Final Exam Example Problems

Exam on April 30 from 10:30-12:30 pm on Zoom

1. If the energy density of dark energy, ε_Λ, were $100\times$ larger than what has been measured, how (if at all) would the abundance of elements change? Explain your reasoning.

2. Does η, the ratio of the number densities of baryons to photons, vary with time? Explain your reasoning. What is the order of magnitude of η at the current time, and what unsolved problem in physics does it represent?

3. Why are CMB fluctuations encoded as hot and cold spots (i.e., what makes them appear hotter or colder than average)? What physically causes the first peak in the CMB power spectrum? What aspect of the first peak tells us that the geometry of the universe is flat, and why?

4. Consider an edge-on spiral galaxy at a redshift $z = 0.4$ with a maximum rotational velocity of 150 km/s, angular scale length $\theta_s = 2$", and V band apparent magnitude $m_V = 22$. Assume the Benchmark cosmology.

   (a) When the light we observe from the galaxy was emitted, what was the scale factor a of the universe?

   (b) What was the temperature of the cosmic background radiation?

   (c) What was the value of the Hubble parameter?

   (d) What is the current proper distance to the galaxy? (HINT: do not attempt to calculate the exact *theoretical* value.)

(e) What is the angular diameter distance and luminosity distance to the galaxy?

(f) What is the age of the universe when the light was emitted? (HINT: the contribution of radiation does not affect the answer significantly.)

(g) If the maximum velocity is maintained out to $\sim 10\times$ the scale length determined from the surface brightness of the stars, what's the total mass of the galaxy? Is it likely the galaxy contains dark matter? Why or why not?

5. Imagine the universe contained radiation, matter, and dark energy with equation of state parameter $w_{\rm DE} = -0.5$ and current density parameters $\Omega_{\rm DE,0} = 0.9$, $\Omega_{\rm m,0} = 0.09$, and $\Omega_{\rm rad,0} = 0.01$. (a) Over what redshift ranges does dark energy, matter, and radiation each dominate? (b) What is the current deceleration parameter q_0? Is this universe currently accelerating or decelerating? (c) What is the curvature (positive, negative, or flat) of this universe? Explain your reasoning. (d) If $H_0 = 50$ km/s/Mpc, what is the current proper distance to a galaxy at a redshift $z = 0.4$?