Homework 4

Due **February 25 at 10:45am via Canvas**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Consider a flat, single component universe.

   (a) For a light source at redshift z that is observed at time t_0, show that z changes at a rate

   $$\frac{dz}{dt_0} = H_0(1+z) - H_0(1+z)^{3(1+w)/2}\,. \tag{1}$$

   (b) For what values of w does the observed redshift increase with time?

   (c) Assuming the single component is matter and $H_0 = 68$ km/s/Mpc, you observe a galaxy at $z = 1$. Using Equation 1, determine how long you will have to keep observing the galaxy in order to see its redshift change by 1 part in 10^6.

2. Although current data are consistent with dark energy being due to a cosmological constant $w = -1$, other values of w are acceptable depending on the exact nature of dark energy. One possibility is called "phantom energy," where $w < -1$. Suppose that the universe is spatially flat and contains matter with a density parameter $\Omega_{\rm m,0}$, and phantom energy with a density parameter $\Omega_{\rm p,0} = 1 - \Omega_{\rm m,0}$ and equation of state parameter $w_{\rm p} < -1$.

   (a) At what scale factor $a_{\rm mp}$ are the energy densities of phantom energy and matter equal?

   (b) Write down the Friedmann equation for this universe in the limit that $a \gg a_{\rm mp}$. Integrate this equation to show that the scale factor a goes to infinity at a finite cosmic time $t_{\rm rip}$, given by the relation

   $$H_0(t_{\rm rip} - t_0) \approx \frac{2}{3\mid 1 + w_p \mid}(1-\Omega_{\rm m,0})^{-1/2}\,. \tag{2}$$

   (c) Current observations of our own universe are consistent with $H_0 = 68\,$km/s/Mpc, $\Omega_{\rm m,0} = 0.3$, and $w_{\rm p} = -1.1$. If these numbers are correct, how long do we have remaining until the Big Rip?

3. In the Benchmark Model, what is the total mass of all the matter within our horizon? What is the total energy of all the photons within our horizon? How many baryons are within the horizon?