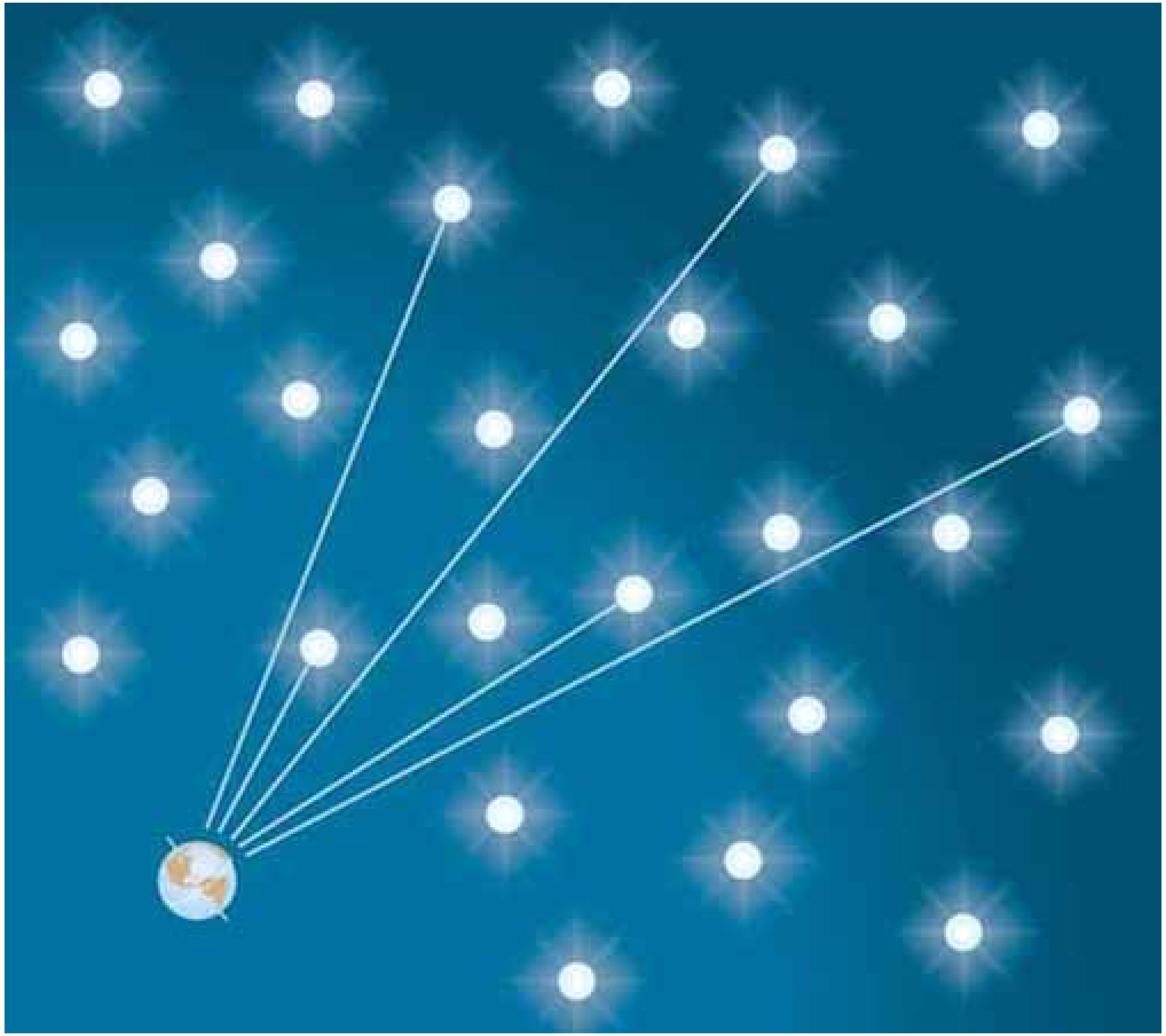
Olber’s Paradox (1823)

Infinitely old, infinitely large universe full of stars

Sky should be as bright as the disk of the Sun!



Getting distances to the nebulae

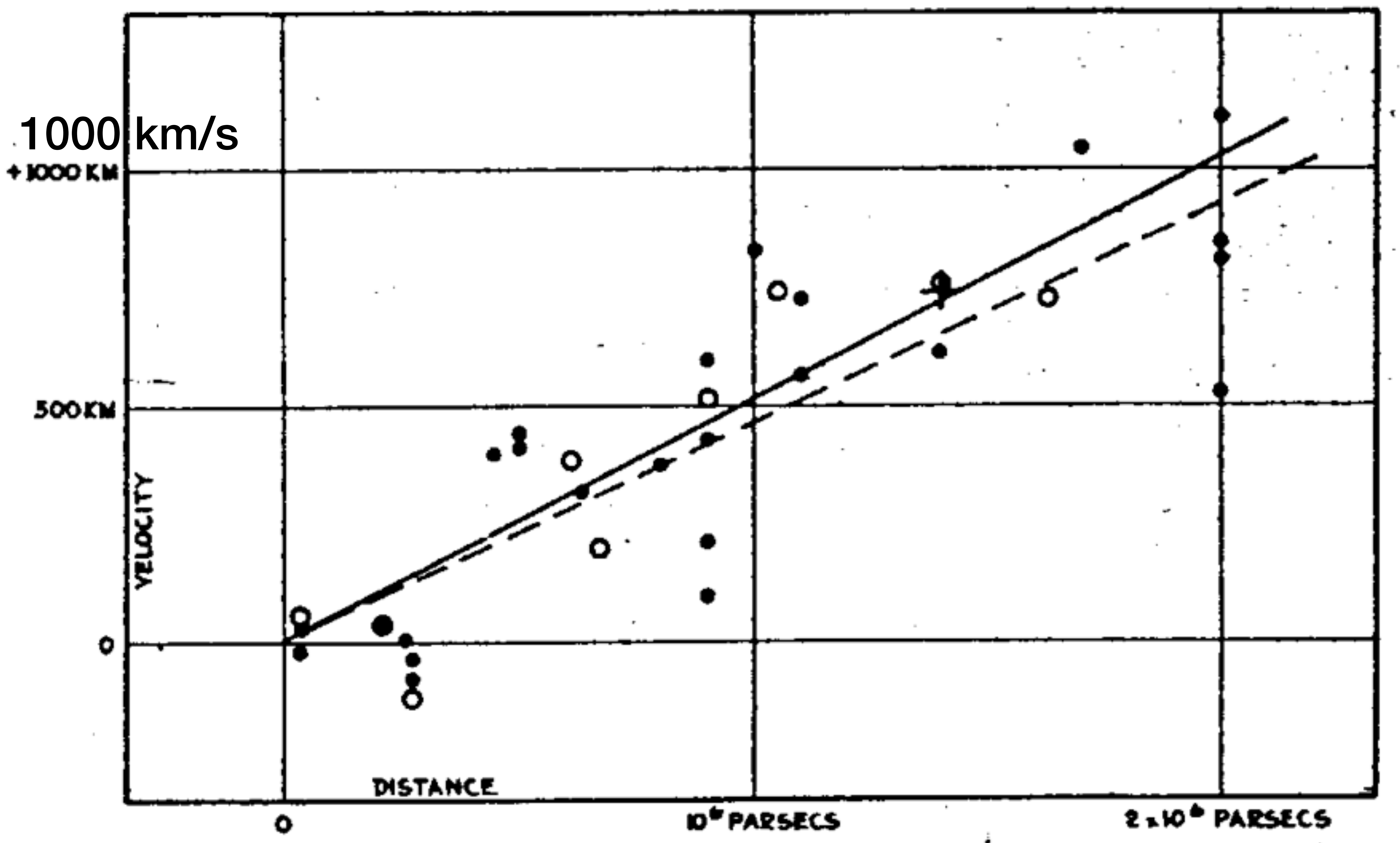
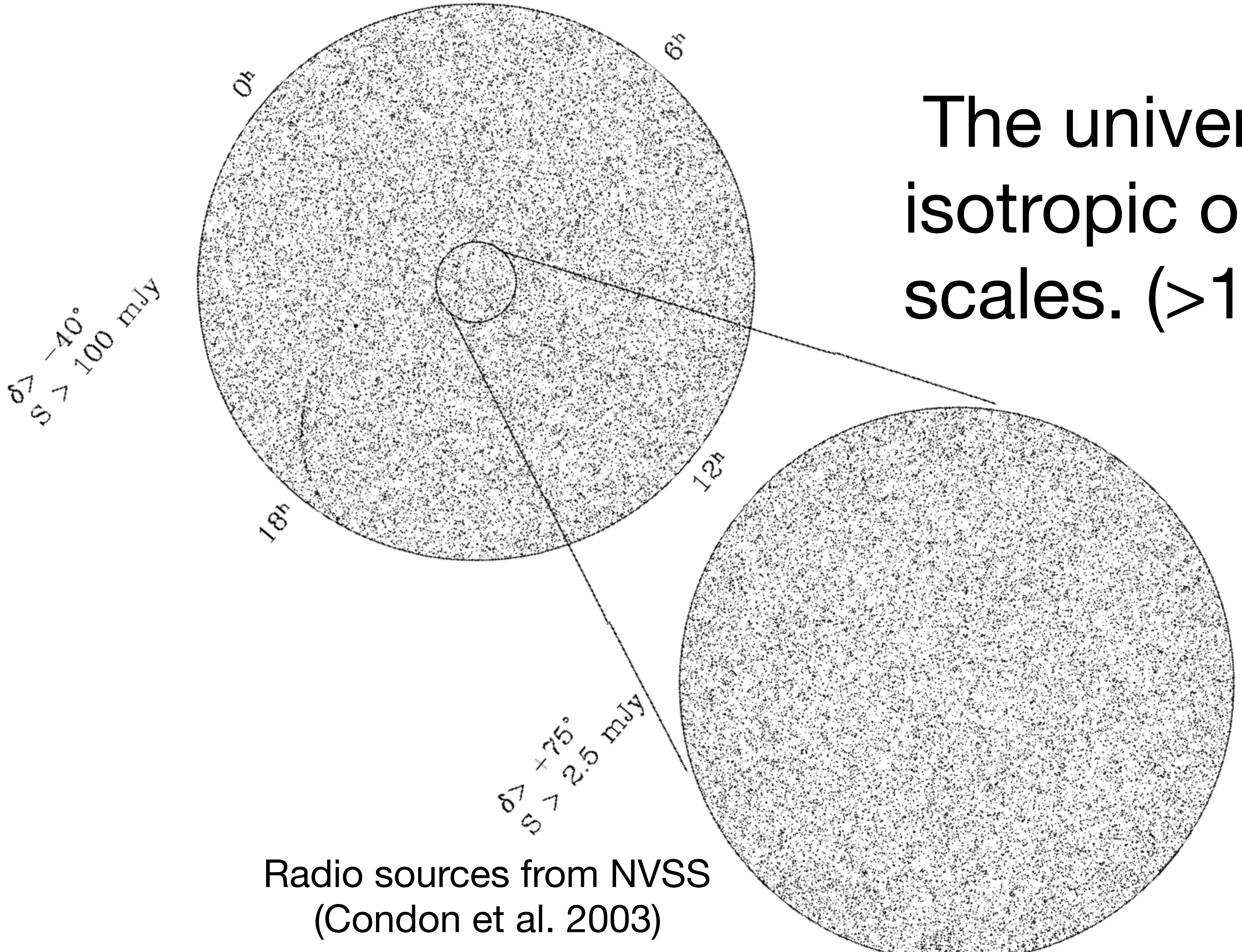


FIGURE 1

Hubble estimated distances to the nebulae, resolved in favor of Curtis and the island universe theory

Also, measurements of line shifts in spectra, interpreted as Doppler velocity shifts, demonstrated that farther away galaxies are “moving” away from us faster

Cosmological Principle



Radio sources from NVSS
(Condon et al. 2003)

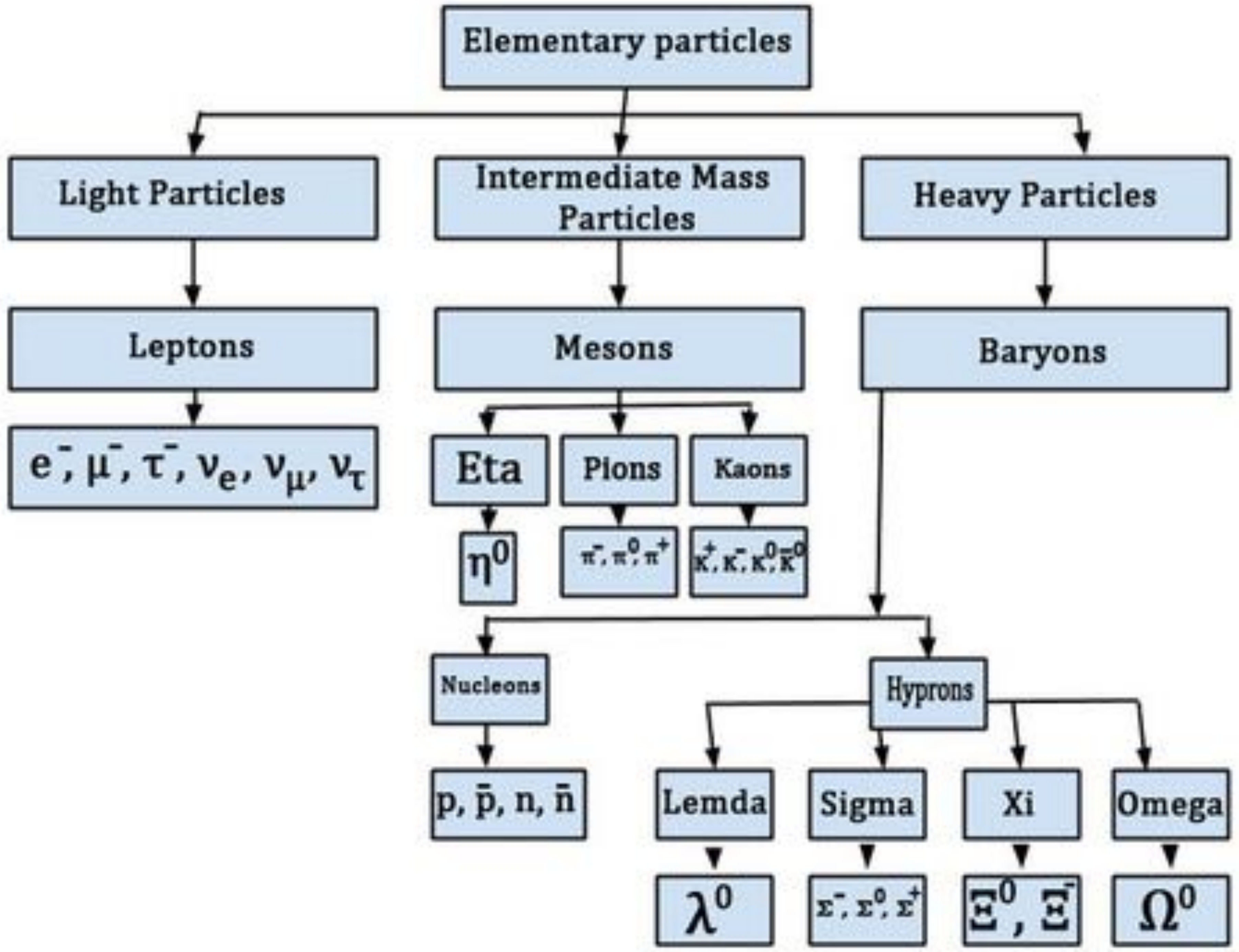
The universe is isotropic on very large scales. (>100Mpc).

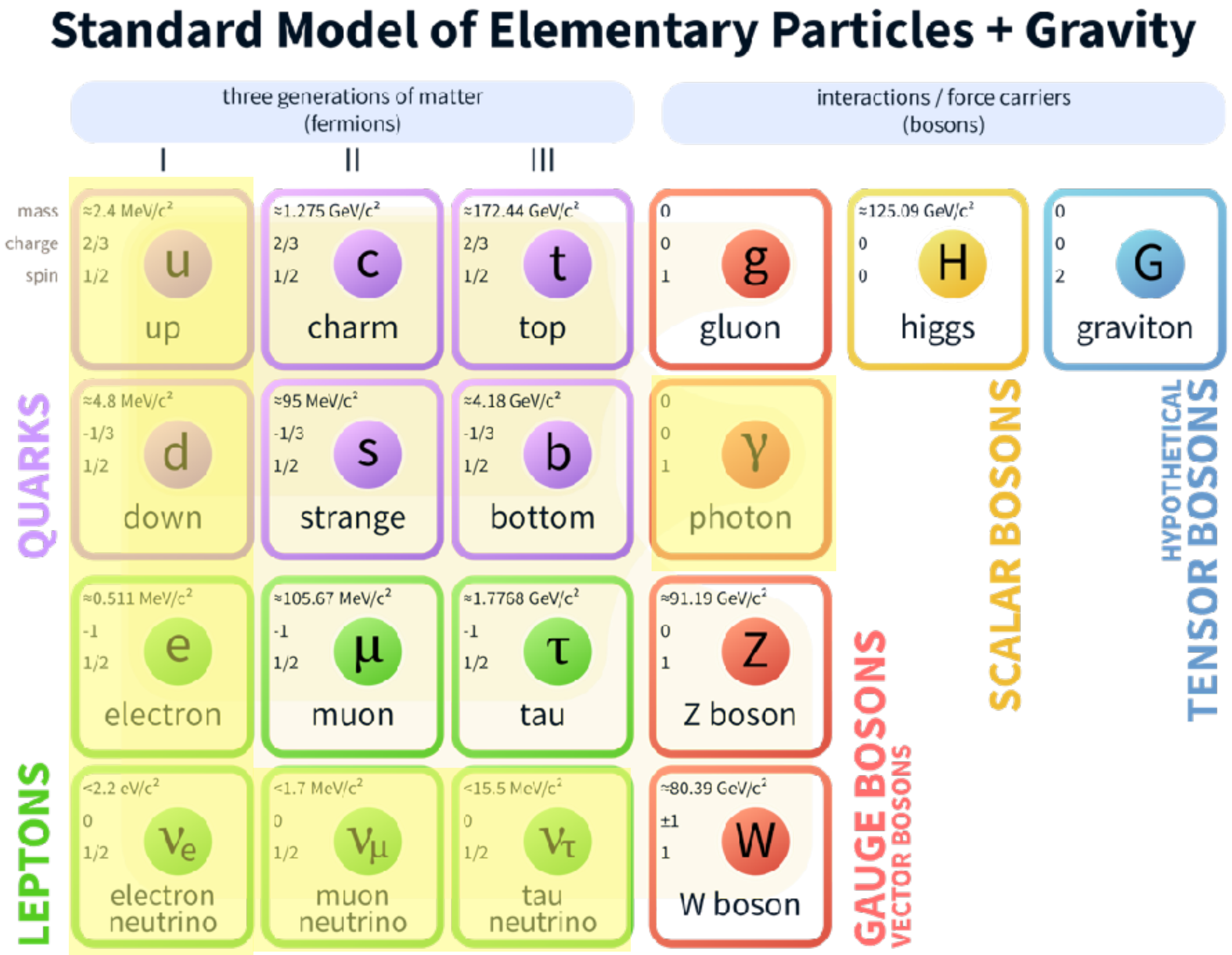
Copernican Principle
=> homogeneous & isotropic

(Cosmological Principle)

Elementary Particles

particle	symbol	rest energy (MeV)	charge
proton	p	938.3	+1
neutron	n	939.6	0
electron	e^-	0.511	-1
neutrino	ν_e, ν_μ, ν_τ	?	0
photon	γ	0	0
dark matter	?	?	0

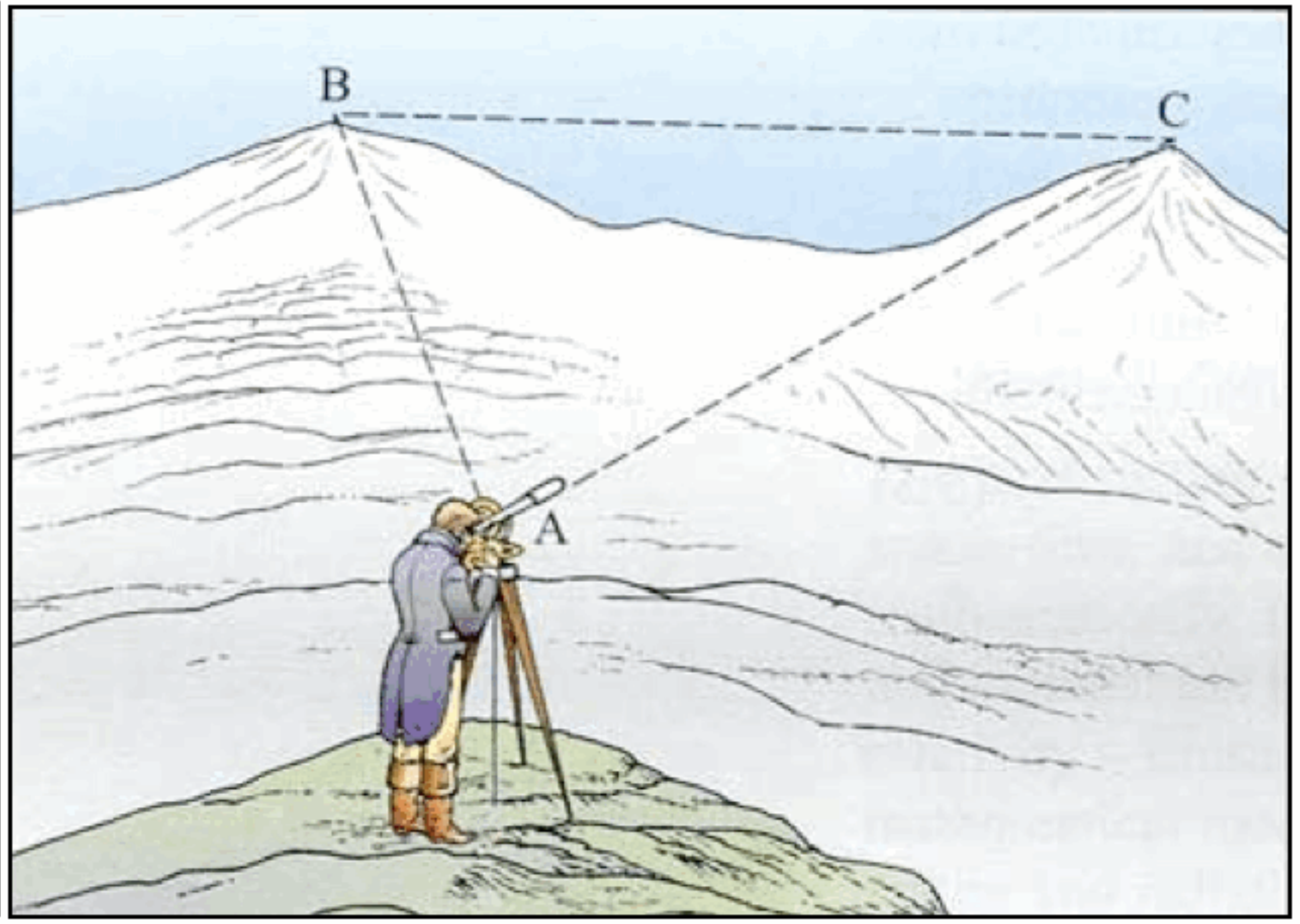




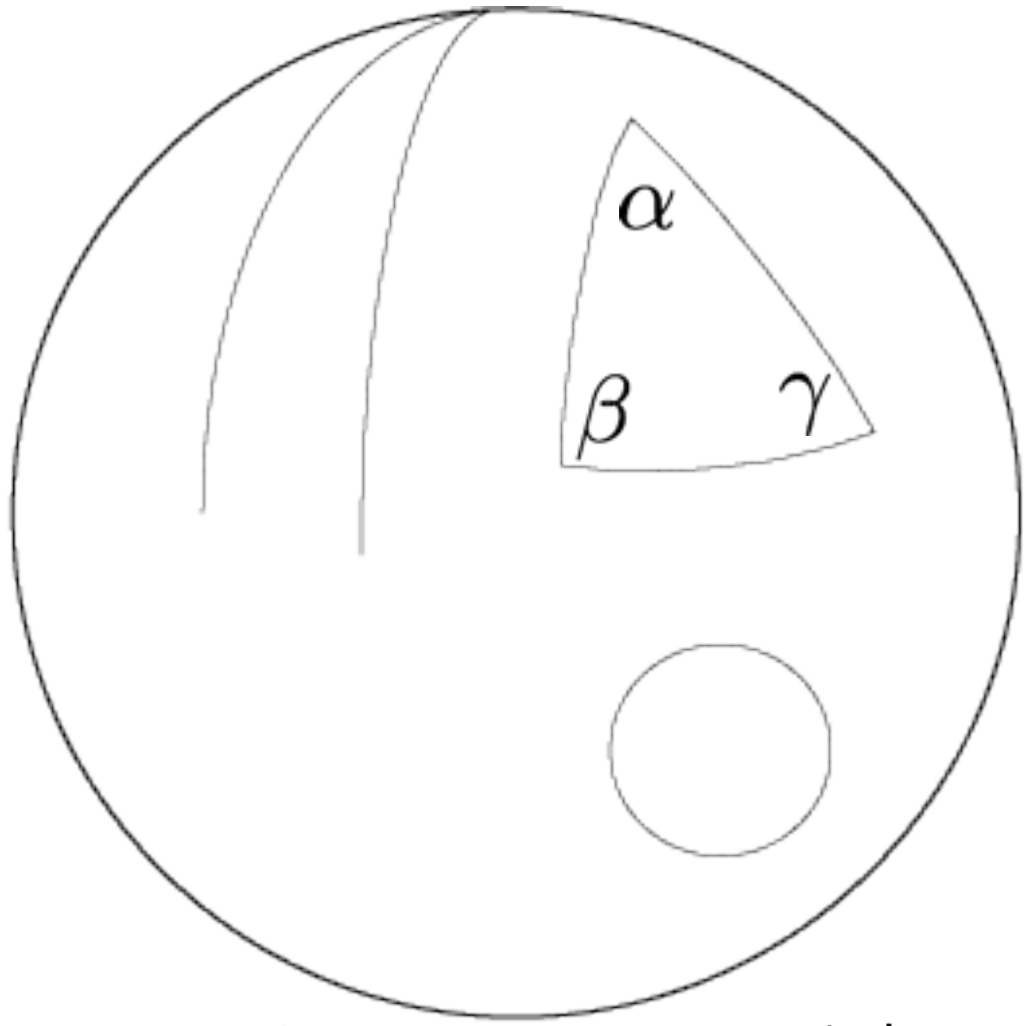
Curvature

How can we measure the curvature of spacetime?

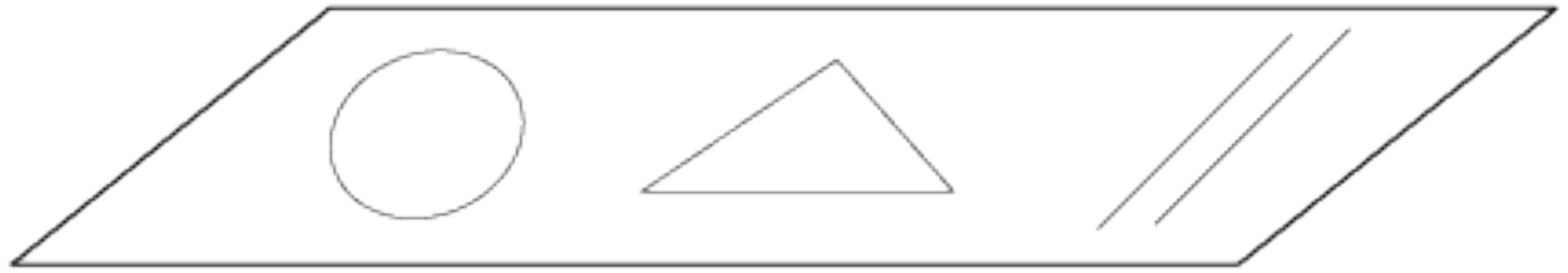
Carl Friedrich Gauss
1777 - 1855



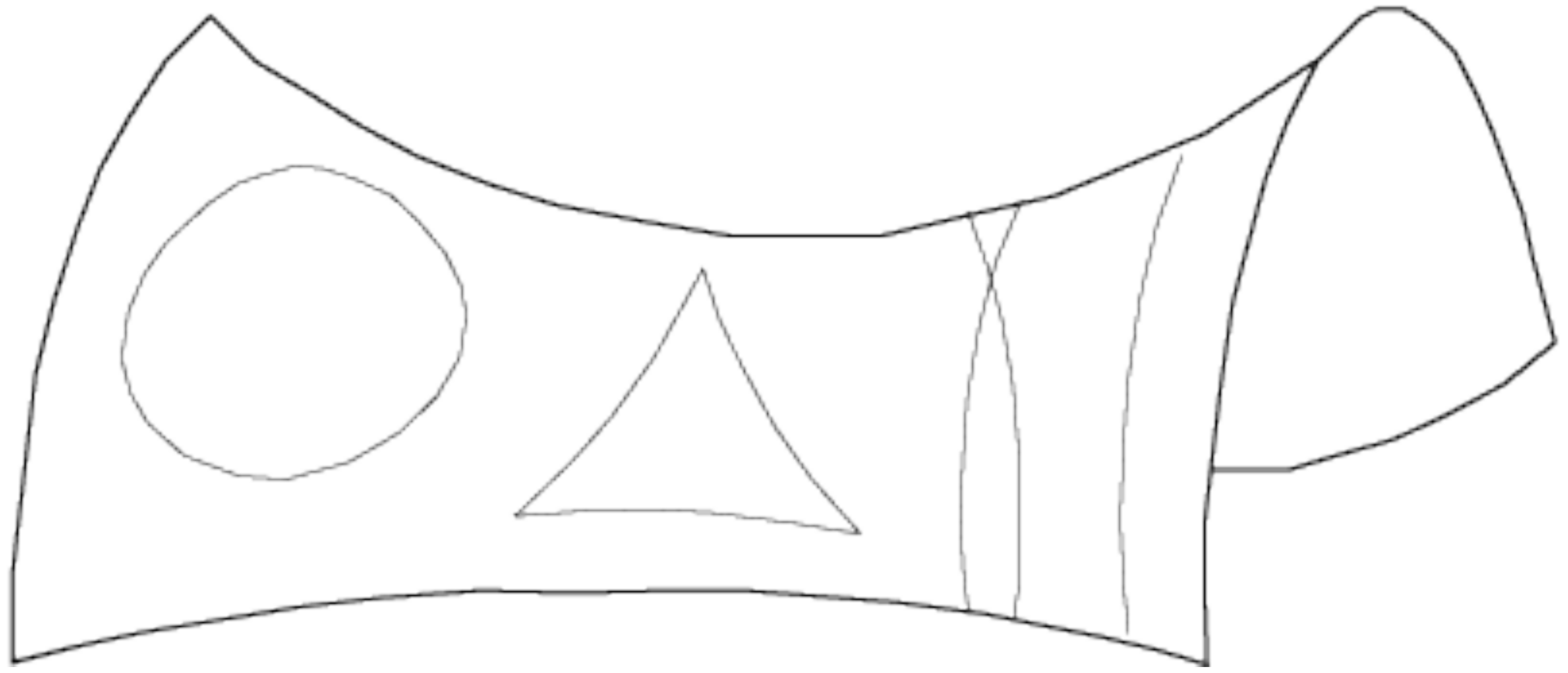
Gauss finds 180 degrees in large survey triangles:
Space is not (grossly) non-Euclidean



$$\alpha + \beta + \gamma = \pi + A/R$$



$$\alpha + \beta + \gamma = \pi$$



$$\alpha + \beta + \gamma = \pi - A/R$$

A = area of triangle R = Radius of Curvature

Only possible geometries that are homogeneous/isotropic

Lengths of Geodesics (3D, polar coords)

↳ straight lines in a given geometry

<OR>

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\phi^2$$

flat or Euclidean space:

$$d\ell^2 = dr^2 + r^2 d\Omega^2$$

elliptical or spherical space:

$$d\ell^2 = dr^2 + R^2 \sin^2 \frac{r}{R} d\Omega^2$$

hyperbolic space:

$$d\ell^2 = dr^2 + R^2 \sinh^2 \frac{r}{R} d\Omega^2$$

$$d\ell^2 = dr^2 + S_\kappa(r)^2 d\Omega^2$$

$$S_\kappa(r) = \begin{cases} R\sin\frac{r}{R} & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh\frac{r}{R} & (\kappa = -1) \end{cases}$$

Minkowski & Robertson-Walker Metrics

metrics define the distance between events in spacetime

Minkowski (no gravity: metric in SR)

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2$$

Robertson-Walker (with gravity, if spacetime is homogeneous & isotropic)

$$ds^2 = -c^2dt^2 + a(t)[dr^2 + S_\kappa(r)^2d\Omega^2]$$

light travels along null geodesics, i.e.: $ds^2 = 0$

cosmological proper time or cosmic time

(r, θ, ϕ) comoving coordinates

Proper Distance

In an expanding universe, how do we define the distance to something at a cosmological distance?

The distance between 2 objects at the same instant of time is given by the RW metric: called the "proper distance"

$$ds = a(t)dr$$

$$d_p(t) = a(t)\int_0^r dr = a(t)r$$

$$\dot{d}_p = \dot{a}r = \frac{\dot{a}}{a}d_p$$

$$v_p(t_0) \equiv H_0 d_p(t_0) \rightarrow d_H(t_0) \equiv c/H_0$$

Redshift and Scale Factor

Proper distance is not usually a practical distance measure.
For example, you might rather want to know the distance light has traveled from a distant object so you know the "lookback time" or how far you're looking into the past.

Relatedly, we measure redshift, but would like to know how redshift is related to the change in scale factor between emission and observation, which is:

$$1 + z = \frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)}$$

Relativistic equation similar

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{Kc^2}{R_0^2}\frac{1}{a(t)^2}$$

$$H(t)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{Kc^2}{R_0^2 a(t)}$$

Today, have: $H_0^2 = \frac{8\pi G}{3c^2}\varepsilon_0 - \frac{Kc^2}{R_0^2}$

Boundary case is $K=0$, so the critical (energy) density is

$$\varepsilon_{crit,0} = \frac{3c^2 H_0^2}{8\pi G}$$

$$\Omega(t) \equiv \frac{\varepsilon(t)}{\varepsilon_c(t)}, \quad \Omega_0 = \frac{\varepsilon(t_0)}{\varepsilon_c(t_0)}$$

Friedmann Eq:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2}$$

Fluid Eq:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

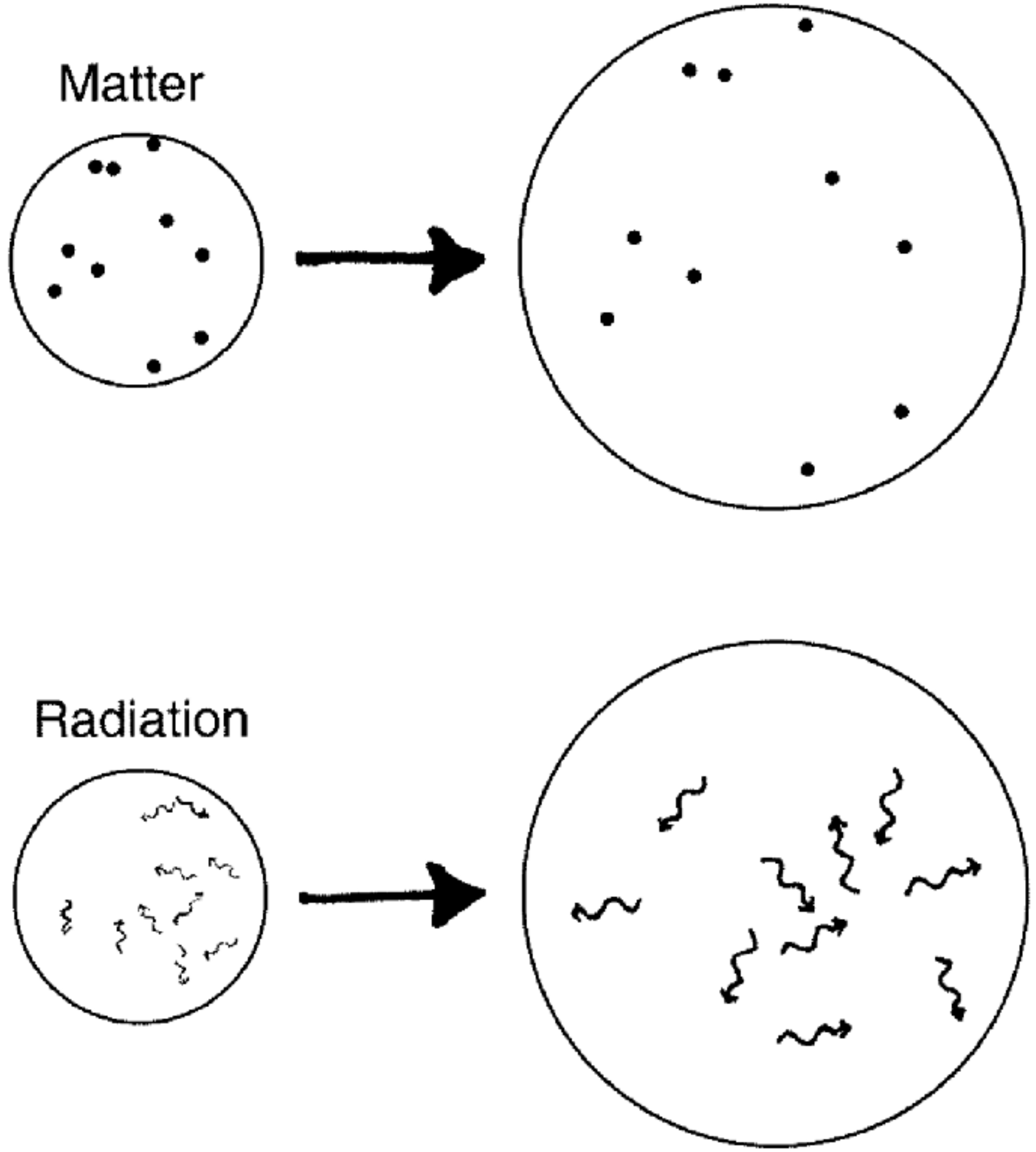
Eq. of State: $P = w\varepsilon$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left[\varepsilon + 3P\right]$$

Evolution of Components

$$\varepsilon = \sum_i \varepsilon_i$$

$$P = \sum_i w_i \varepsilon_i$$



Model Universes

Can now solve for a(t) generically, if not necessarily analytically:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_i \varepsilon_{i,0} a^{-1-3w_i} - \frac{\kappa c^2}{R_0^2}$$

- Empty
- Matter only
  - classic case: open, closed, flat
- Radiation only (+curvature)
- Lambda only (+curvature)
- Various Combinations!

$$\underline{w=0} \qquad \underline{w=\frac{1}{3}} \qquad \underline{w=-1}$$

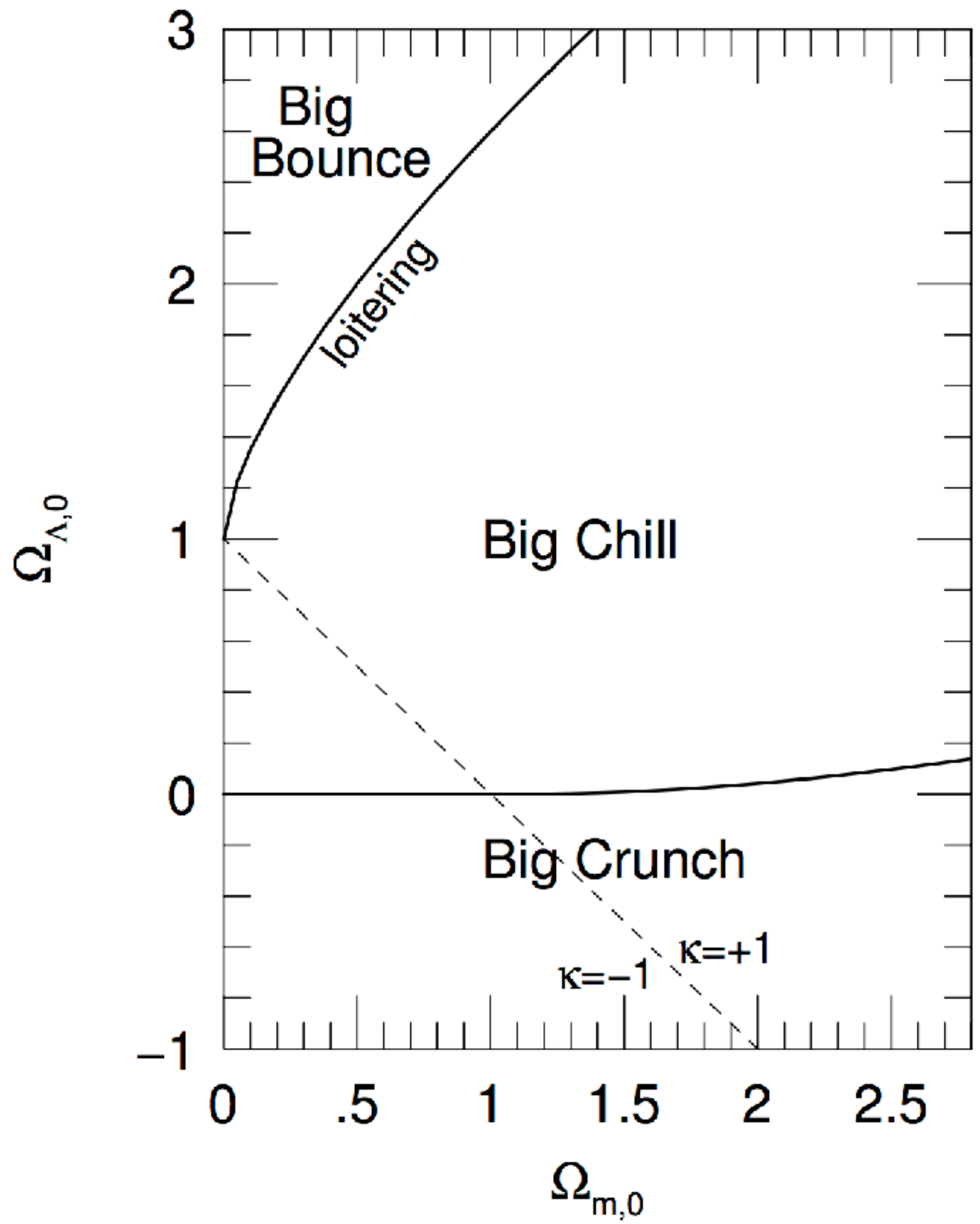
$$\varepsilon_m = \varepsilon_{m,0}/a^3 \qquad \varepsilon_r = \varepsilon_{r,0}/a^4 \qquad \varepsilon_\Lambda = \varepsilon_{\Lambda,0}$$

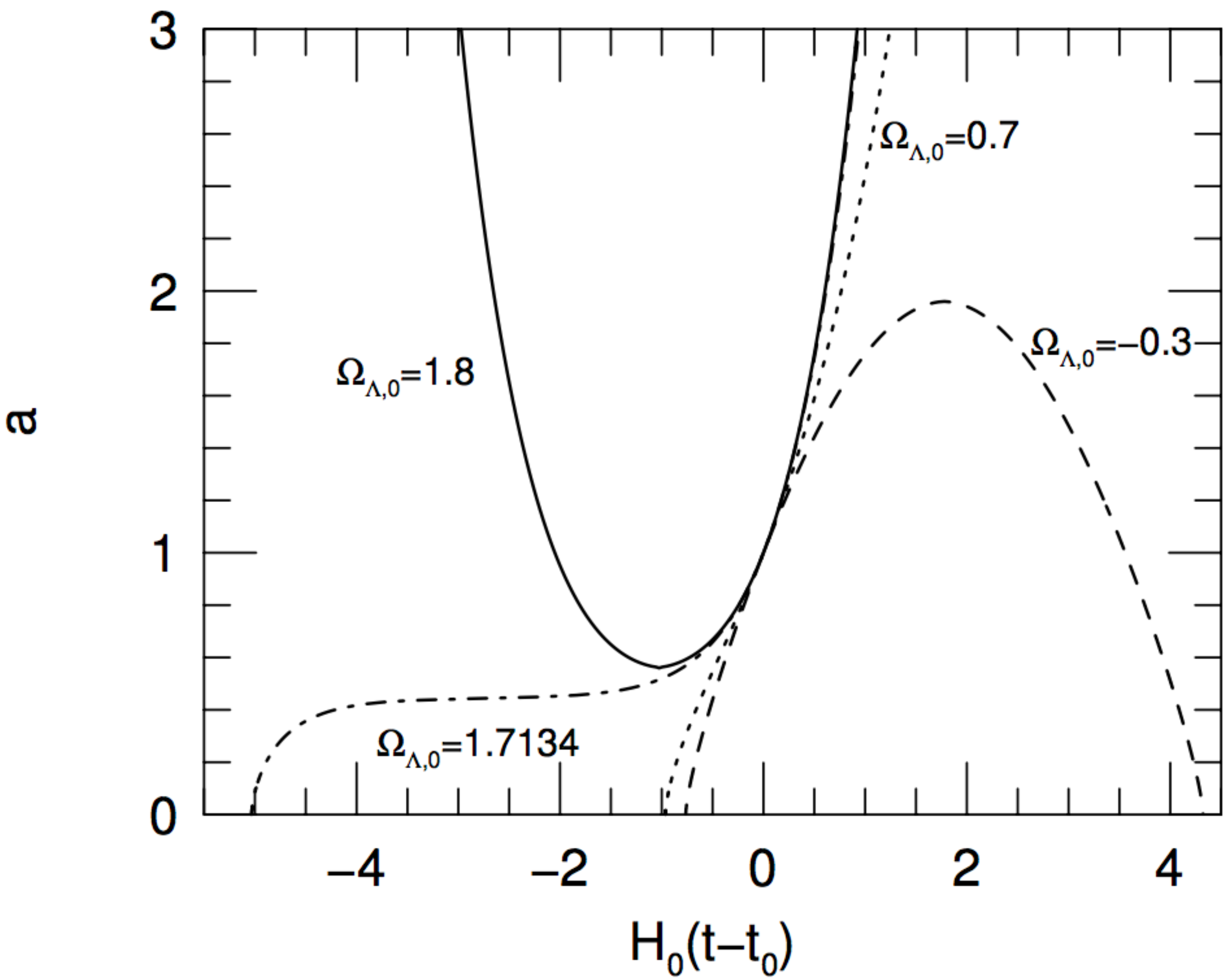
$$\boxed{d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}} \qquad \boxed{d_p(t_e) = \frac{1}{1+z} d_p(t_0)}$$

$$H_0^{-1}\dot{a} = \left[\Omega_{r,0}\, a^{-2} + \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + (1-\Omega_0)\right]^{1/2}$$

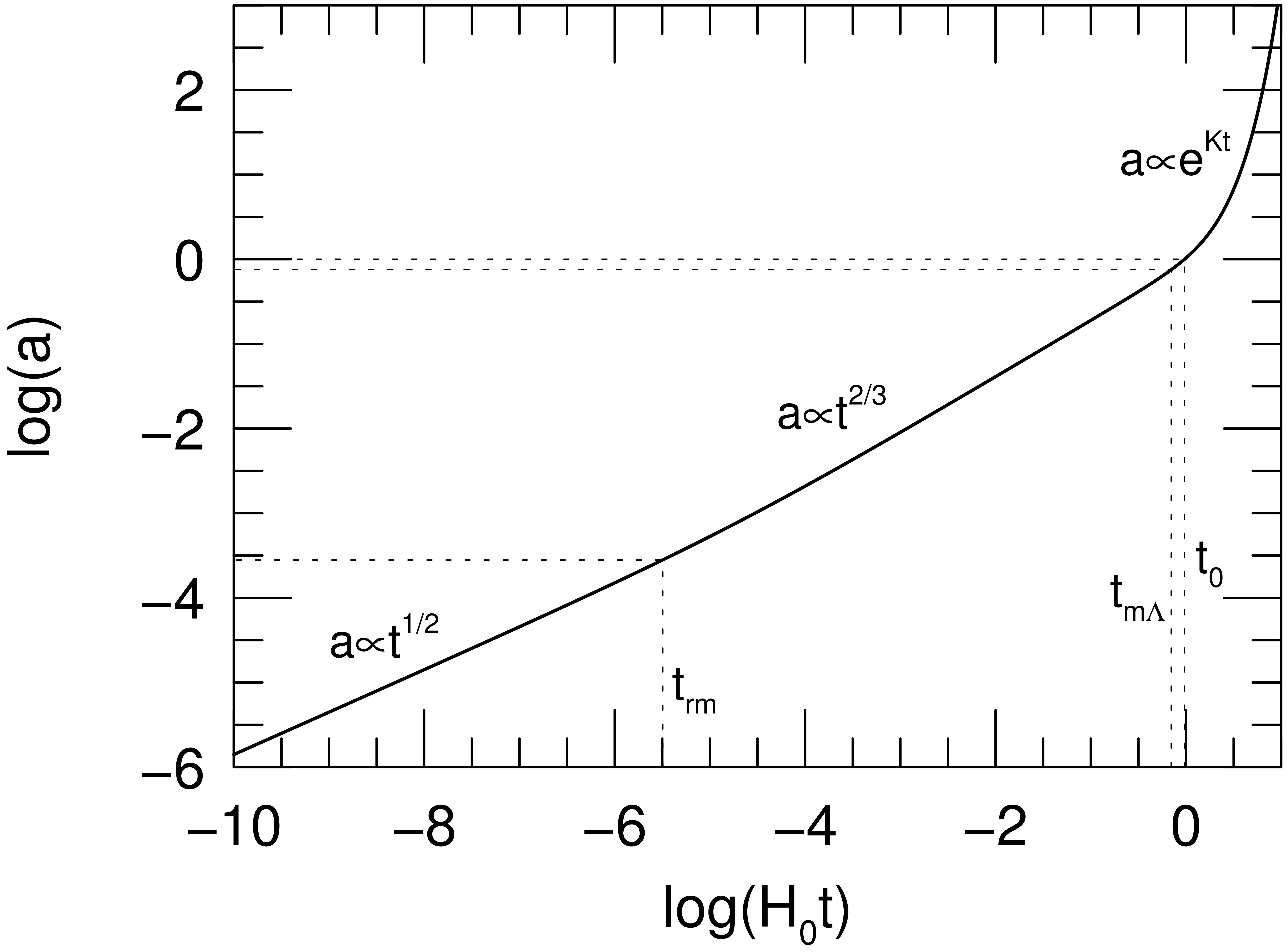
$$\boxed{t = \frac{1}{H_0}\int_0^a \frac{da}{E(a)}}$$

Matter + Lambda + Curvature





Benchmark Model



$$\kappa = 0$$

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

$$\Omega_r = \Omega_\gamma + \Omega_\nu$$

$$\Omega_m = \Omega_{bary} + \Omega_{dm}$$

$$\Omega_{r,0} = 0.31$$
$$\Omega_{bary,0} = 0.048$$
$$\Omega_{dm,0} = 0.262$$

$$\Omega_{r,0} = 9.0 \times 10^{-5}$$
$$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$$
$$\Omega_{\nu,0} = 3.65 \times 10^{-5}$$

$$\Omega_{\Lambda,0} \approx 0.69$$

Rad. – Matter :	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 50$ kyr
Matter – Λ :	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now :	$a_0 = 1$	$t_0 = 13.7$ Gyr