Homework 9

Due **April 8 at 10:45am via Canvas**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. What upper limit is placed on $\Omega(t_P)$ by the requirement that the universe not end in a Big Crunch between the Planck time, $t_P \sim 5 \times 10^{-44}$ s, and the start of the inflationary epoch at t_i? Compute the maximum value of $\Omega(t_P)$, first assuming $t_i \sim 10^{-36}$ s, then assuming $t_i \sim 10^{-26}$ s. (Hint: prior to inflation, the Friedmann equation will be dominated by the radiation term and the curvature term.)

2. Current observational limits on the density of magnetic monopoles tells us that their density parameter is currently $\Omega_{\rm M,0} < 10^{-6}$. If monopoles formed at the GUT time, with one monopole (mass $m_{\rm M} = m_{\rm GUT}$) per horizon, how many e-foldings of inflation would be required to drive the current density of monopoles below the bound $\Omega_{\rm M,0} < 10^{-6}$? Assume that inflation took place immediately after the formation of monopoles.

3. **Presentation Prep**: Submit a slide-by-slide outline of your talk. Each slide should have a brief (1 sentence) specific description (e.g., "Slide 1: introduction" isn't very useful, but "Slide 1: remind everyone what the evidence for dark matter is" is). Feel free to sketch what some slides will look like if that would be useful.

4. What if the dark energy that dominates our universe today is just the manifestation of another round of inflation? It has been speculated that the present day acceleration of the universe is due to the existence of a false vacuum, much like the false vacuum that may be responsible for inflation between time t_i and time t_f in the early universe. Just as inflation is postulated to occur as a significant cooling phase during a period between a radiation dominated regime (or maybe magnetic monopole dominated?) and another radiation dominated regime, this model for the cosmological constant is postulated to eventually decay and release energy back into the universe in the form of radiation. Suppose that the energy density of this false vacuum today is $\varepsilon_\Lambda = 0.69\varepsilon_{c,0} = 3360$ MeV m^{-3}, and the current matter density is $\varepsilon_{m,0} = 0.31\varepsilon_{c,0} = 1510$ MeV m^{-3}.

   (a) What will be the value of the Hubble parameter once the false vacuum becomes strongly dominant?

   (b) Suppose that the false vacuum is fated to decay instantaneously to radiation at a time $t_f = 50t_0$. Assume that the radiation takes the form of blackbody photons. To what temperature will the universe be reheated at $t = t_f$?

   (c) At what time will the universe again be dominated by matter? (Remember, when matter was created during baryogenesis, the temperature was high enough that quarks were not able to be bound up in nucleons.)