ASTR 4080 - Week 14

Recall the scattering eqn. for atoms to interact w/ light

$$\Gamma = n_e \sigma_e c \quad (\text{per sec})$$

As CMB photons travel, they can still get scattered by intervening e^-s

The prob. a photon is scattered is expressed by optical depth

$$\tau_* = \int_{t_*}^{t_0} \Gamma(t)\,dt$$

$$= c\sigma_e \int_{t_*}^{t_0} n_e(t)\,dt$$

$$= \underbrace{0.066 \pm 0.0016}_{\text{from Planck}}$$

$\boxed{\text{1/15 } \gamma\text{s get scattered out of l.o.s.}}$

Can estimate t_* knowing this.

Assume everything is ionized H + e^-

$$n_e = n_p = n_{bary,0}\, a^{-3}$$

define $\Gamma_0 = c\sigma_e n_{bary,0} = 0.0023 H_0$

$$\tau_* = \Gamma_0 \int_{t_*}^{t_0} \frac{dt}{a(t)^3}$$

change to a from t, $\frac{d}{dt} a(t) = \dot{a}$ so $dt = \frac{da}{\dot{a}}$

$$\tau_* = \Gamma_0 \int_{a(t_*)}^{1} \frac{da}{\dot{a} a^3} = \Gamma_0 \int_{a(t_*)}^{1} \frac{da}{H(a)\, a^4}$$

change from a to z:

$$= \Gamma_0 \int_0^{z_*} \frac{(1+z)^2\, dz}{H(z)}$$

$a = (1+z)^{-1}$

$da = \frac{-dz}{(1+z)^2}$

$$= \Gamma_0 \int_0^{z_*} \frac{(1+z)^2\, dz}{H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}}}$$

(flat, matter + Λ dominated)

$$\tau_* = \frac{2}{3\Omega_{m,0}} \frac{\Gamma_0}{H_0} \left(\left[\Omega_{m,0}(1+z_*)^3 + \Omega_{\Lambda,0}\right]^{1/2} - 1 \right)$$

$$= 0.00485 \left(\left[0.31 (1+z_*)^3 + 0.69\right]^{1/2} - 1 \right)$$

BACK TO SLIDES

$\boxed{z_* = 7.8 \pm 1.3}$, $t_* = 650\,\text{Myr}$

Obs. universe: $d_p(t_0) \approx 14$ Gpc

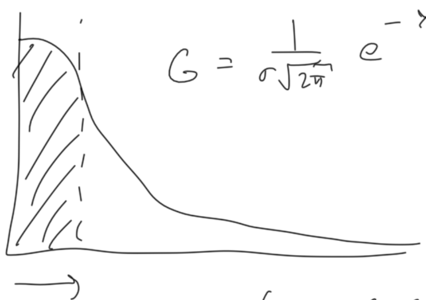
total mass inside is $M = \rho_{m,0} \frac{4\pi}{3} d_p(t_0)^3$

$\approx 4.3 \times 10^{23} M_\odot$

☆ There are 4.3×10^9 regions $\sim / 10^{14} M_\odot$,

so the $\underline{1^{ST}}$ $10^{14} M_\odot$ region to collapse

is a statistical outlier: $\frac{1}{4.3 \times 10^9}$ chance



$G = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$

$p = \mathrm{erf}\left(\frac{n/\sigma}{\sqrt{2}}\right)$

$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

1σ 68% $(2\sigma : 95\%)$

$(3\sigma : 99.7\%)$

$\boxed{2.3 \times 10^{-10} \rightarrow 6.2\sigma}$

$\delta_{coll} = 6.2 \rightarrow 1 + z_{coll} \approx \delta_{rm}(1 + z_{rm}) = \delta_{coll}$

$\boxed{z = 5.2}$

$T_{gas} \approx 2.3 \times 10^7$ K

$\bar{\rho}_{gas} \approx 1.5 \times 10^{-24}$ kg m^{-3}

$\hookrightarrow \boxed{t_{cool} \approx 42 \text{ Gyr}}$

Hierarchical Fragmentation

- if 1% inefficient, start w/ 100 cores each w/ mass M

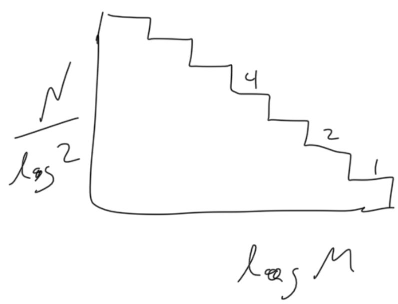
$1\ M,\ 198\ M/2$

$1\ M,\ 2\ M/2,\ 392\ M/4$

$1\ M,\ 2\ M/2,\ 4\ M/4,\ 776\ M/8$

after n steps, have $1\ M,\ 2\ M/2, \ldots, 2^n\ M/2^n$

log-spaced histogram: $\log M/2^n = \log M - n \log 2$



$$\frac{dN}{d\log M} \propto \frac{1}{M}$$

Equivalent to $\xi(M) \propto M^{-2}$ (close to obs. $M^{-2.3}$)