Homework 2

Due **February 4 at 2pm in class**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Awhile back (when in-person encounters were possible—those were the days), my office was visited by someone interested in entering our graduate program. Most of the conversation centered around his pet dark matter theory (for the record, not a good strategy, even if your idea has some merit). In essence, he was suggesting that you could explain dark matter in the Milky Way as entirely being due to charge separation; all you need to explain the missing mass (which we call dark matter) is about a Jupiter-mass of electrons separated from protons over galaxy scales (with, say, all the protons in galaxy centers and electrons in galaxy halos). The electrostatic energy between the separated charges would have the same mass-energy as an equivalent mass-energy of dark matter. Should I have lobbied to have him accepted into the graduate program, knowing we'll one day share a Nobel Prize, or are there problems with his idea? Be as specific and clear in your arguments as you can.

2. Suppose you are a two-dimensional being, living on the surface of a sphere with radius R. Show that if you draw a circle of radius r, the circle's circumference will be
$$C = 2\pi R \sin\frac{r}{R}\,. \tag{1}$$
Idealize the Earth as a perfect sphere of radius R=6370 km. If you could measure distances with an error of ± 1 meter, how large of a circle would you have to draw on the Earth's surface to convince yourself that the Earth is spherical rather than flat?

3. Show that, for each value of $\kappa = (+1, 0, -1)$, equations 3.33-3.35 are equivalent to equation 3.36 under the substitution $x \equiv S_\kappa(r)$.

4. Assuming the scale factor $a(t)$ evolves as a power law with time as

$$a(t) = \left(\frac{t}{t_0}\right)^\Gamma \tag{2}$$

where the power law index $\Gamma > 0$, derive an expression for the Hubble parameter $H(z)$ as a function of the age of the universe t_0, Γ, and the redshift z at time t. What is the age of the universe if $\Gamma = 1/2$ and $H_0 = 70$ km/s/Mpc? For what value of Γ is the age of the universe equal to the Hubble time?