Homework 3

Due **February 11 at 10:45am via Canvas**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Suppose the energy density of the cosmological constant is comparable to the present critical density $\varepsilon_\Lambda = \varepsilon_{\rm crit,0} \sim 5$ GeV m^{-3}. What is the total energy of the cosmological constant within a sphere of 1 AU radius? Compare this value to the rest energy of the Sun. Should the cosmological constant have a significant effect on the motion of planets within the solar system given the energies at play? Given the rest mass of the Milky Way galaxy (a hundred billion stars, although 90% of the total mass is in the form of dark matter), how far away would another galaxy need to be for the influence of the cosmological constant and the Milky Way's gravity to be comparable?

2. Consider Einstein's static universe, in which the attractive force of the matter density is exactly balanced by the repulsive force of the cosmological constant ($\Lambda = 4\pi G\rho$). Suppose that some matter is converted into radiation (by stars for instance, since that's precisely what they do!). Will the universe start to expand or contract? Explain your answer.

3. Again consider Einstein's static universe. If $\rho = 2.7 \times 10^{-27}$ kg m^{-3}, what is the radius of curvature R_0? How long would it take a photon to circumnavigate such a universe?

4. The principle of wave-particle duality tells us that a particle with momentum p has an associated de Broglie wavelength of $\lambda = h/p$; just like with light, this wavelength is tied to the expansion of space: $\lambda \propto a$. The total energy density of a gas of particles can be written as $\varepsilon = nE$, where n is the number density of particles and E is the energy per particle. For simplicity, let's assume that all the gas particles have the same mass m and momentum p. The energy per particle then simply follows the relation:

$$E^2 = (m^2c^4 + p^2c^2) = (m^2c^4 + h^2c^2/\lambda^2) \tag{1}$$

Compute the equation-of-state parameter w for this gas as a function of the scale factor a. Show that $w = 1/3$ in the highly relativistic limit ($a \to 0$, $p \to \infty$) and that $w = 0$ in the highly non-relativistic limit ($a \to \infty$, $p \to 0$).