Homework 1

Due **January 28 at 10:45pm via Canvas upload**
Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Cosmic strings are hypothetical ∼1-dimensional topological defects in space-time that may have formed in the early universe. In the novel *Tides of Light* by Gregory Benford (also a physicist), an advanced alien civilization uses a cosmic string to bore a hole through the center of a planet to mine heavy metals in its core. The main character is tossed down the shaft through the planet and calculates how long the journey will take him, which is given as this equation in the 1989 paperback edition:

$$\text{time} = \left[\frac{\pi}{2} - \tan^{-1}\frac{v}{R\sqrt{\frac{4\pi}{3}G\rho}}\right]\frac{3}{4\pi G\rho}\,, \tag{1}$$

where v is his initial velocity, R is the radius of the planet, G is the gravitational constant, and ρ is the planet's average density. Use this equation (and common sense reasoning) to estimate the length of the journey for an astronaut falling through an Earth-like planet.

2. Suppose you are in an infinitely large, infinitely old universe in which the average number density of stars is $n_* = 10^9$ Mpc^{-3} and the average stellar radius is equal to the Sun's radius of $R_* = R_\odot = 7 \times 10^8$ m.

   (a) How far, on average, could you see in any direction before your line of sight struck a star? (Assume standard Euclidean geometry holds true in this universe.) Put your answer in units of Mpc.

   (b) If the stars are clumped into galaxies with a number density $n_g = 1$ Mpc^{-3} and average radius $R_g = 2000$ pc, how far, on average, could you see in any direction before your line of sight hit a galaxy? Put your answer in units of Mpc.

3. A hypothesis once used to explain the Hubble relation is the "tired light hypothesis." The tired light hypothesis states that the universe is not expanding, but that photons simply lose energy as they move through space (by some unexplained means), with the energy loss per unit distance being given by the law

$$\frac{dE}{dr} = -KE\,, \tag{2}$$

where K is a constant.

   (a) Show that this hypothesis gives a distance-redshift relation that is linear in the limit $z \ll 1$.

   (b) What must the value of K be in order to yield a Hubble constant of $H_0 = 70$ km s^{-1} Mpc^{-1}?

4. In a Steady State Universe, H_0 is predicted to be constant with time and matter must be created at a constant rate to maintain a constant density. In this model, matter must be created in the solar system at the same constant rate. The solar system is virialized (gravitationally bound) and does not follow cosmological expansion. Instead, whatever is created in the solar system stays in the solar system and contributes to the mass of the Sun. For a Hubble constant of $H_0 = 70$ km s^{-1} Mpc^{-1}, assume that all matter created in a sphere with radius 1000 AU was used to create the Sun. How many years does it take to generate enough material in this steady state universe to form the Sun?