Homework 5

Due **March 4 at 10:45am via Canvas**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Let's say some apocalyptic event has taken place (so many options!), and you're reduced to a single 100 watt LED you use to mark the location of your encampment so you can find it after foraging at night (because daytime temperatures are too hot, of course). What is the bolometric absolute magnitude of the light? What is its bolometric apparent magnitude at a luminosity distance $d_L = 5$ km? What is the maximum luminosity distance at which you could see this light (use the magnitude of the faintest visible stars as a reference)? What is the maximum luminosity distance at which you could see a naked-eye supernova ($L \sim 3 \times 10^9\ L_\odot$)?

2. Suppose you are in a spatially flat universe containing a single component with a unique equation of state parameter w.

   (a) For $w \neq -1/3$, what is the current proper distance $d_p(t_0)$ as a function of z and w? What is the expression for $d_p(t_0)$ if $w = -1/3$?

   (b) For $w \neq -1/3$, what is the luminosity distance d_L and the angular-diameter distance d_A as a function of z and w in units of the Hubble distance?

   (c) At what redshift will d_A have a maximum value? What will this maximum value be, in units of the Hubble distance?

3. The surface brightness Σ of an astronomical object is defined as its observed flux f divided by its observed angular area $\Delta\Omega$, $\Sigma = f/\Delta\Omega$. Consider a class of objects that are both standard candles and standard yardsticks. The bolometric luminosity of the standard candle is L. Suppose that the comoving distance is r to an object at redshift z. For simplicity and without losing generality, you can assume that the object appears to be a square with proper (physical) side length of $\Delta\ell$.
   (a) What is the observed flux f of the object?
   (b) What does the observed angular area $\Delta\Omega$ of the object appear to be?
   (c) What is the surface brightness Σ of the object?
   (d) Would observing the surface brightness of this class of objects be a useful way of determining the value of the deceleration parameter q_0? Why or why not?

4. Let's say you want to determine whether the universe is accelerating or decelerating using some kind of standard candle, and you can detect that standard candle out to a redshift $z \sim 0.4$. What precision do you need in your distance measurements at this redshift to distinguish the Benchmark Model from an Einstein-de Sitter Model (flat, matter-only universe)? What precision do you need to distinguish between the Benchmark Model and a flat universe dominated by the cosmological constant?