ASTR 4080 - Week 7

Rotation Curves of Galaxies

$$a = \frac{v^2}{R}$$ angular acceleration of circular motion

$$a_g = \frac{GM(<R)}{R^2}$$ assumes spherical symmetry, so

all interior mass can be assumed to be @ the center

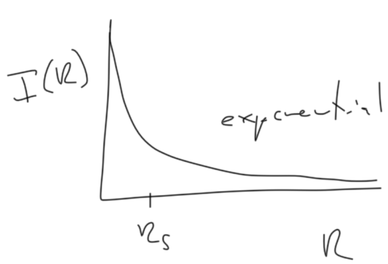
Therefore $$\frac{v^2}{R} = \frac{GM(<R)}{R^2}$$

or $$\boxed{v = \sqrt{\frac{GM(<R)}{R}}}$$

SB of a disk galaxy typically falls off

as $$\boxed{I(R) = I(0)\, e^{-R/R_s}}$$

$R_s \sim 4$ kpc (MW) & $R_s \sim 6$ kpc (M31)



- if $M \propto L$, beyond few $\times R_s$, mass const.

so $\underline{v \propto R^{-1/2}}$

"Keplerian"

Slide on rotation curve

- rearrange eq. $\rightarrow$ $M(R) = \frac{v^2 R}{G}$

MW: $L_{gal,V} \simeq 2.0 \times 10^{10} L_{\odot,V}$

so $\langle M/L_{gal,V} \rangle \simeq \underline{\underline{64}}\, M_\odot / L_{\odot,V} \left(\frac{R_{halo}}{100\,kpc} \right)$

Avg. M/L of gal. (given their stars) is $\sim 4\, M_\odot / L_{\odot,V}$, so clearly lots of non-stellar mass

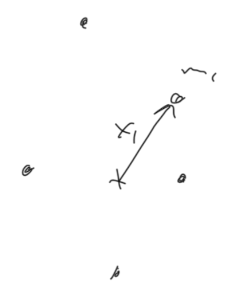
$$\left[\begin{array}{l} R_{halo} \simeq 75\,kpc - 300\,kpc \\ M_{tot} \simeq (1-4) \times 10^{12} M_\odot \\ M/L \simeq (50-200)\, M_\odot / L_{\odot,V} \end{array} \right]$$

Dark Matter in Galaxy Clusters

→ Galaxy Velocities

Need the virial theorem

– start w/ moment of inertia



$$I \equiv \sum_i m_i |\vec{x}_i|^2$$

– can link to W + K by taking 2nd derivative

$$\ddot{I} = 2 \sum_i m_i \left(\vec{x}_i \cdot \ddot{\vec{x}}_i + \dot{\vec{x}}_i \cdot \dot{\vec{x}}_i \right)$$

position × accel. term, equiv. to potential E
→ due to grav.

K term

$$K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2$$

$$\ddot{\vec{x}}_i = G \sum_{j \neq i} m_j \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^3}$$

$$\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = G \sum_{\substack{i,j \\ (i \neq j)}} m_i m_j \frac{\vec{x}_i (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3} = \sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j)$$

$$\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = \frac{1}{2}\left[\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) + \sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j)\right]$$

$$= \frac{G}{2}\left[\sum_{i,j} m_i m_j \frac{\vec{x}_i (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3} + \sum_{j,i}\right]$$

$$= \frac{G}{2}\left[\sum \frac{m_i m_j}{|\vec{x}_i - \vec{x}_j|^3} \underbrace{\left(\vec{x}_i \vec{x}_j - \vec{x}_i^{\,2} + \vec{x}_j \vec{x}_i - \vec{x}_j^{\,2}\right)}_{-(\vec{x}_i - \vec{x}_j)^2}\right]$$

$$= -\frac{G}{2}\sum_{i,j} \frac{m_i m_j}{|\vec{x}_i - \vec{x}_j|} = W$$ pot. energy

So $\boxed{\ddot{I} = 2W + 4K}$

Steady state, $\ddot{I} = 0$ & $K = -\frac{W}{2}$

Can write $W = -\alpha \frac{GM^2}{r_h}$ (simple $\sim$ of 1 part. w/ corr. factor α)

& $K = \frac{1}{2} M \langle v^2 \rangle$

where $\langle v^2 \rangle$ is mass-weighted velocities of galaxies

So this gives $\frac{1}{2} M \langle v^2 \rangle = \frac{\alpha}{2} \frac{G M^2}{R}$

$$\boxed{M = \frac{\langle v^2 \rangle r_h}{\alpha G}}$$ ← half-mass radius

velocity dispersion σ_r is what can be measured, which is l.o.s. spread in velocities

→ since true vel. are 3D

$$\langle v^2 \rangle = 3 \sigma_r^2$$

☆ Slides for Coma

$$\left.\begin{array}{l} \langle v^2 \rangle \approx 3 \, (880 \, \text{km/s})^2 \\ r_h \sim 1.5 \, \text{Mpc} \end{array}\right\} \quad M_{Coma} \approx 2 \times 10^{15} M_\odot$$

$$[L_{Coma,V} \approx 5 \times 10^{12} L_{\odot,V}]$$

$$\left[\begin{array}{l} M_{*,Coma} \sim 2 \times 10^{13} M_\odot \\ M_{gas,Coma} \sim 2 \times 10^{14} M_\odot \end{array}\right]$$

$$\boxed{\langle M/L \rangle \sim 400 \, M_\odot / L_{\odot,V}}$$

ASTR 4080 - Week 6