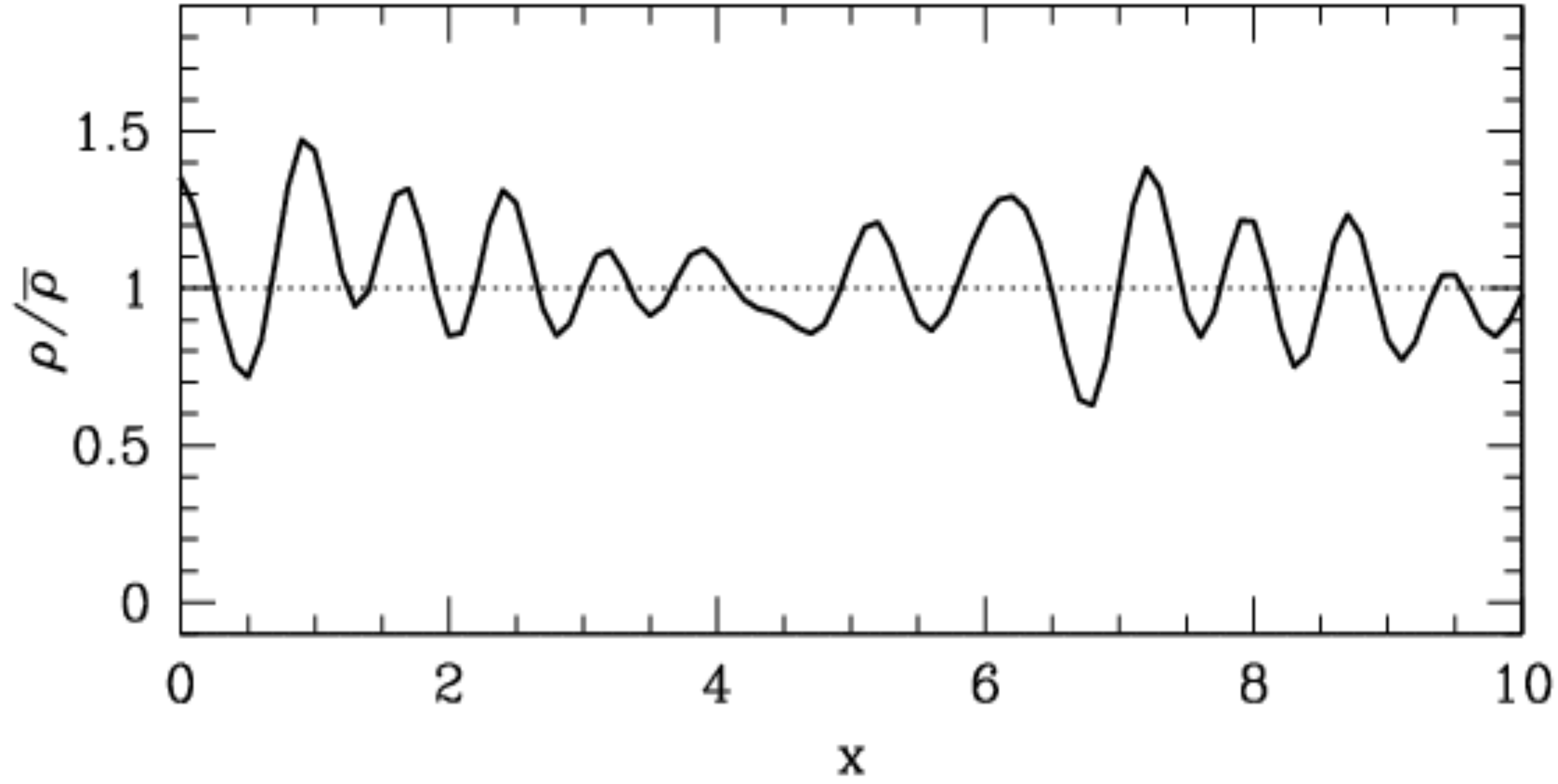



ASTR/PHYS 4080:
Intro to Cosmology
Week 13

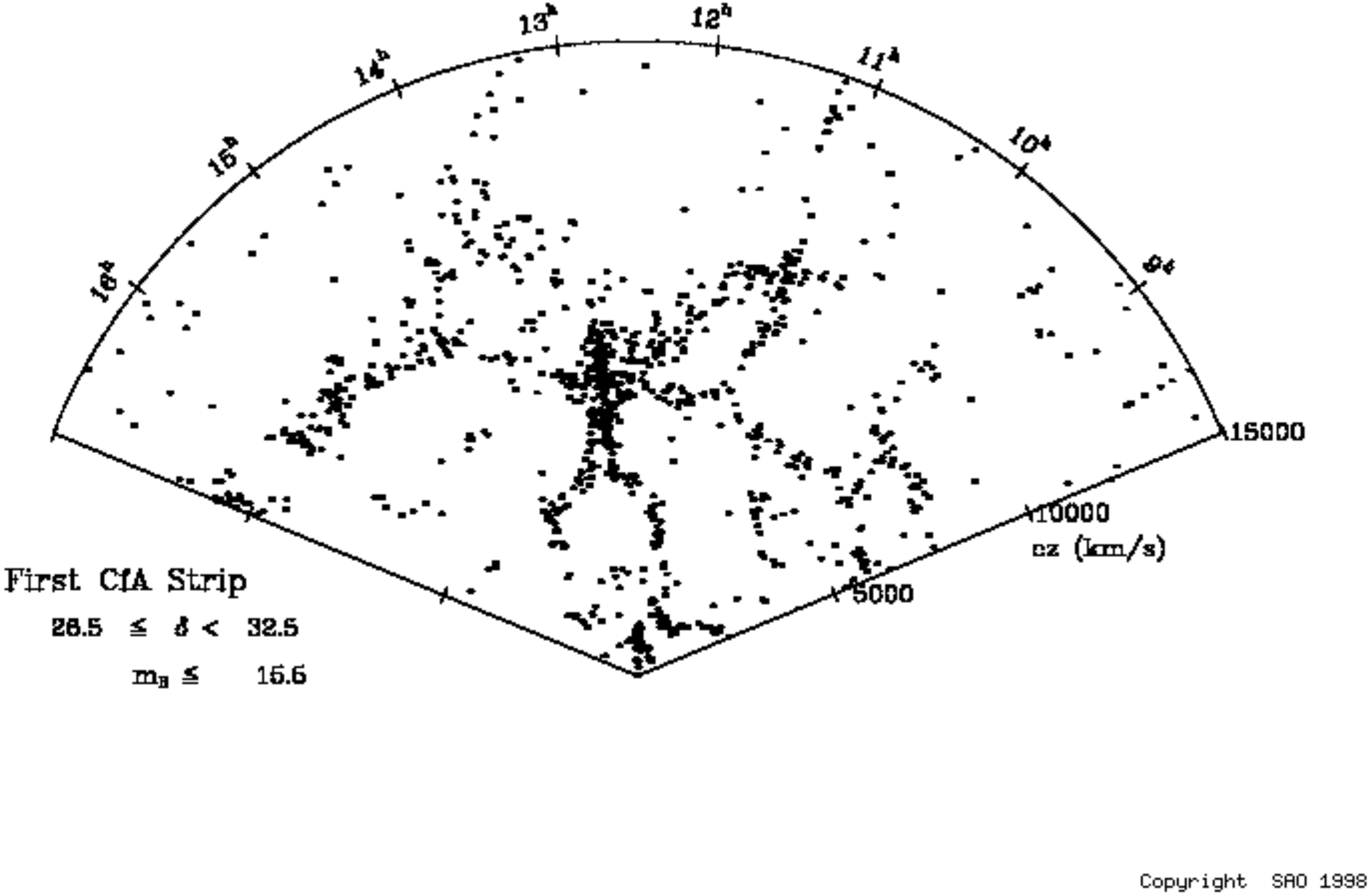
# Primordial Density Fluctuations from Inflation

- isentropic/adiabatic fluctuation, equal fluctuation in all forms of energy (photons, neutrinos, DM, baryons)
  ⇒ perturbation to spacetime curvature
- quantum fluctuation (of a weakly coupled field)
  ⇒ Gaussian fluctuation
  - distribution of fluctuation in space P(δ), Gaussian
  - joint distribution $P(\delta_1, \delta_2, \ldots, \delta_n)$ at points $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$, multi-variate Gaussian

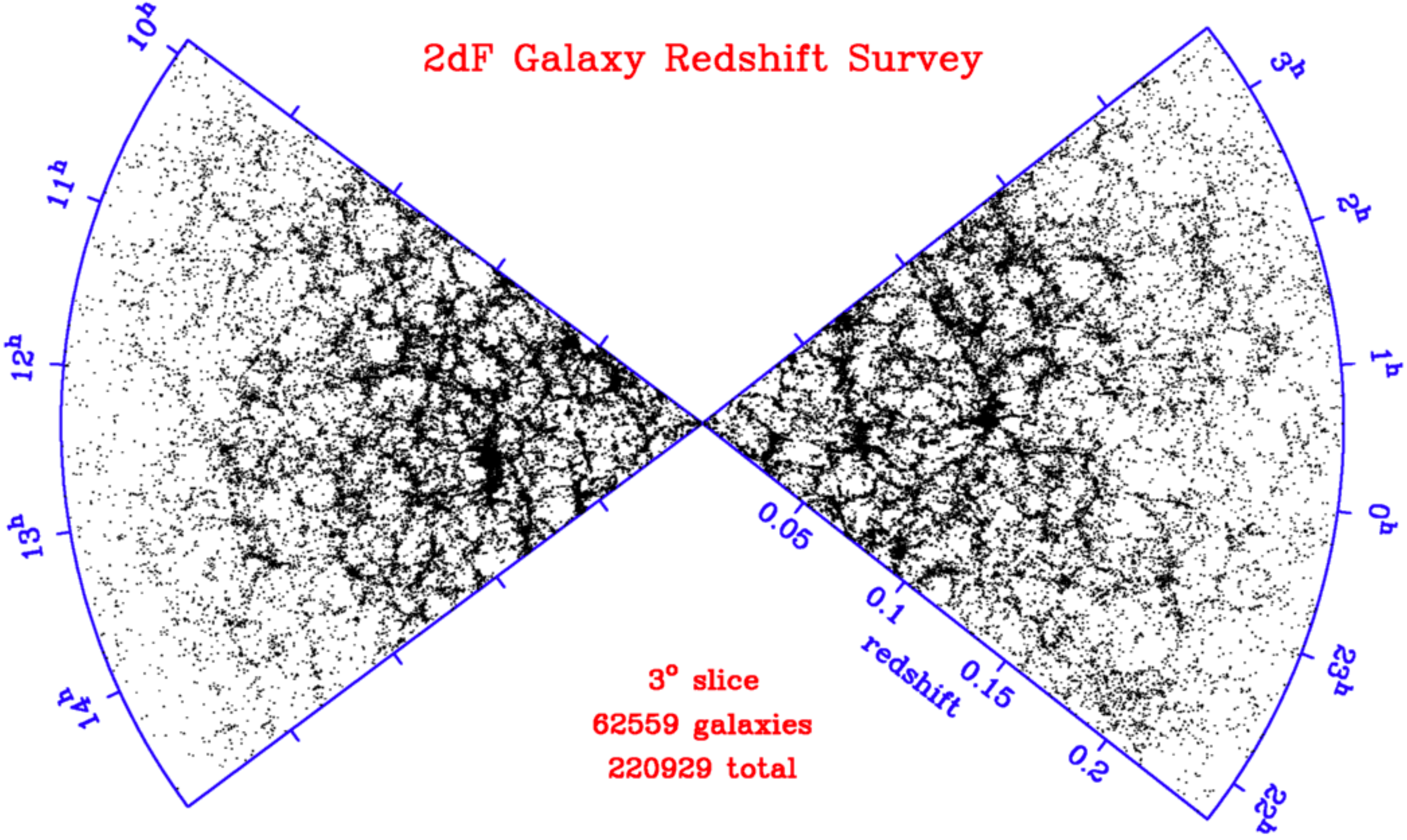


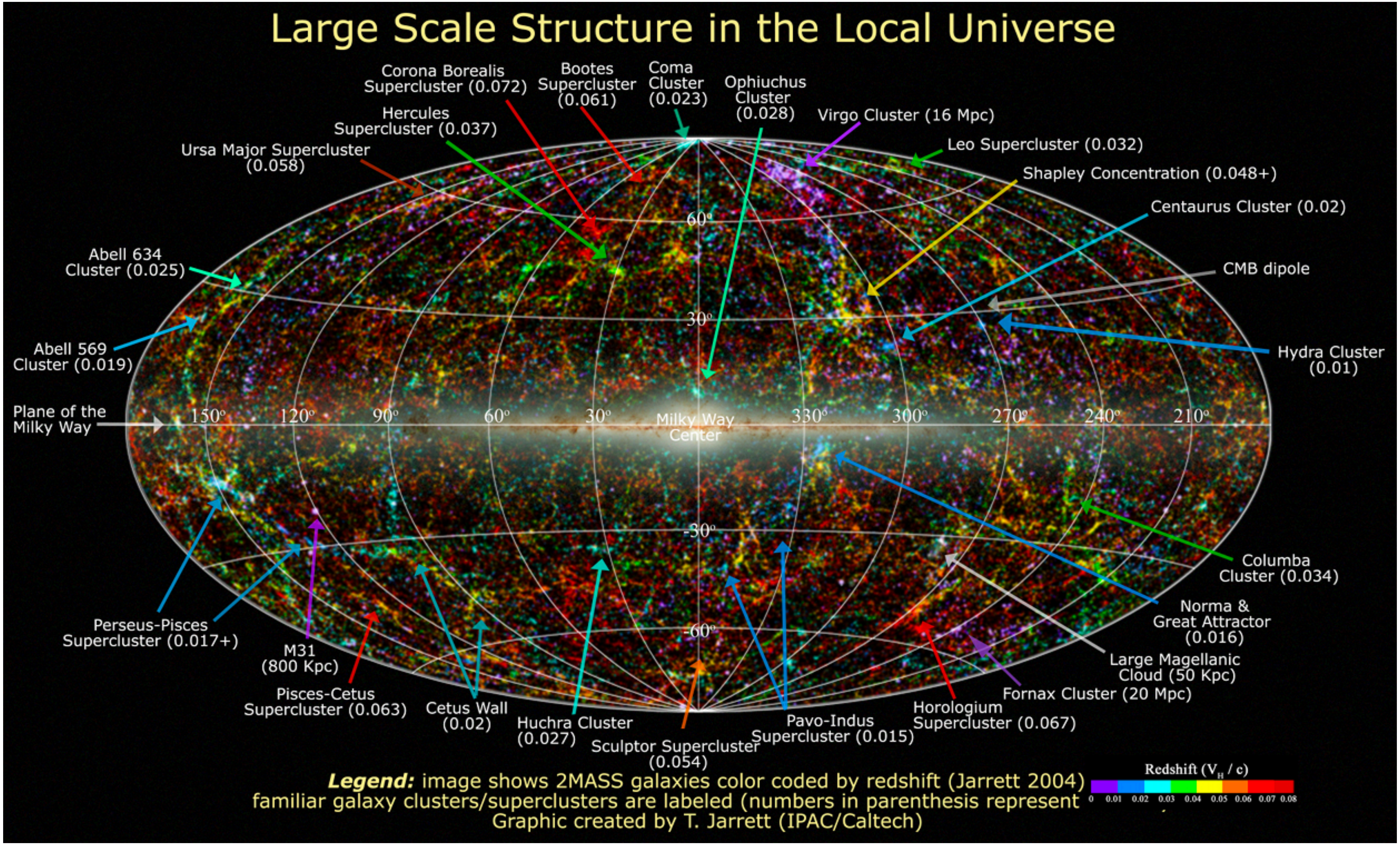



# Coma Cluster



# Galaxy Surveys

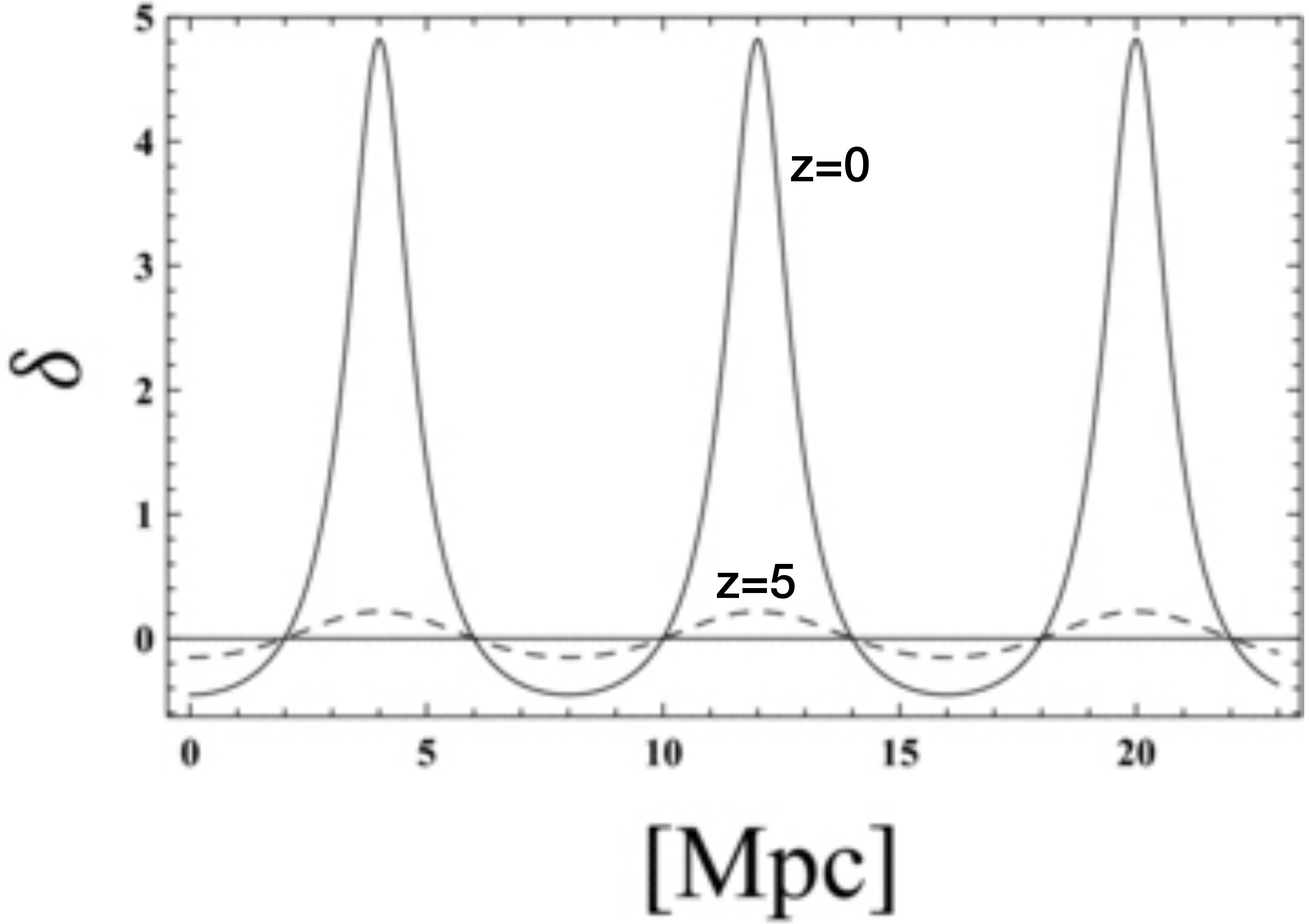


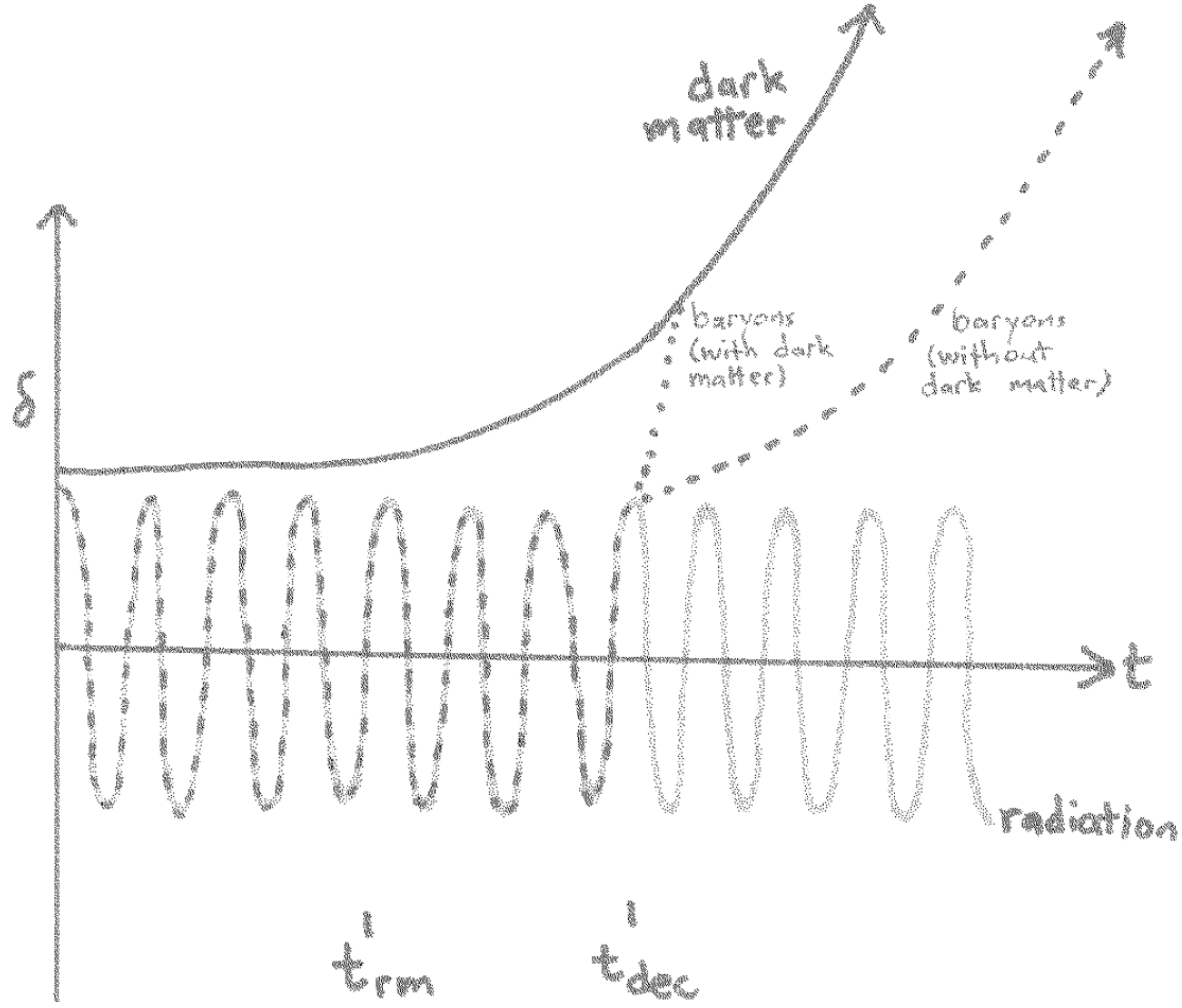

Large Scale Structure in the Local Universe
Corona Borealis Supercluster (0.072)
Bootes Supercluster (0.061)
Coma Cluster (0.023)
Ophiuchus Cluster (0.028)
Virgo Cluster (16 Mpc)
Hercules Supercluster (0.037)
Leo Supercluster (0.032)
Ursa Major Supercluster (0.058)
Shapley Concentration (0.048+)
Centaurus Cluster (0.02)
Abell 634 Cluster (0.025)
CMB dipole
Abell 569 Cluster (0.019)
Hydra Cluster (0.01)
Plane of the Milky Way
Milky Way Center
60°
30°
-30°
-60°
150°
120°
90°
60°
30°
330°
300°
270°
240°
210°
Columba Cluster (0.034)
Norma & Great Attractor (0.016)
Perseus-Pisces Supercluster (0.017+)
M31 (800 Kpc)
Large Magellanic Cloud (50 Kpc)
Fornax Cluster (20 Mpc)
Pisces-Cetus Supercluster (0.063)
Cetus Wall (0.02)
Huchra Cluster (0.027)
Sculptor Supercluster (0.054)
Pavo-Indus Supercluster (0.015)
Horologium Supercluster (0.067)
Legend: image shows 2MASS galaxies color coded by redshift (Jarrett 2004)
familiar galaxy clusters/superclusters are labeled (numbers in parenthesis represent
Graphic created by T. Jarrett (IPAC/Caltech)
Redshift ($V_H$ / c)
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08

# Consider small initial fluctuations in density

$$\bar{\varepsilon}(t) \equiv \frac{1}{V}\int_V \varepsilon(\vec{r},t)d^3r$$

$$\delta(\vec{r},t) \equiv \frac{\varepsilon(\vec{r},t) - \bar{\varepsilon}(t)}{\bar{\varepsilon}(t)}$$




dark matter
baryons (with dark matter)
baryons (without dark matter)
δ
t
radiation
$t_{rm}$
$t_{dec}$

# Power spectrum of density fluctuations

$$ds^2 = -c^2dt^2 + a(t)^2[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

$$(r, \theta, \phi)$$

comoving coordinates

Like CMB temperature fluctuations, can decompose density fluctuations into components: while we used spherical harmonics for the CMB (surface of a sphere), density fluctuations are 3D inside a volume, so more appropriate to use 3D Fourier components

$$\delta(\vec{r}) = \frac{V}{(2\pi)^3} \int \delta_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}} d^3k$$

where each component obeys

$$\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} - \frac{3}{2}\Omega_m H^2 \delta_{\vec{k}} = 0$$

$$\left(|\delta_{\vec{k}}| \ll 1,\ \lambda_{\rm J} < a(t)2\pi/k < c/H\right)$$

- scales must be larger than the Jeans length to collapse
- if they're larger than the Hubble distance, then collapse proceeds differently

# Power spectrum of density fluctuations

Power spectrum defined to be the mean squared amplitude of the Fourier components:

$$P(k) = \langle |\delta_{\vec{k}}|^2 \rangle$$

Gaussian field: each component uncorrelated and random, drawn from the Gaussian distribution

$$p(\delta) = \frac{1}{\sigma_\delta \sqrt{2\pi}} \exp\left(-\frac{\delta^2}{2\sigma_\delta}\right)$$

$$\sigma_\delta^2 = \frac{V}{2\pi^2} \int_0^\infty P(k) k^2 dk$$

Inflation predicts this (random quantum fluctuations) and a power law power spectrum (with n=1)

$$P(k) \propto k^n$$

# Power spectrum of density fluctuations

This scenario means that if you sample the universe at random places within spherical volumes of radius *r* (containing mass *M* on average), the spread in masses behaves like:

$$\frac{\delta M}{M} \equiv \left\langle \left( \frac{M - \langle M \rangle}{\langle M \rangle} \right)^2 \right\rangle^{1/2} \propto r^{-(3+n)/2} \propto M^{-(3+n)/6}$$

sigma-8: amplitude of density fluctuations

$$\frac{\delta M}{M} = \sigma(M) = \sigma_8 \left( \frac{M}{M_8} \right)^{-(3+n)/6}$$

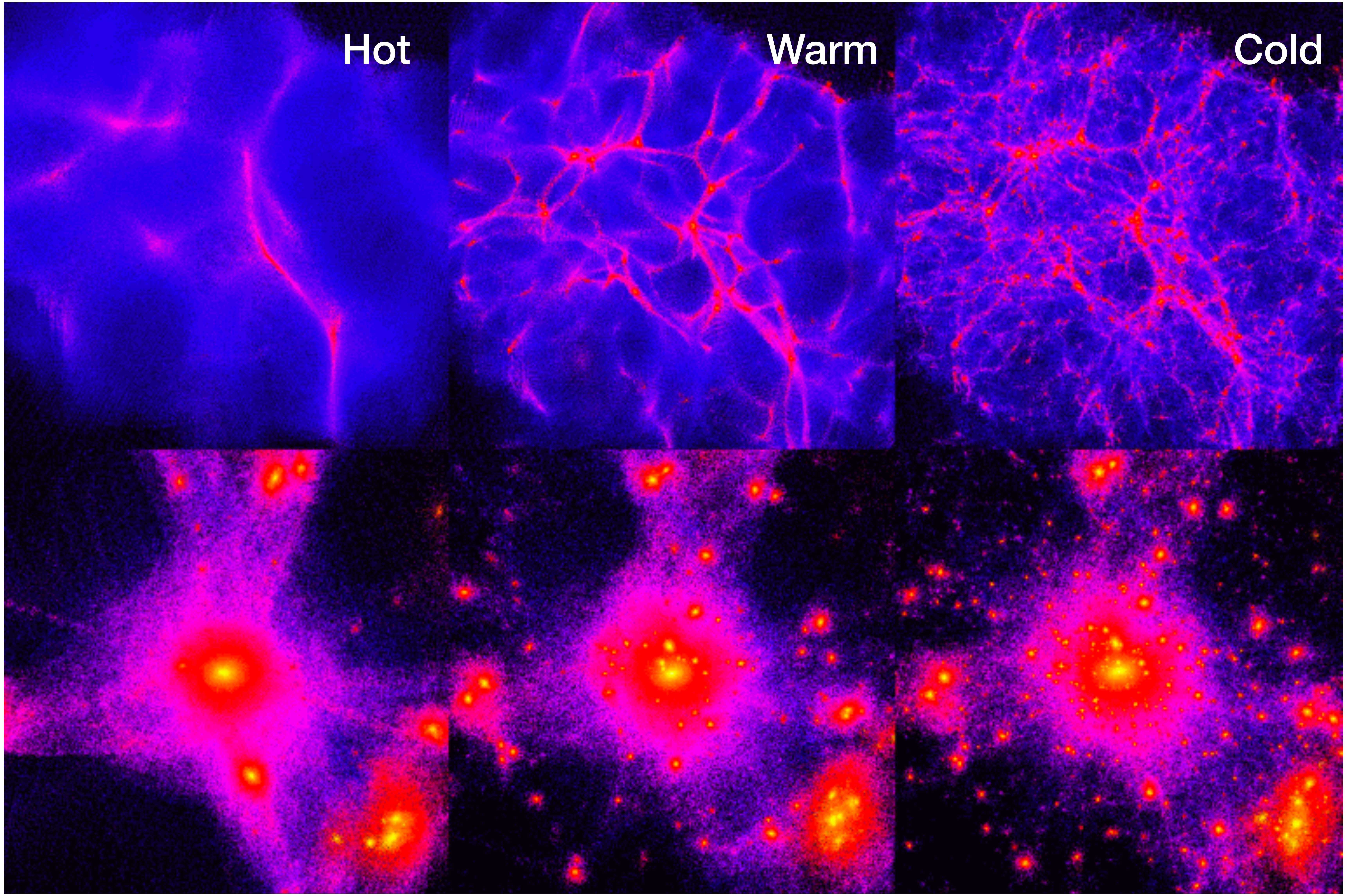
(Not quite the same sigma as on the previous slide)

$$\frac{\delta M}{M}$$

$$M_8 = \frac{4\pi}{3} (8\ h^{-1}\ \mathrm{Mpc})^3 \bar{\rho}$$

$$n = 0.97 \pm 0.01,\ \ \sigma_8 \sim 0.8$$

# Temperature of the Dark Matter




velocity of particles compared to the speed of light

relativistic at time of collapse (like neutrinos): hot

non-relativistic at time of collapse (like WIMPs): cold

fast motions wipe out initial overdensities on small scales: “free-streaming”

# Temperature of the Dark Matter

How do galaxies form if dark matter is hot vs. cold?

MW-like galaxy mass is contained within a Hubble volume before decoupling, in the radiation-dominated era (*a*~4e-6, *t*~12 yr, *kT*~60 eV)

Particle total energies relative to their rest mass determine whether they are relativistic or not

$$\text{hot} : mc^2 \ll 3kT \qquad \text{cold} : mc^2 \gg 3kT$$

For hot DM, density fluctuations are wiped out below a size determined by the horizon distance when the particles become non-relativistic:

$$d_{\rm min} \approx 13 \text{ kpc} \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-2}$$

comoving: $$r_{\rm min} = \frac{d_{\rm min}}{a(t_h)} \approx \left(\frac{T_h}{2.73 \text{ K}}\right) d_{\rm min} \approx 55 \text{ Mpc} \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-1}$$

$$M_{\rm min} = \frac{4\pi}{3} r_{\rm min}^3 \rho_{\rm m,0} \approx 2.7 \times 10^{16} \text{ M}_\odot \left(\frac{m_h c^2}{3 \text{ eV}}\right)^{-3}$$

all structure below superclusters wiped out if DM hot: superclusters old, galaxies younger

# Temperature of the Dark Matter

How do galaxies form if dark matter is hot vs. cold?

MW-like galaxy mass is contained within a Hubble volume before decoupling, in the radiation-dominated era ($a$~4e-6, $t$~12 yr, $kT$~60 eV)

Particle total energies relative to their rest mass determine whether they are relativistic or not

$$\text{hot} : mc^2 \ll 3kT \qquad \text{cold} : mc^2 \gg 3kT$$

For cold DM, density fluctuations cannot grow quickly within a Hubble distance ($\delta \propto \ln t$), but growth at larger scales can proceed once the particles decouple from radiation

For a WIMP-like particle that decouples at 1s (a~3e-10, c/H~2ct~6e8m), scales > 60 pc can grow as long as they remain larger than the Hubble distance, at which time their growth is slowed until the end of the radiation era (90 Mpc scales)

Superclusters grow immediately and never stop, but less massive structures have a pause in their growth until matter dominates, at which point all scales can grow

# Temperature of the Dark Matter

# Temperature of the Dark Matter

Hot dark matter alone gives a bad fit to observations (galaxies are detected up to z~10, superclusters forming now) - top-down scenario doesn't work

Cold dark matter predicts small structures to form first (bottom-up formation), with smaller things merging together to build larger structures: hierarchical structure formation

Some hot DM acceptable - measuring the large scale structure and comparing to the theoretical power spectrum yields the constraint:

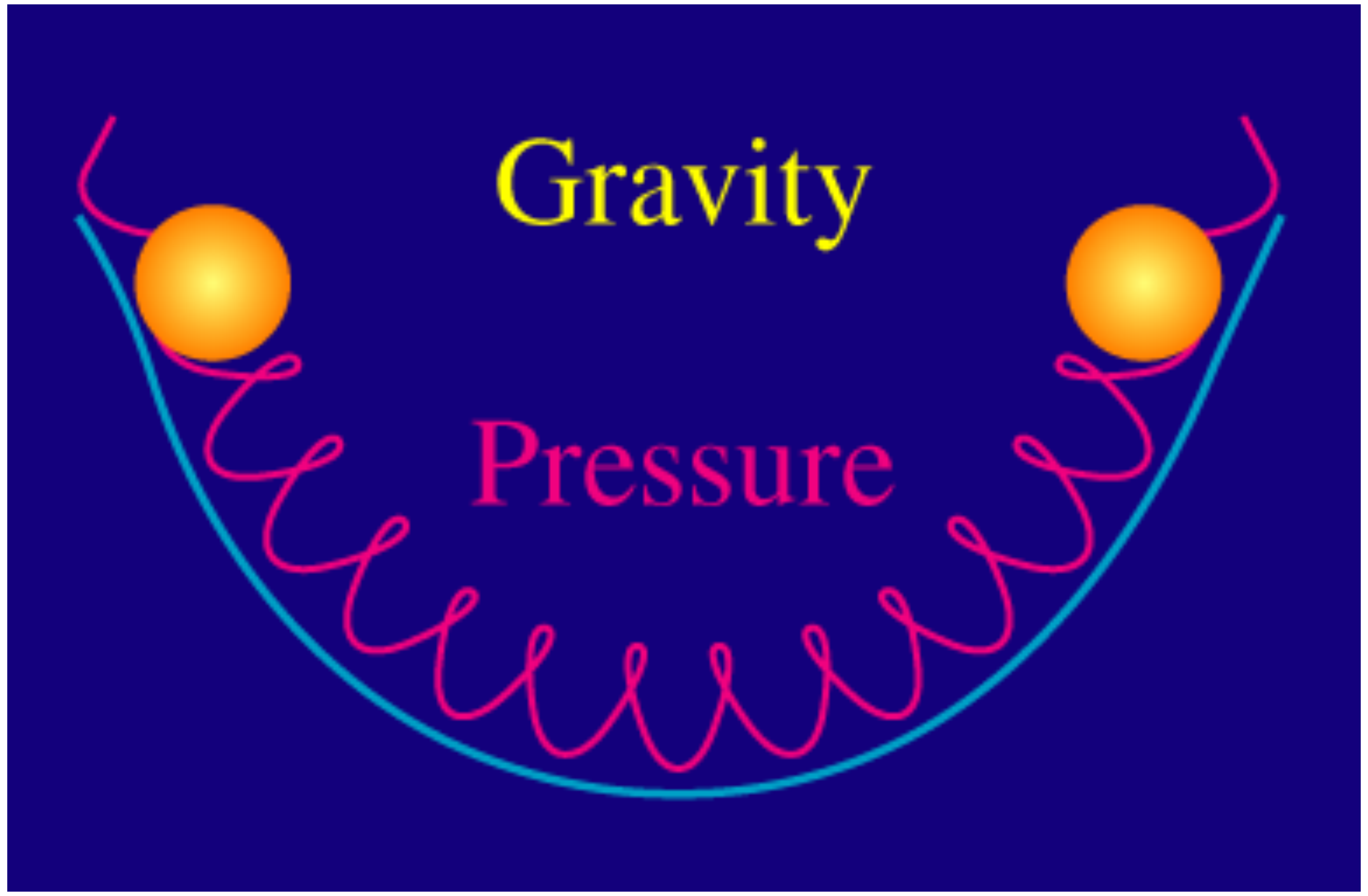
$$\Omega_{\mathrm{HDM},0} \leq 0.007$$

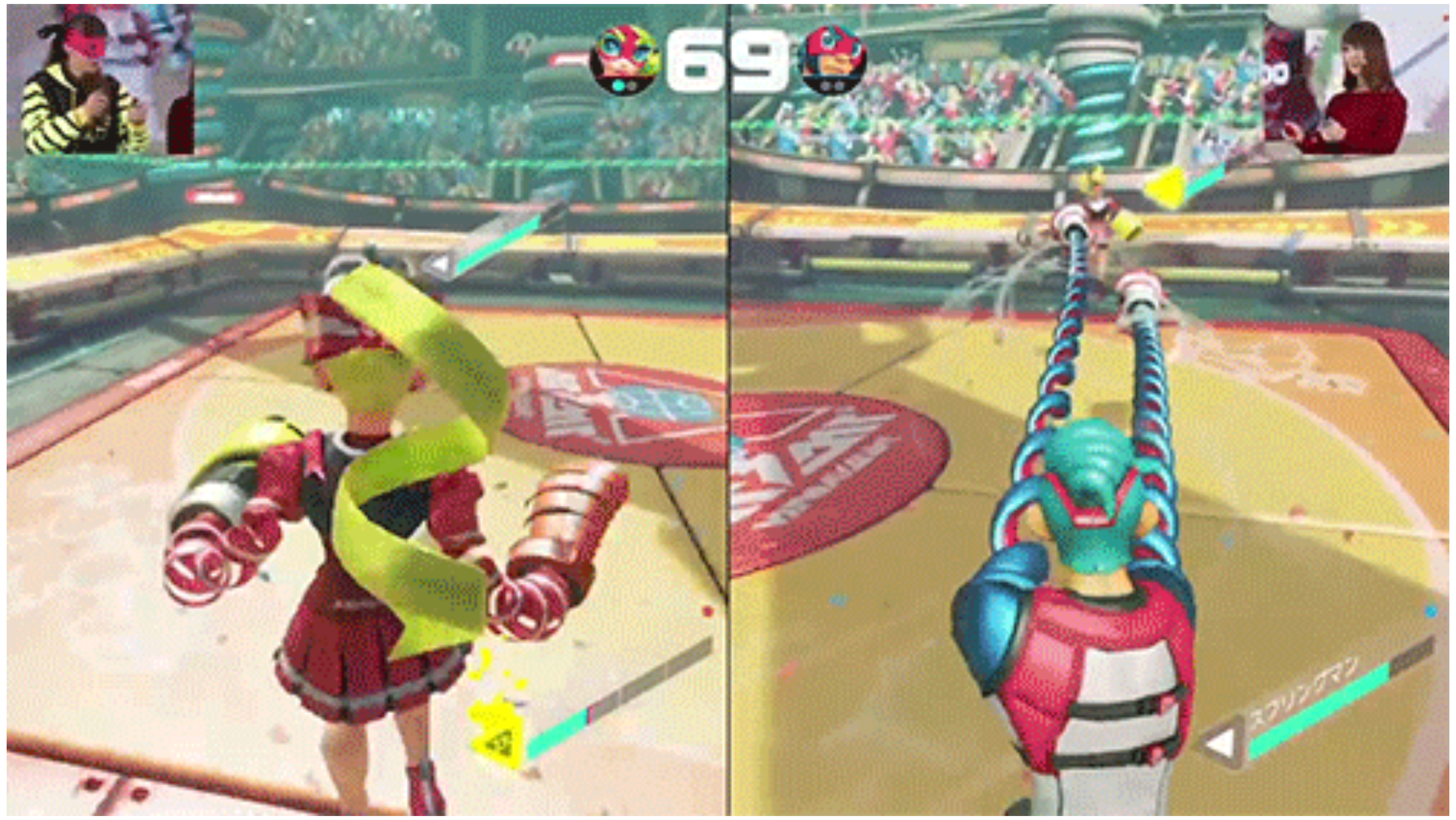
We now know neutrinos have mass, so their summed mass is constrained by this to be:

$$[m(\nu_e) + m(\nu_\mu) + m(\nu_\tau)]c^2 \leq 0.3 \text{ eV}$$

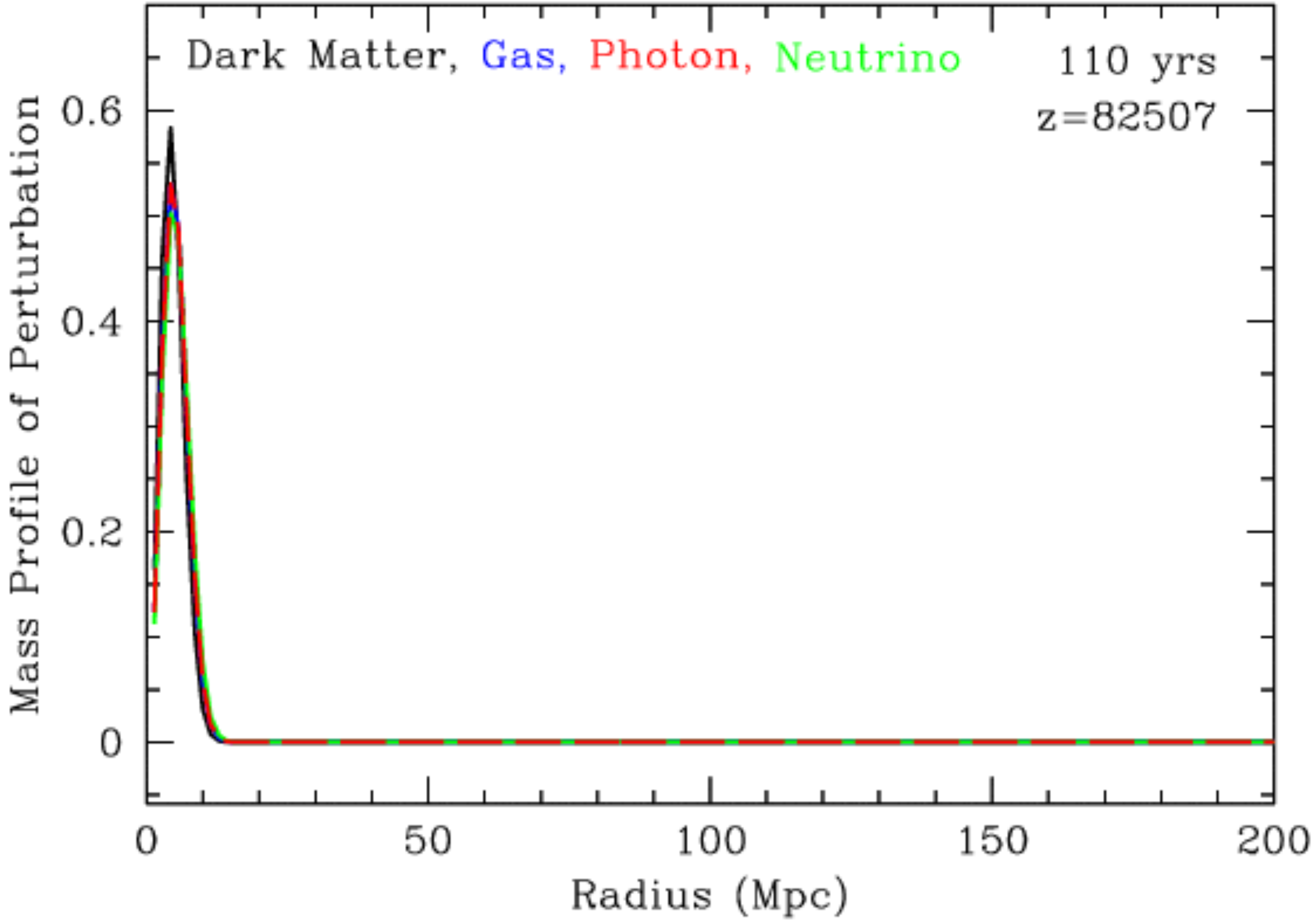
The benchmark model is typically referred to as $\Lambda\mathrm{CDM}$

# Baryon Acoustic Oscillations

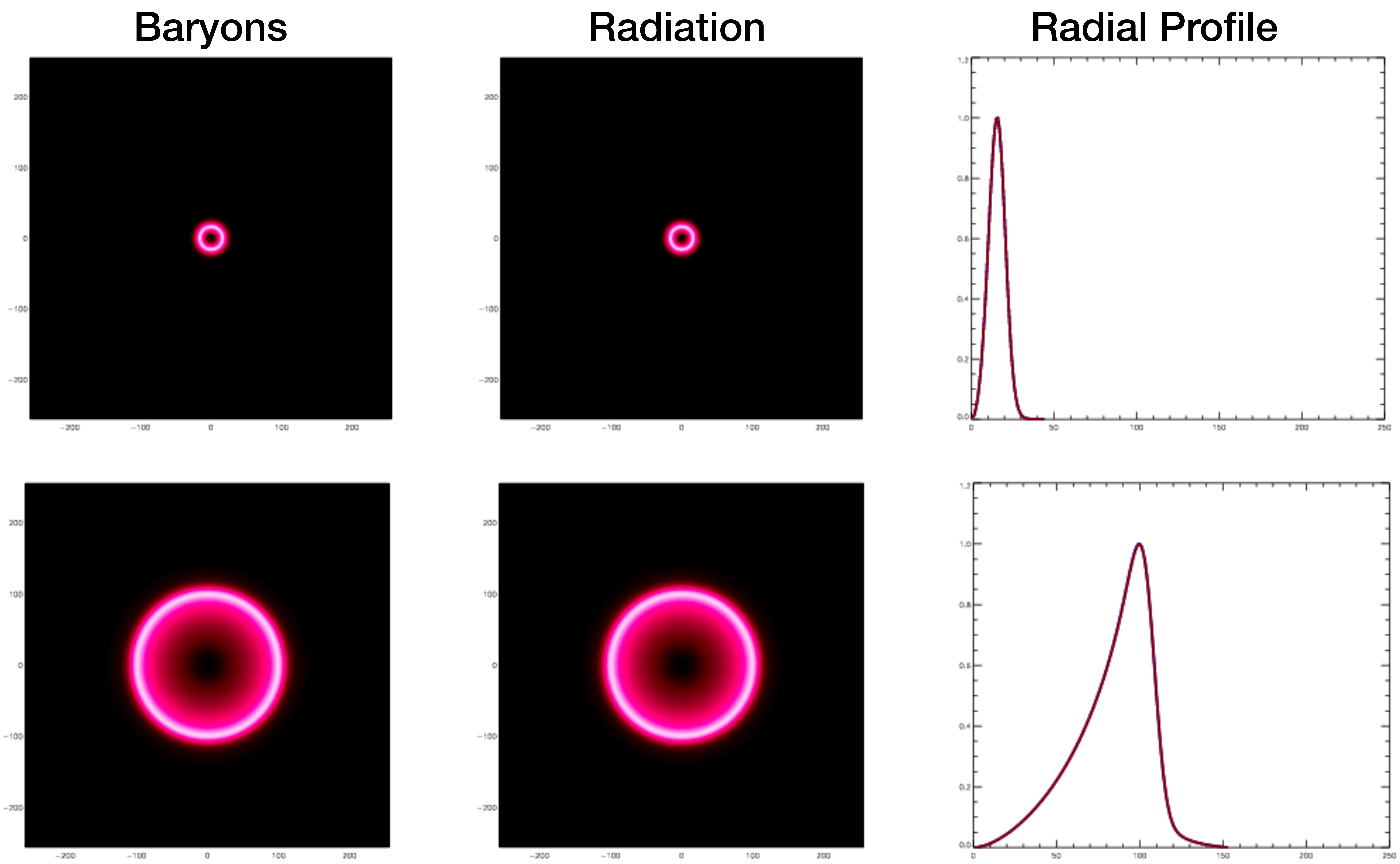


# Baryon Acoustic Oscillations

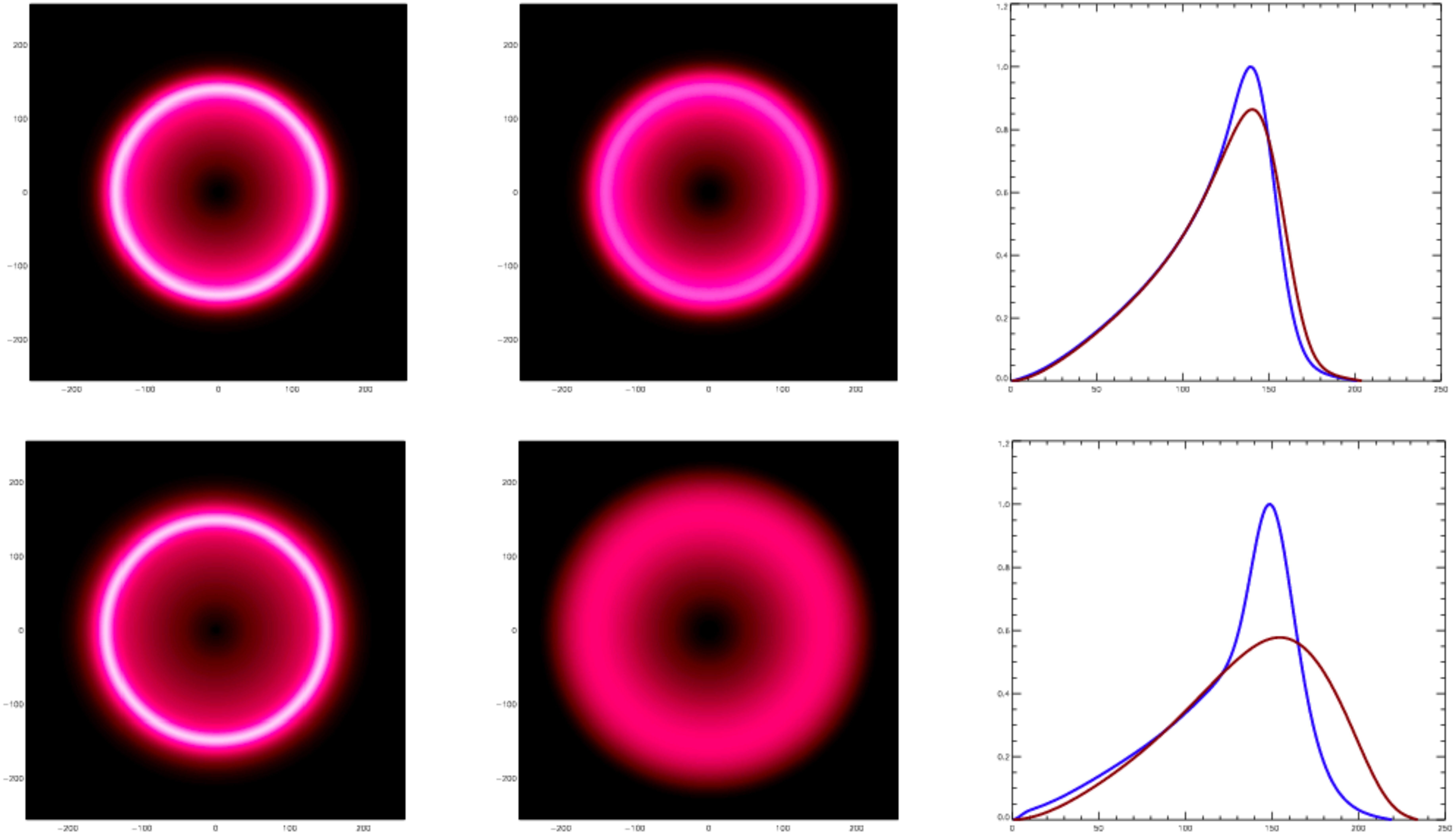


# Baryon Acoustic Oscillations



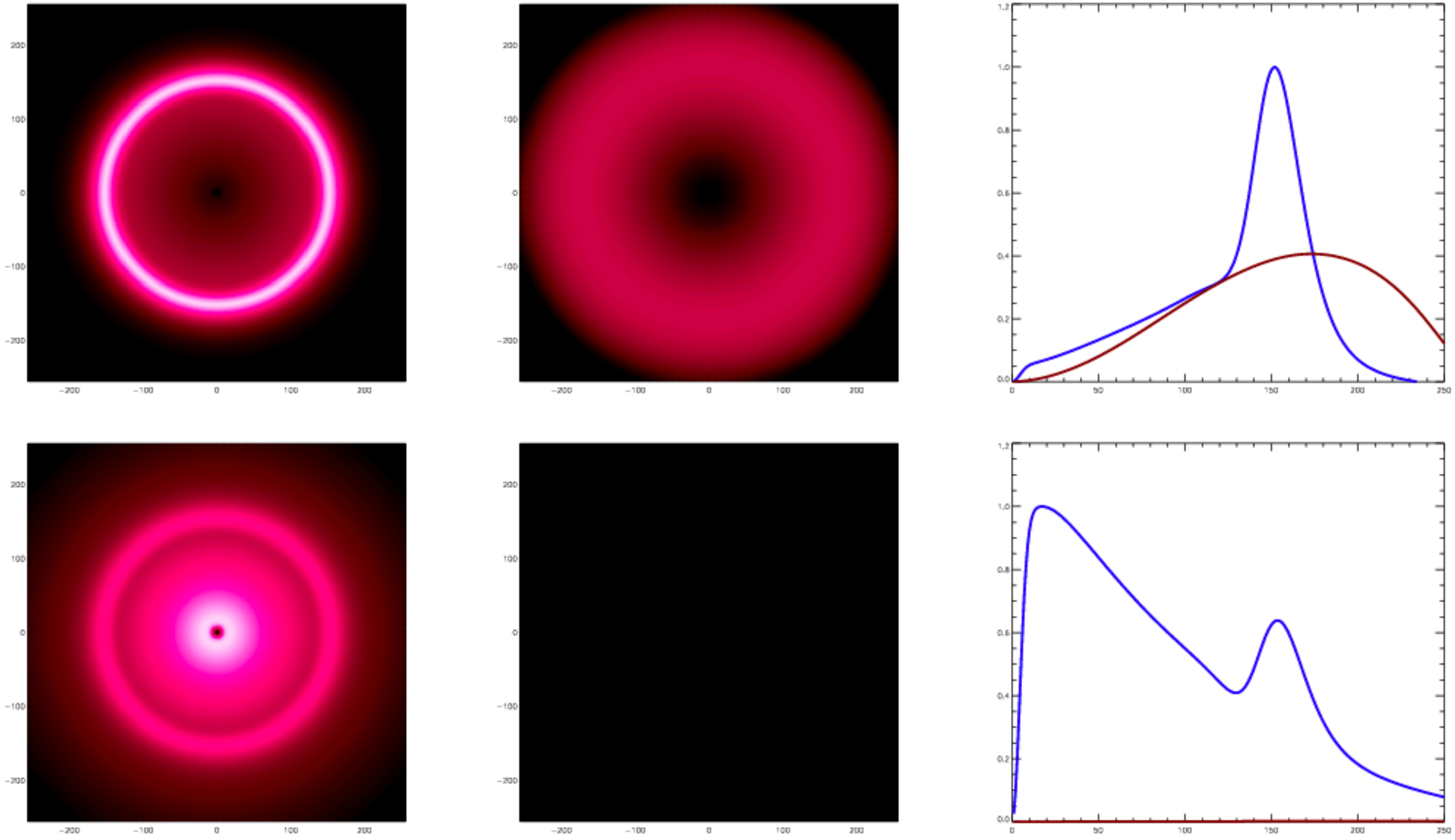
# Baryon Acoustic Oscillations

Baryons | Radiation | Radial Profile

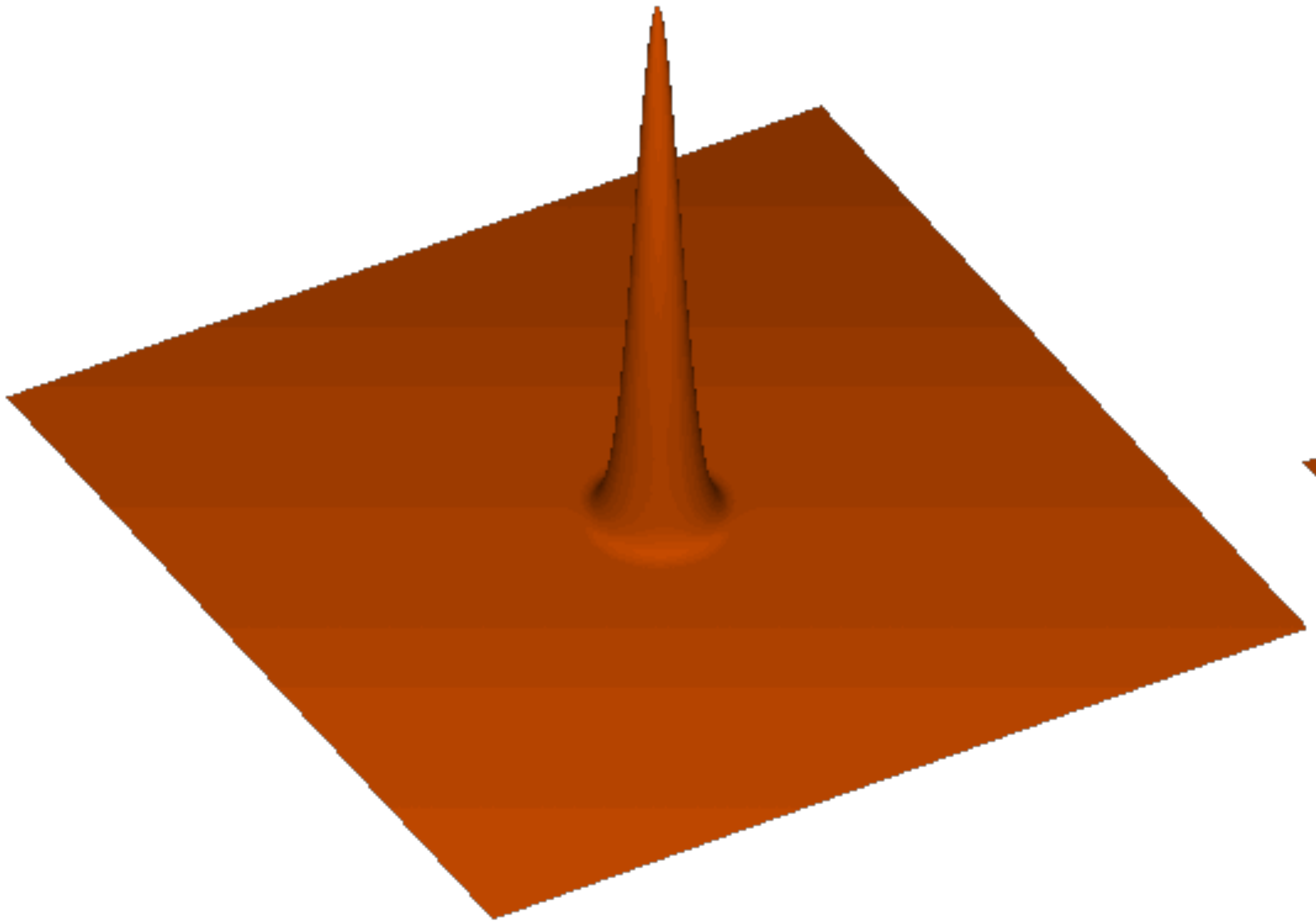


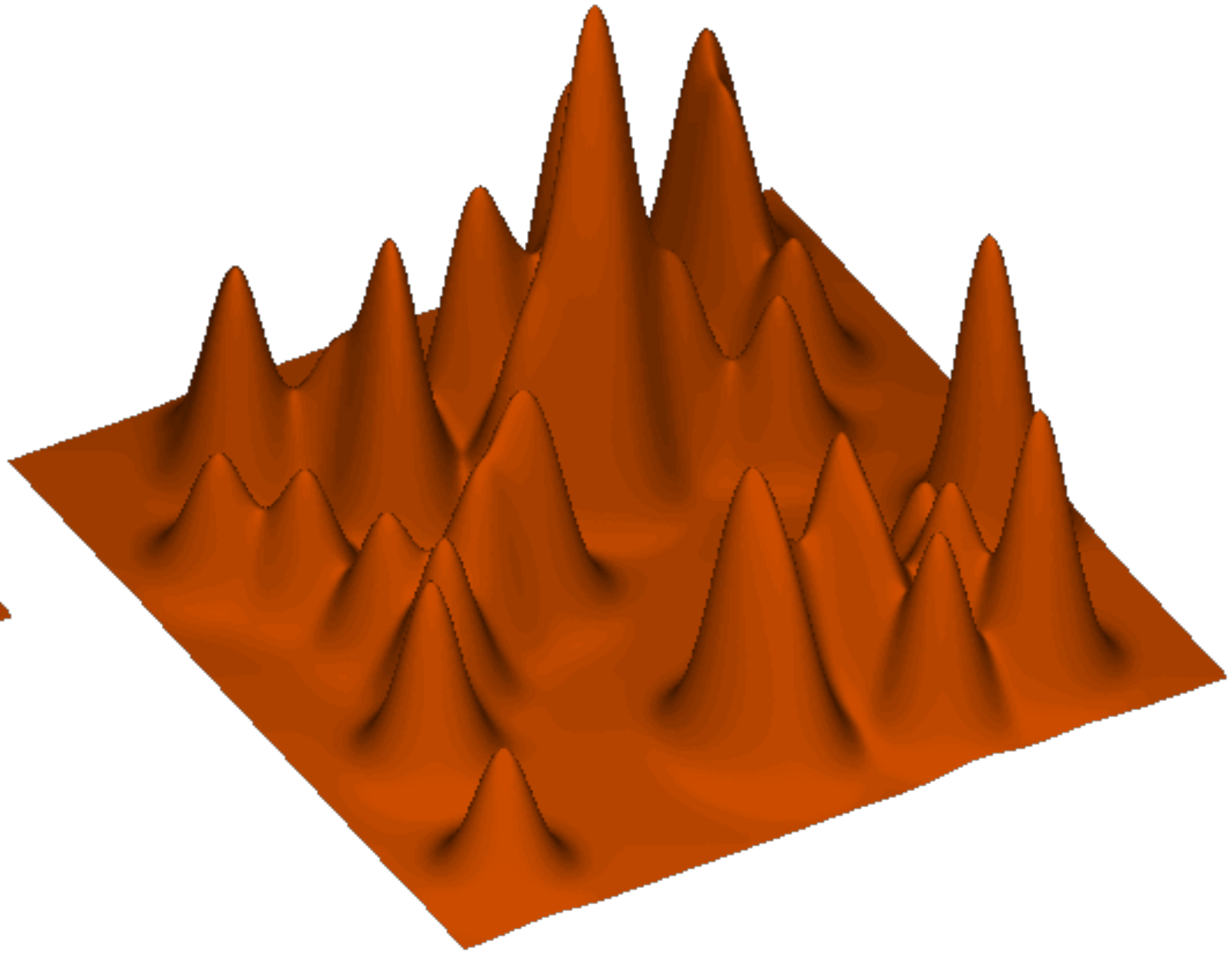
# Baryon Acoustic Oscillations

Baryons | Radiation | Radial Profile

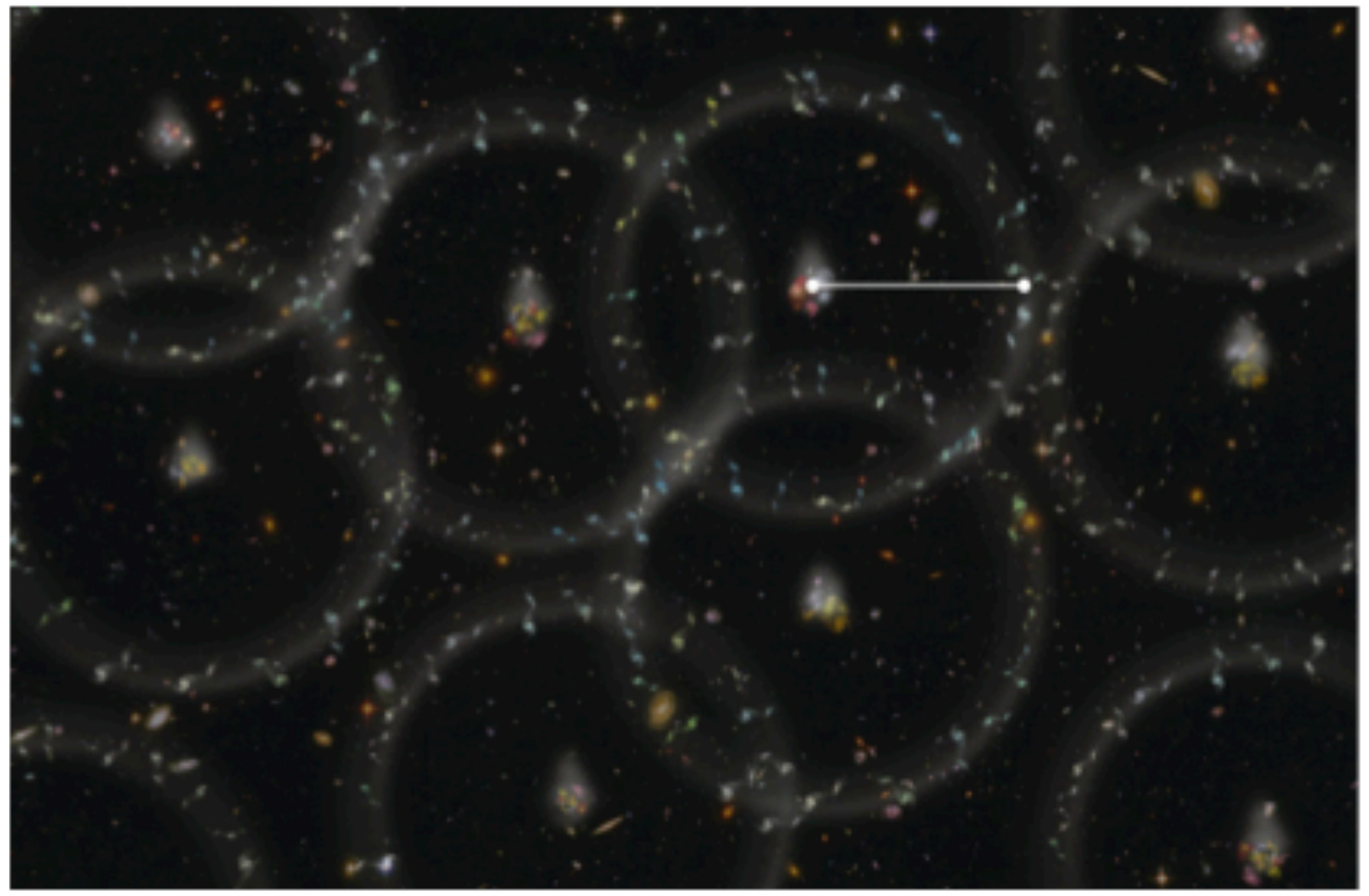


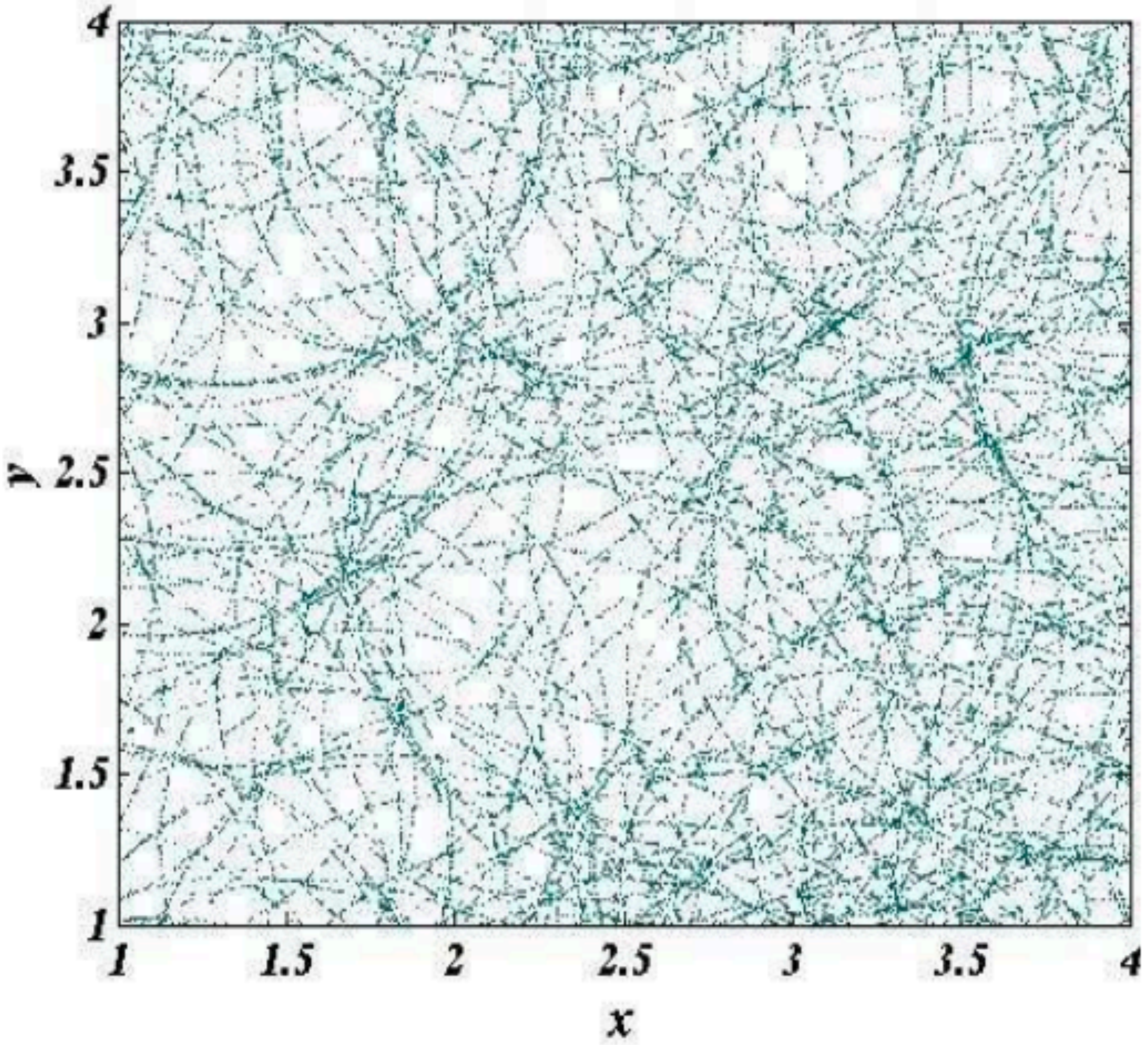
# Baryon Acoustic Oscillations

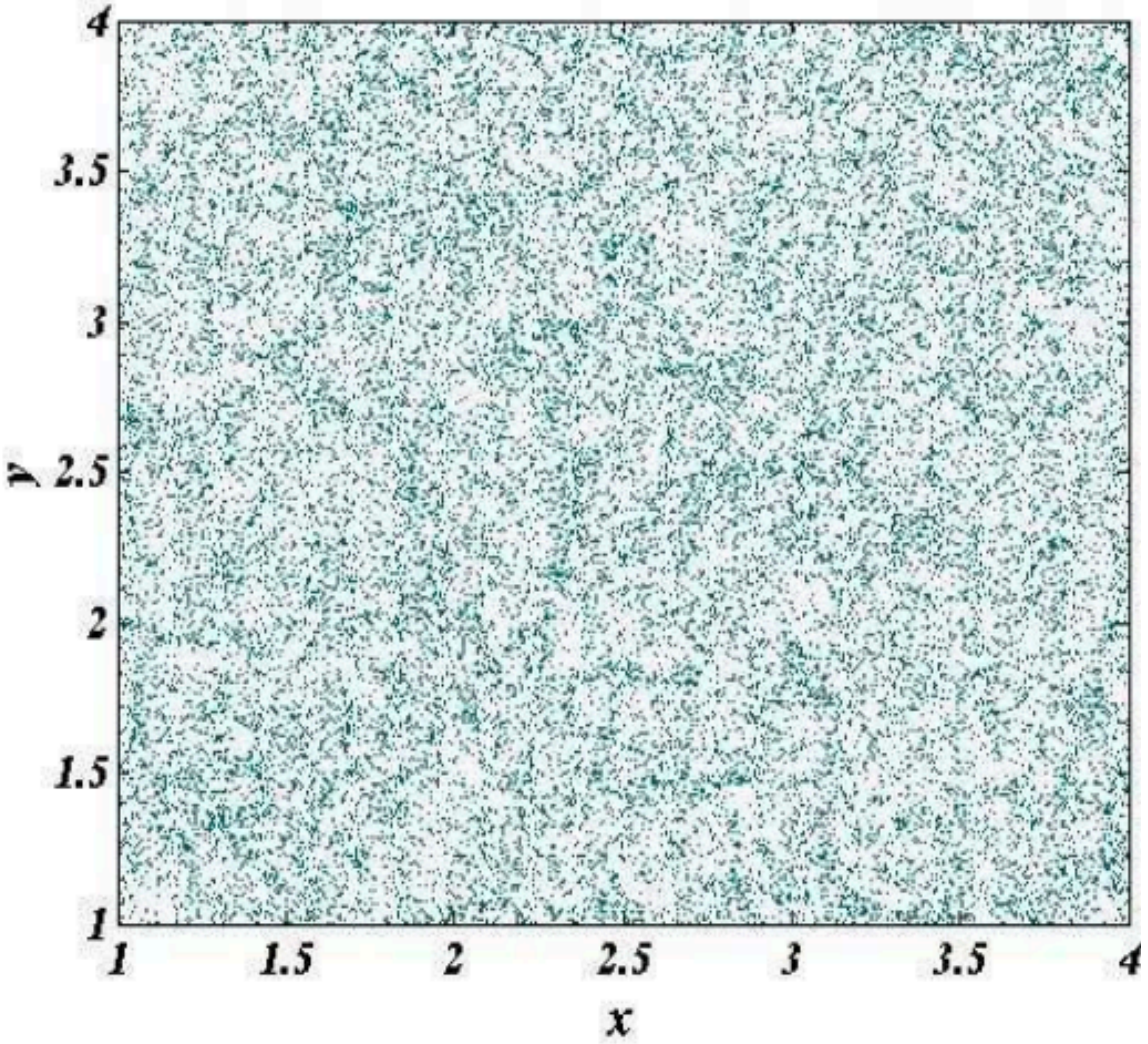




# Baryon Acoustic Oscillations





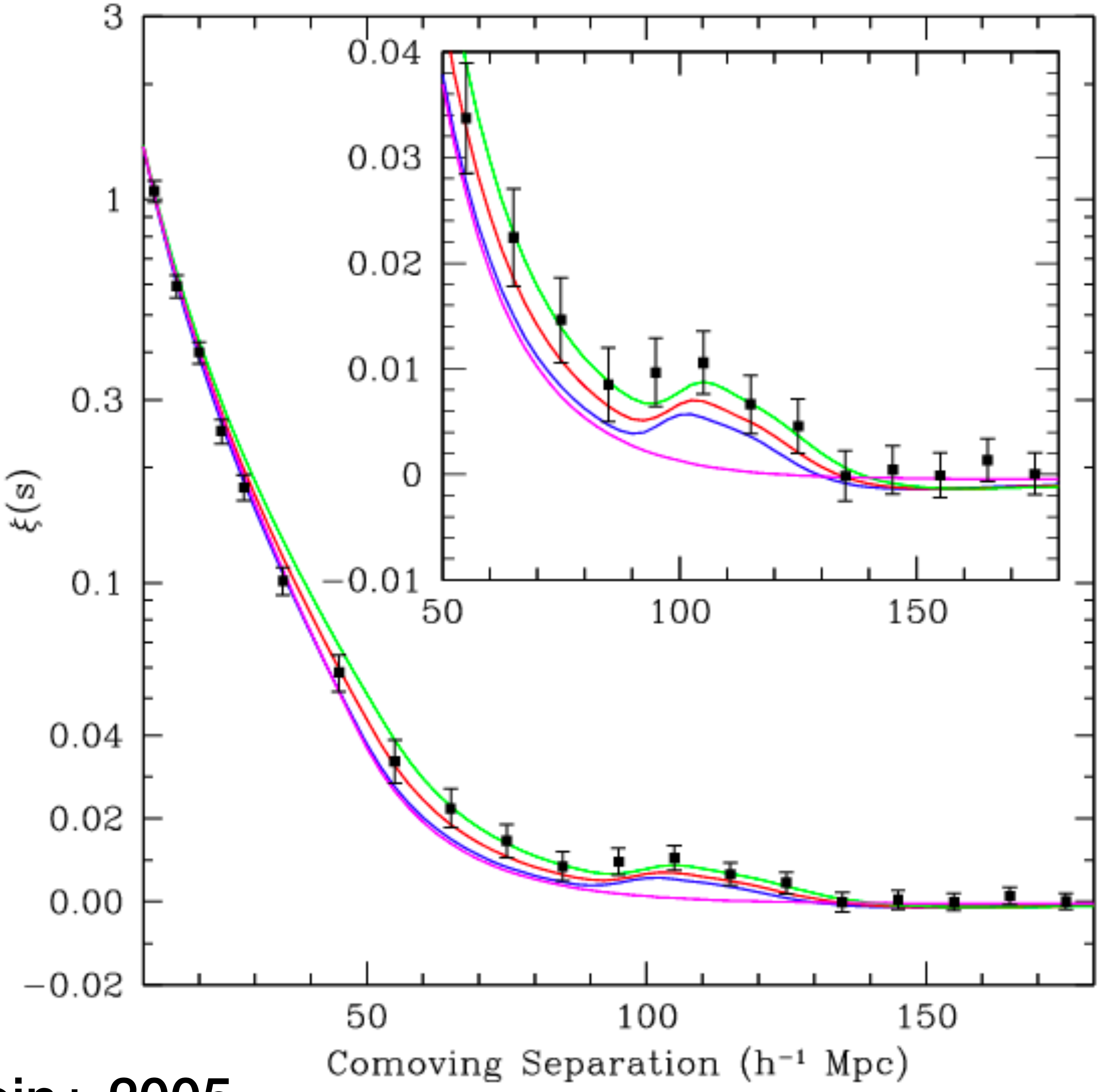
# Baryon Acoustic Oscillations

To measure, use galaxies to trace the signature of these oscillations

The number of galaxies should be correlated with each other on scales comparable to the sound horizon of the largest acoustic peaks (~150 Mpc comoving)

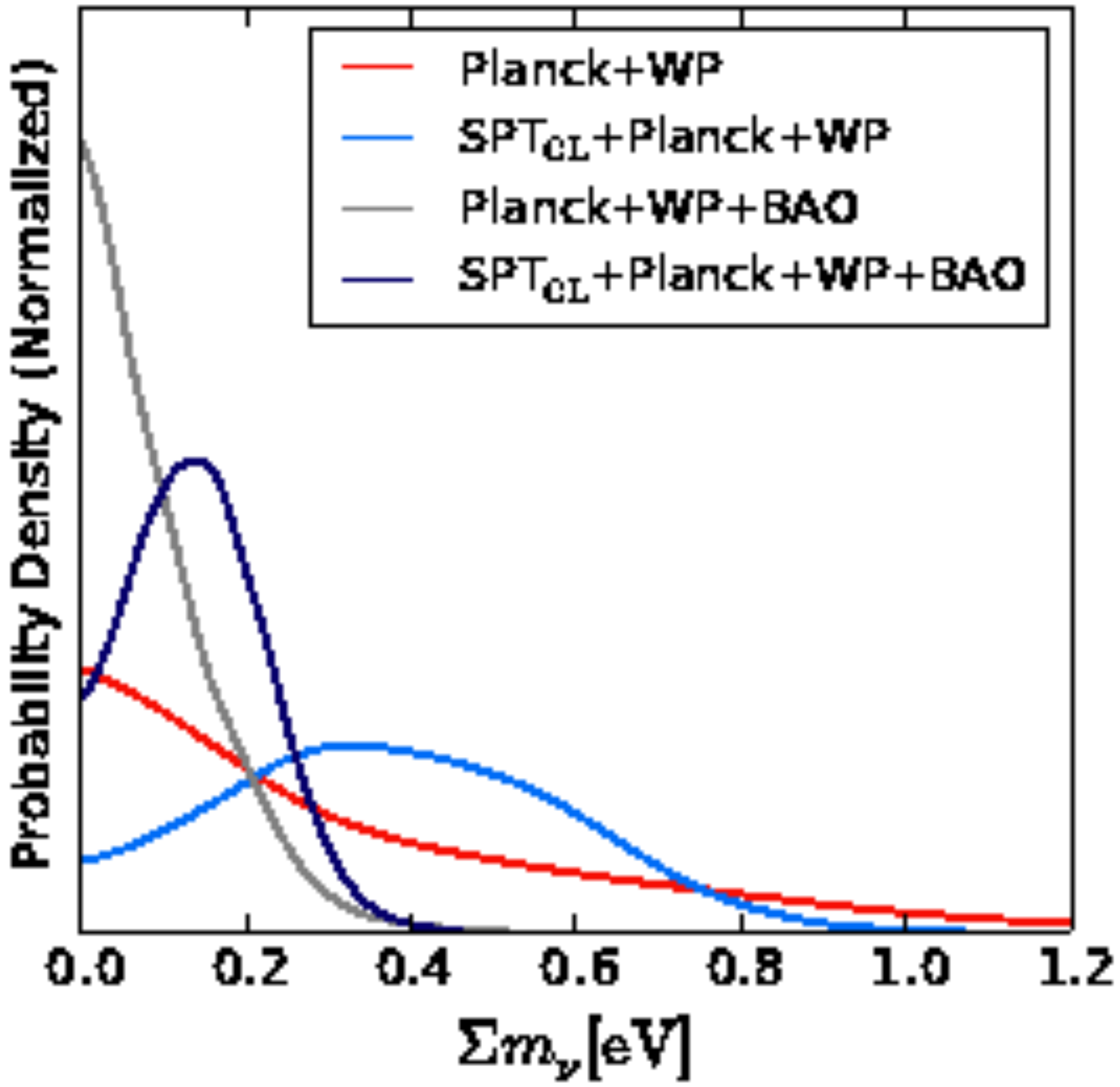
The number of galaxies within a given volume is

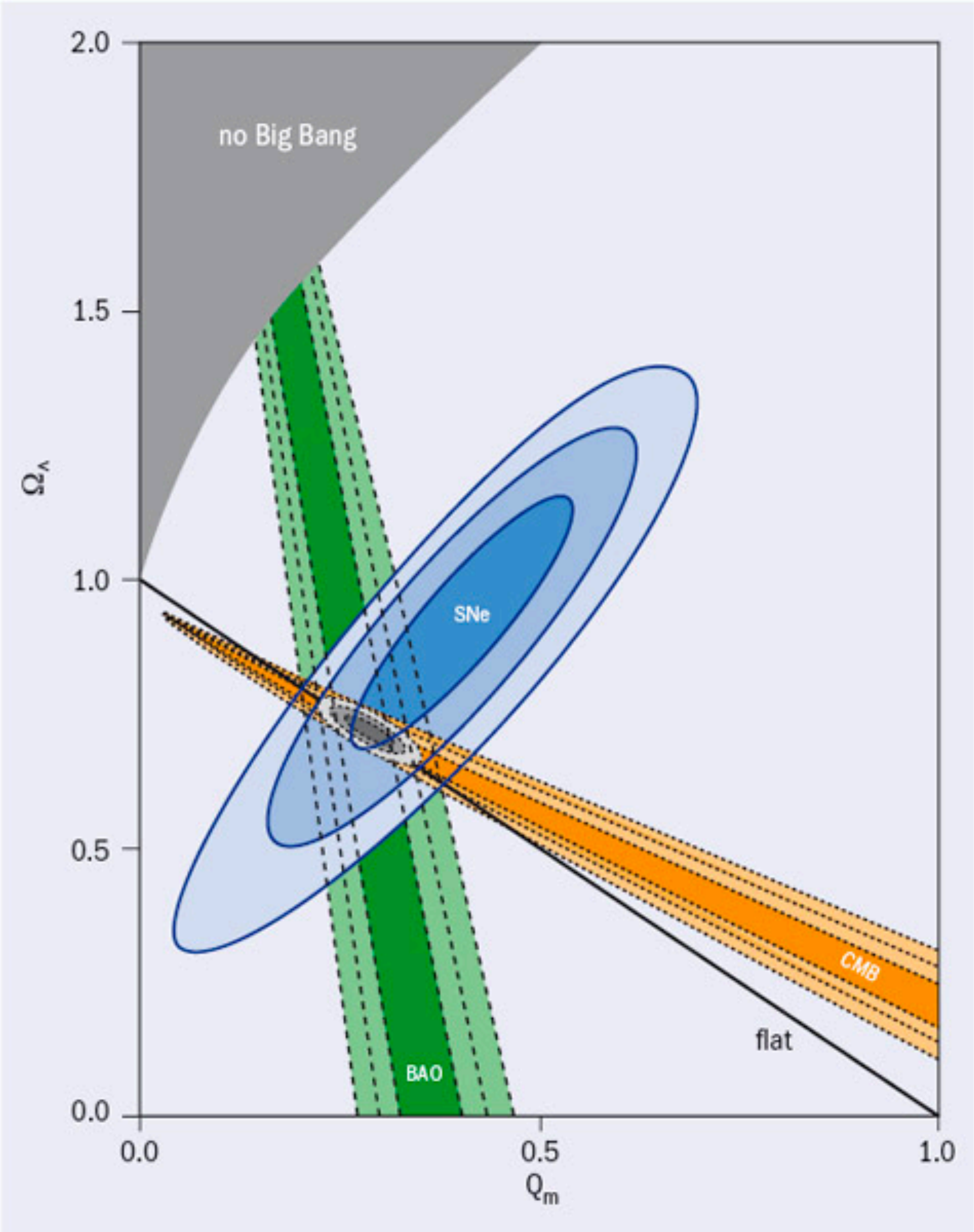
$$dN = n_{\rm gal}[1 + \xi(r)]dV$$

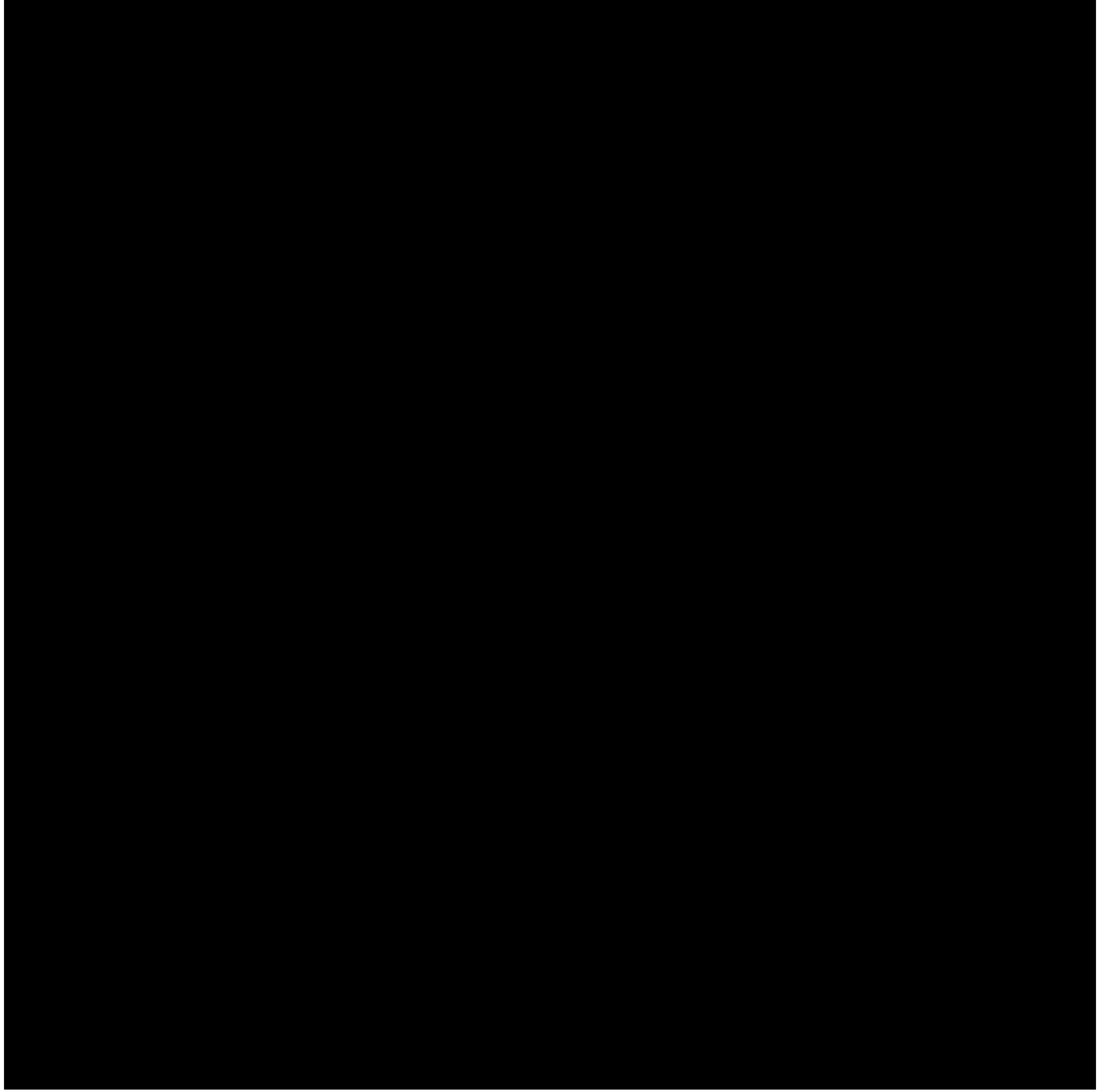



Eisenstein+ 2005

# BAO - Cosmological Constraints





z=4.00
log10(M*)=10.4
SFR=80.0
sSFR=3.07Gyr−1
ILLUSTRIS