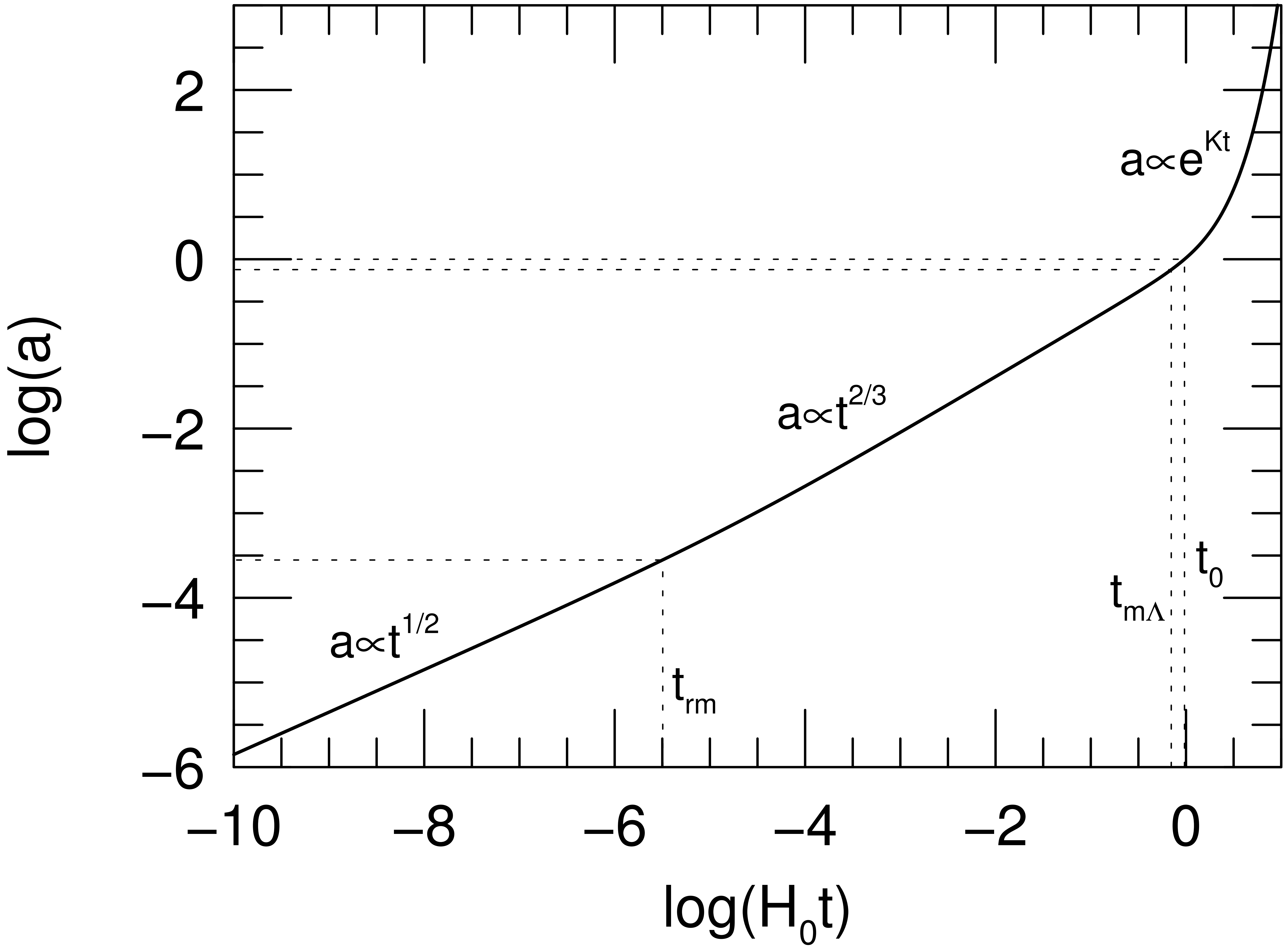
Midterm 2 Brief Review

Benchmark Model



$$\kappa = 0$$

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

$$\Omega_r = \Omega_\gamma + \Omega_\nu$$

$$\Omega_m = \Omega_{bary} + \Omega_{dm}$$

$\Omega_{m,0}$	$=$	0.31
$\Omega_{bary,0}$	$=$	0.048
$\Omega_{dm,0}$	$=$	0.262

$\Omega_{r,0} = 9.0 \times 10^{-5}$

$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$

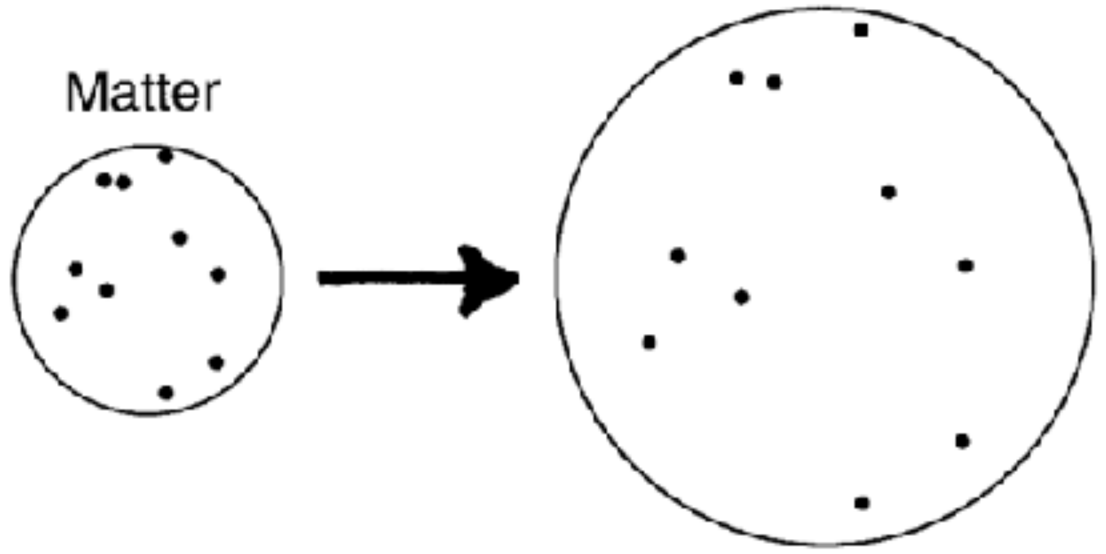
$\Omega_{\nu,0} = 3.65 \times 10^{-5}$

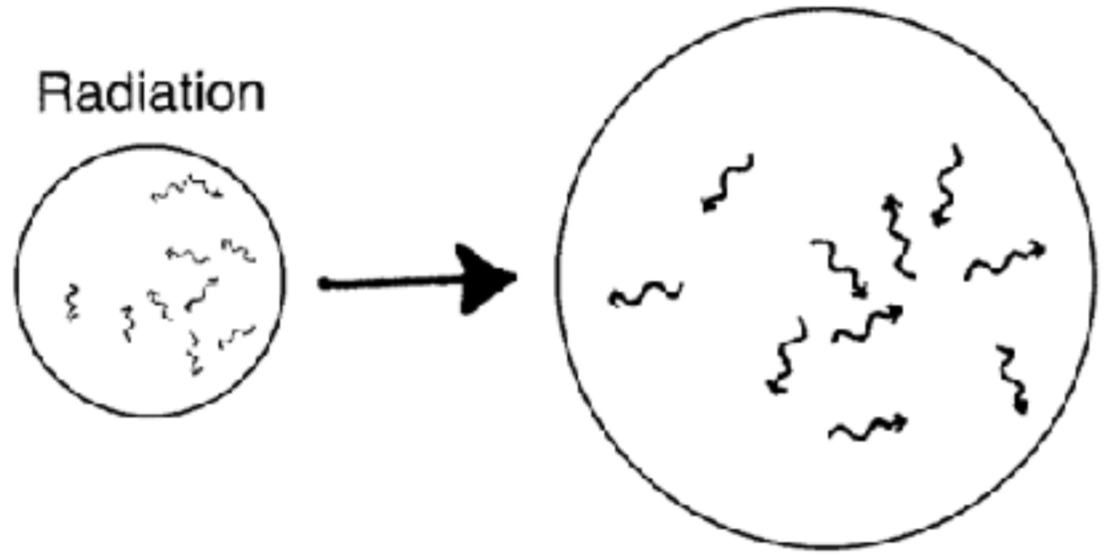
$$\Omega_{\Lambda,0} \approx 0.69$$

Rad. − Matter :	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 50$ kyr
Matter − Λ :	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now :	$a_0 = 1$	$t_0 = 13.7$ Gyr

$w = 0$ $\qquad$ $w = \frac{1}{3}$ $\qquad$ $w = -1$

$\varepsilon_m = \varepsilon_{m,0}/a^3$ $\qquad$ $\varepsilon_r = \varepsilon_{r,0}/a^4$ $\qquad$ $\varepsilon_\Lambda = \varepsilon_{\Lambda,0}$





$$\boxed{d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}}$$

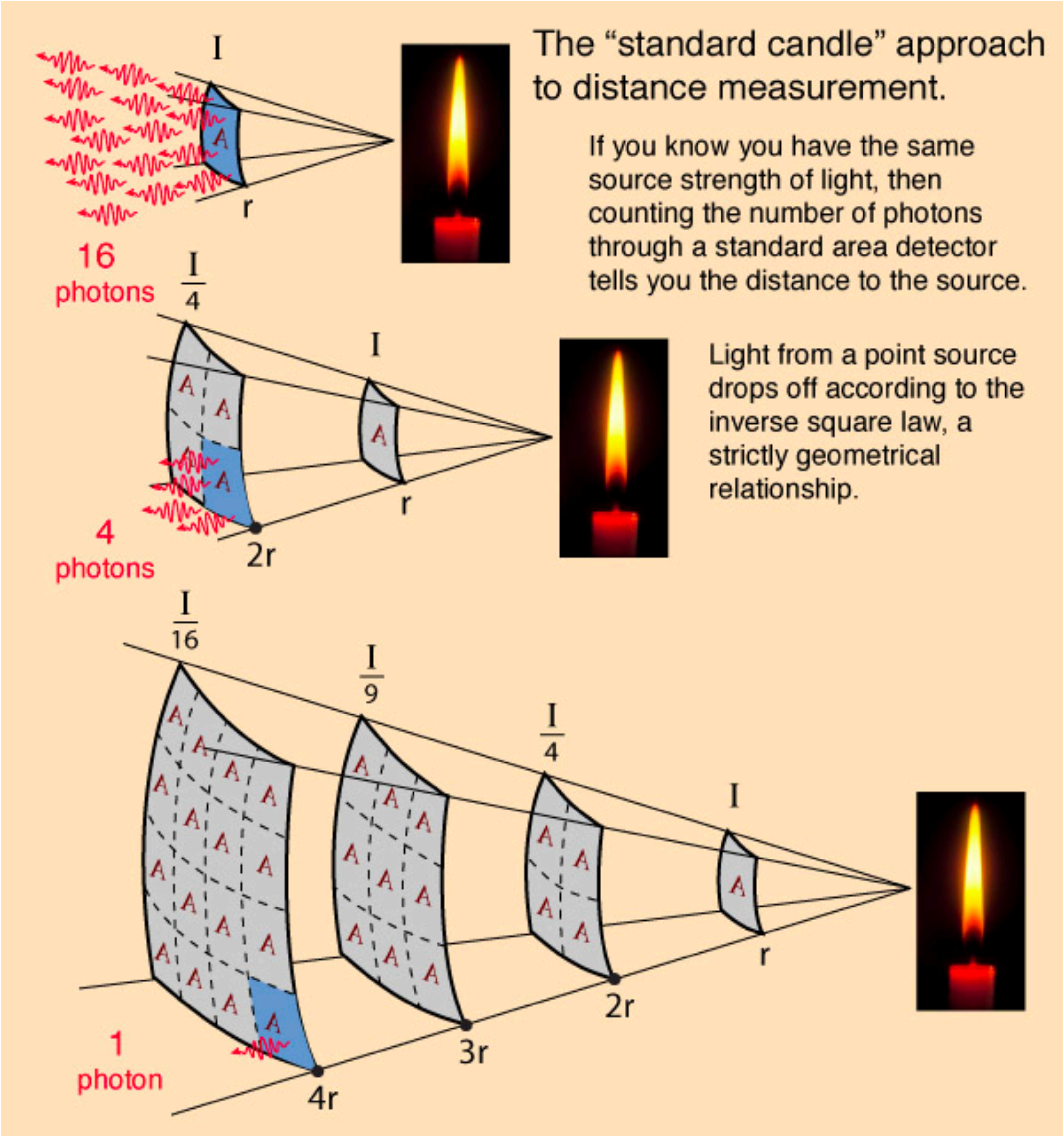
$$H_0^{-1}\dot{a} = \left[\Omega_{r,0}\, a^{-2} + \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + (1-\Omega_0)\right]^{1/2}$$

$$\boxed{d_p(t_e) = \frac{1}{1+z} d_p(t_0)}$$

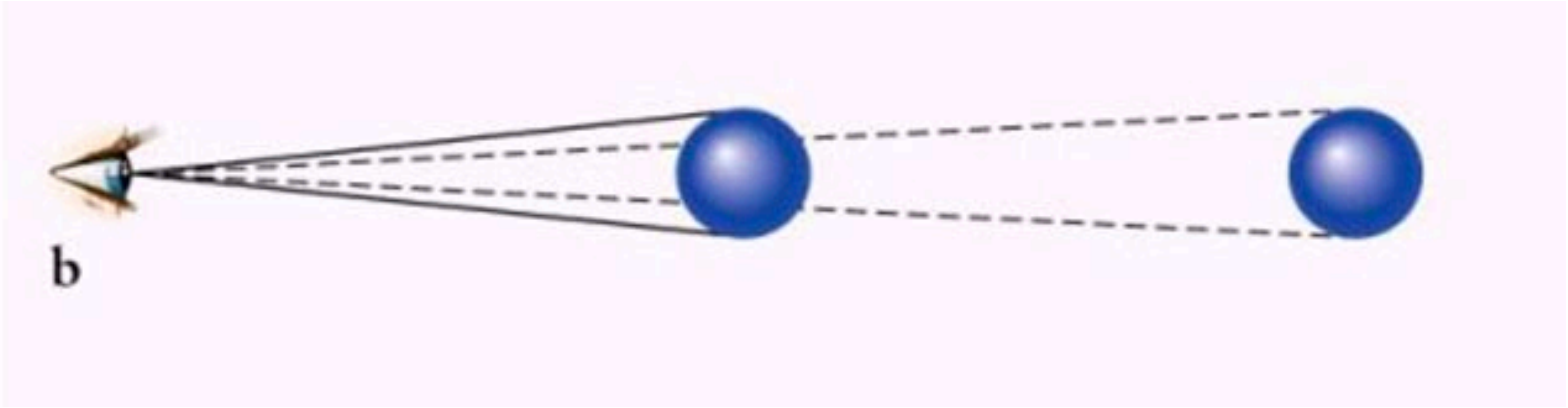
$$\boxed{t = \frac{1}{H_0}\int_0^a \frac{da}{E(a)}}$$

Practical Distance Measures

Luminosity Distance



Angular Diameter Distance



Practical Distance Measures

$$d_L = \sqrt{\frac{L}{4\pi f}}$$

$$d_A = \frac{D}{\theta}$$

in flat, static universe,

$$d_L = d_A = d_p$$

$$d_p \rightarrow ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

$$ds^2 = -c^2 dt^2 + a^2 \left[dr^2 + S_k(r)^2 d\Omega^2 \right]$$

$$S_k \begin{cases} R_0 \sin r/R_0 & k = +1 \\ r & k = 0 \\ R_0 \sinh r/R_0 & k = -1 \end{cases}$$

$$\frac{S_k(r)}{1+z} = d_A$$

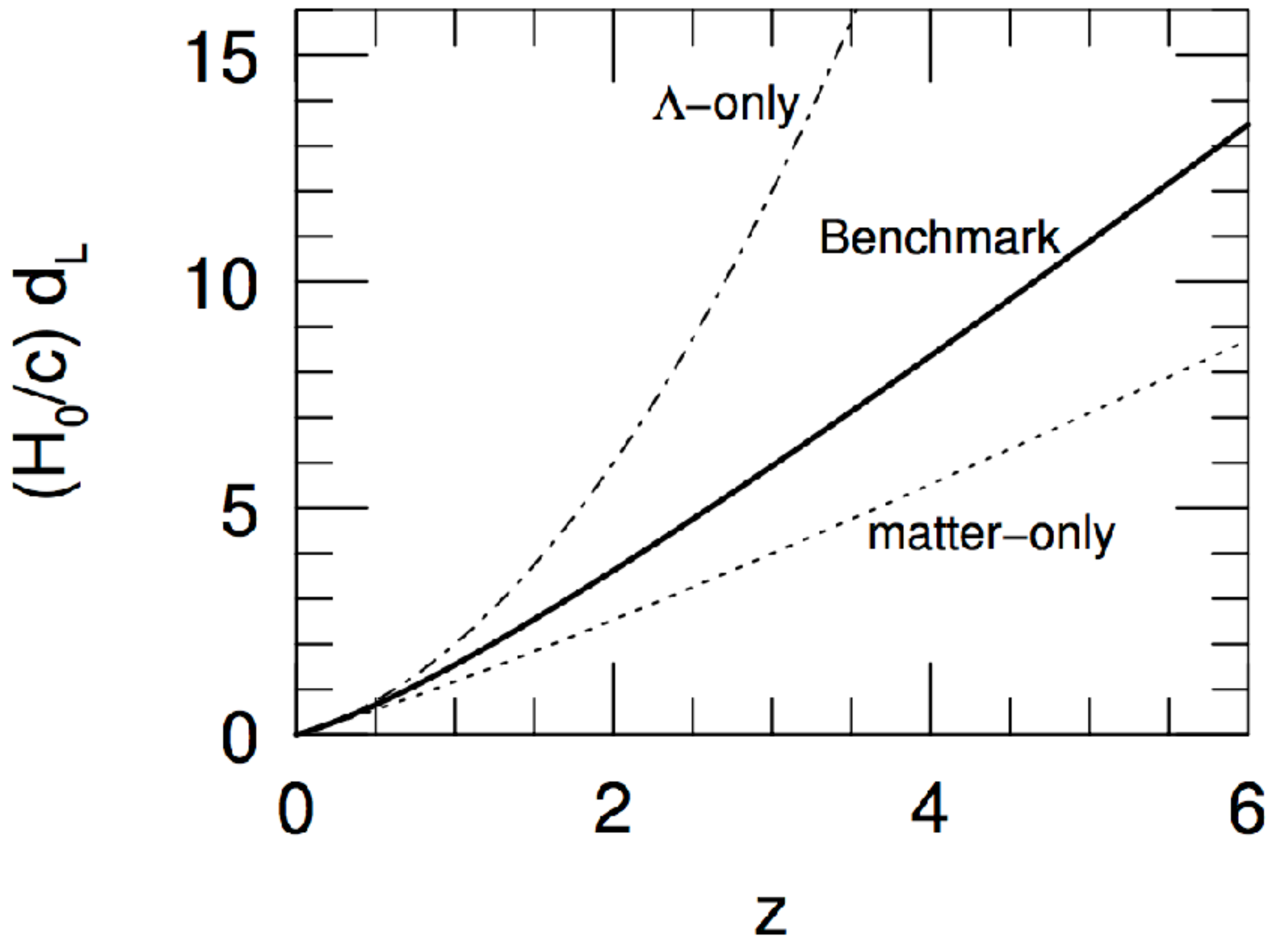
$$d_L = S_k(r)(1+z)$$

$$k = 0, \quad d_A = \frac{d_p(t_0)}{1+z} = \frac{d_L}{(1+z)^2}$$

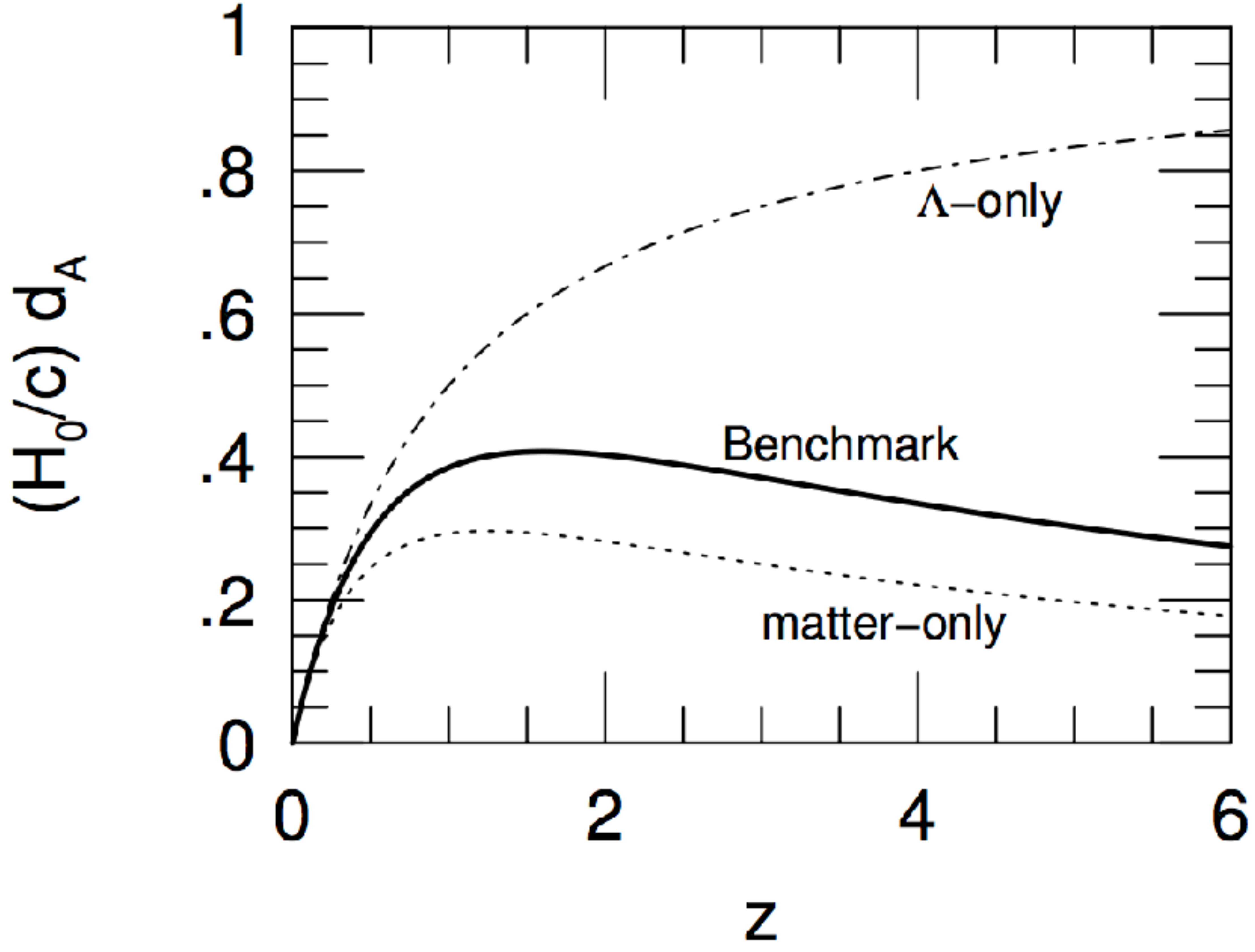
How distances are affected by underlying cosmology

$$d_L \approx \frac{cz}{H_0}\left(1 + \frac{1 - q_0}{2} z\right) \qquad q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0} \qquad d_A \approx \frac{cz}{H_0}\left(1 - \frac{3 + q_0}{2} z\right)$$

Luminosity Distance

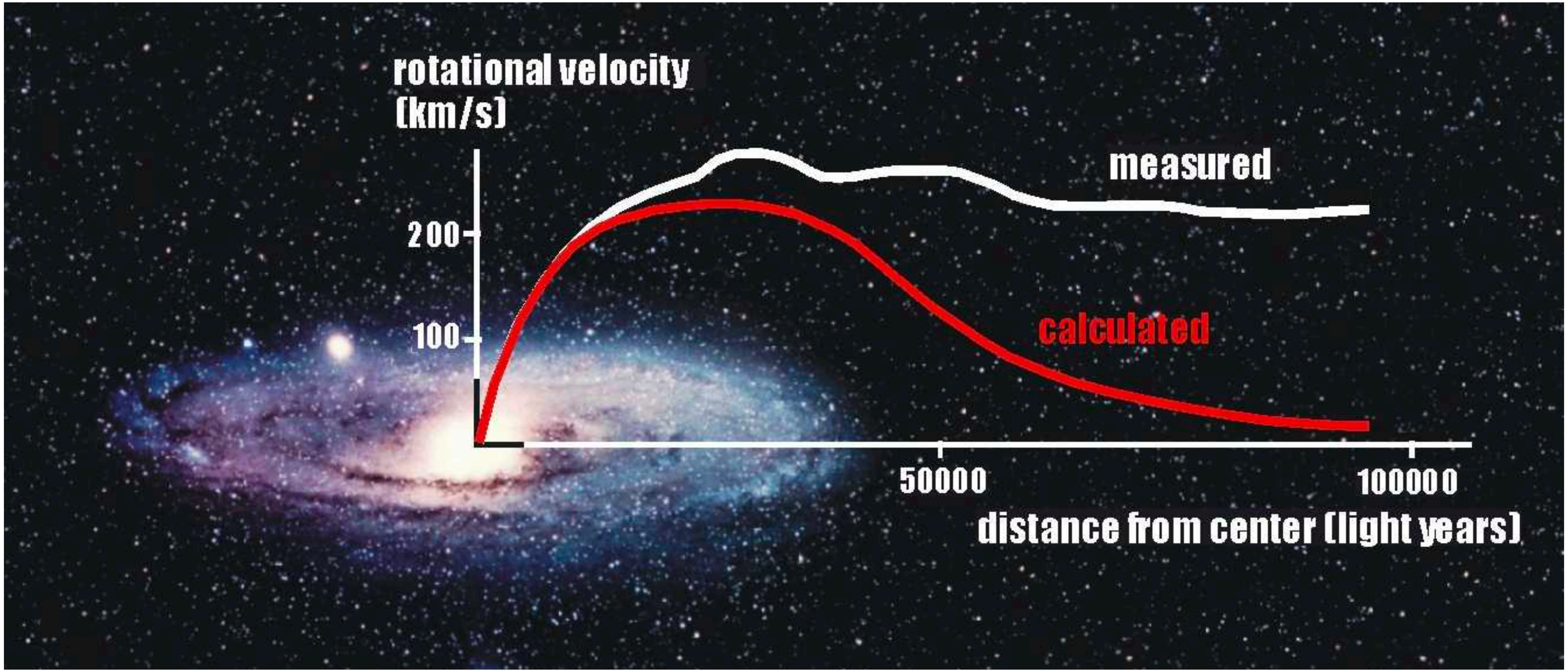


Angular Diameter Distance



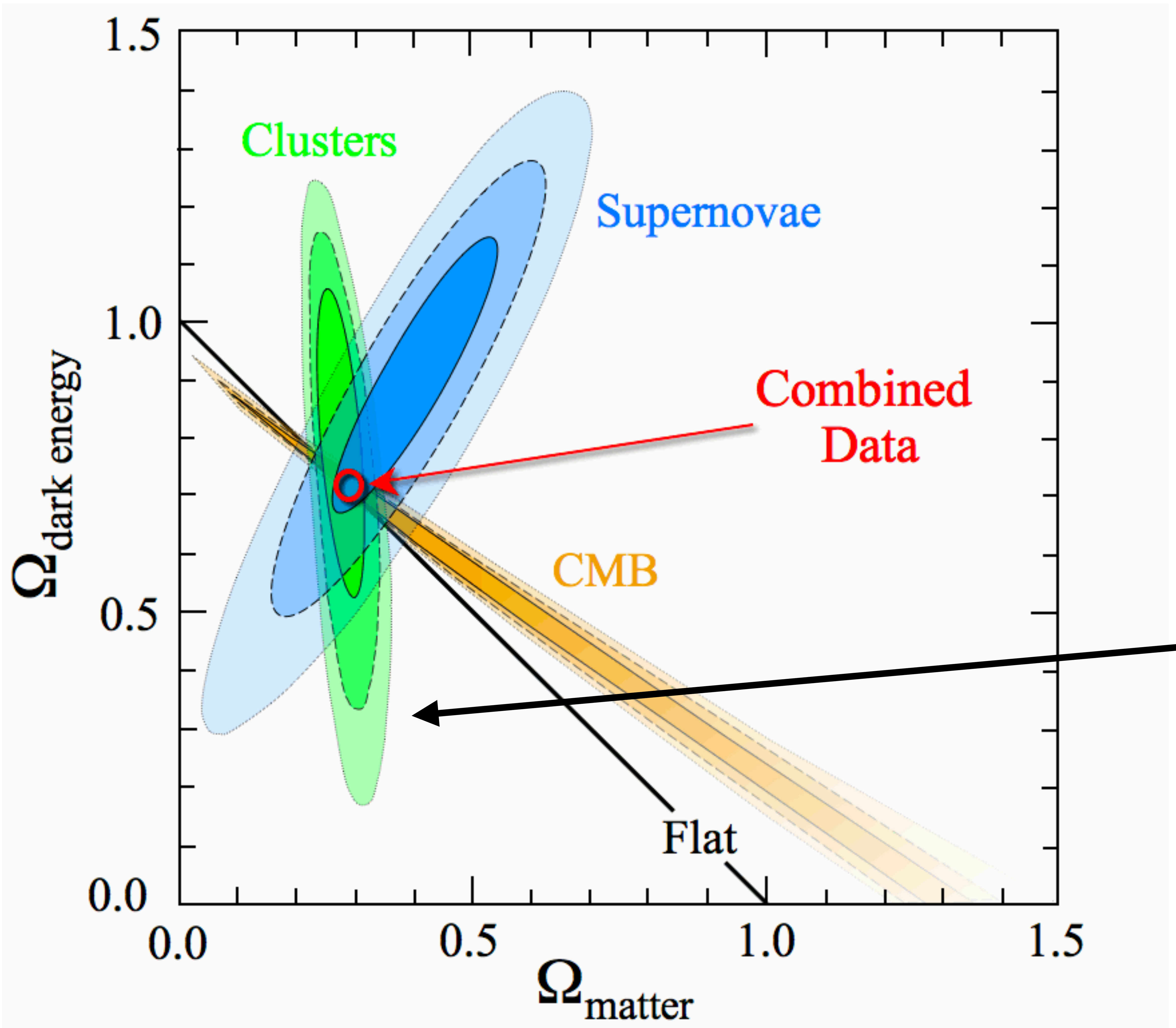
Clusters
Supernovae
Combined Data
CMB
Flat
$\Omega_{\text{dark energy}}$
Ω_{matter}
0.0
0.5
1.0
1.5
HST Discovered
Ground Discovered
μ
z
30
35
40
45
Binned Gold data
high-z gray dust
Evolution ~ z
pure acceleration: q(z)=-0.5
w=-1.2, dw/dz=-0.5
w=-0.8, dw/dz=+0.5
Ω_M=1.0, Ω_Λ=0.0
~ pure deceleration: q(z)=0.5
Empty (Ω=0)
Ω_M=0.29, Ω_Λ=0.71
Δ(m-M) (mag)
2.0
-0.5

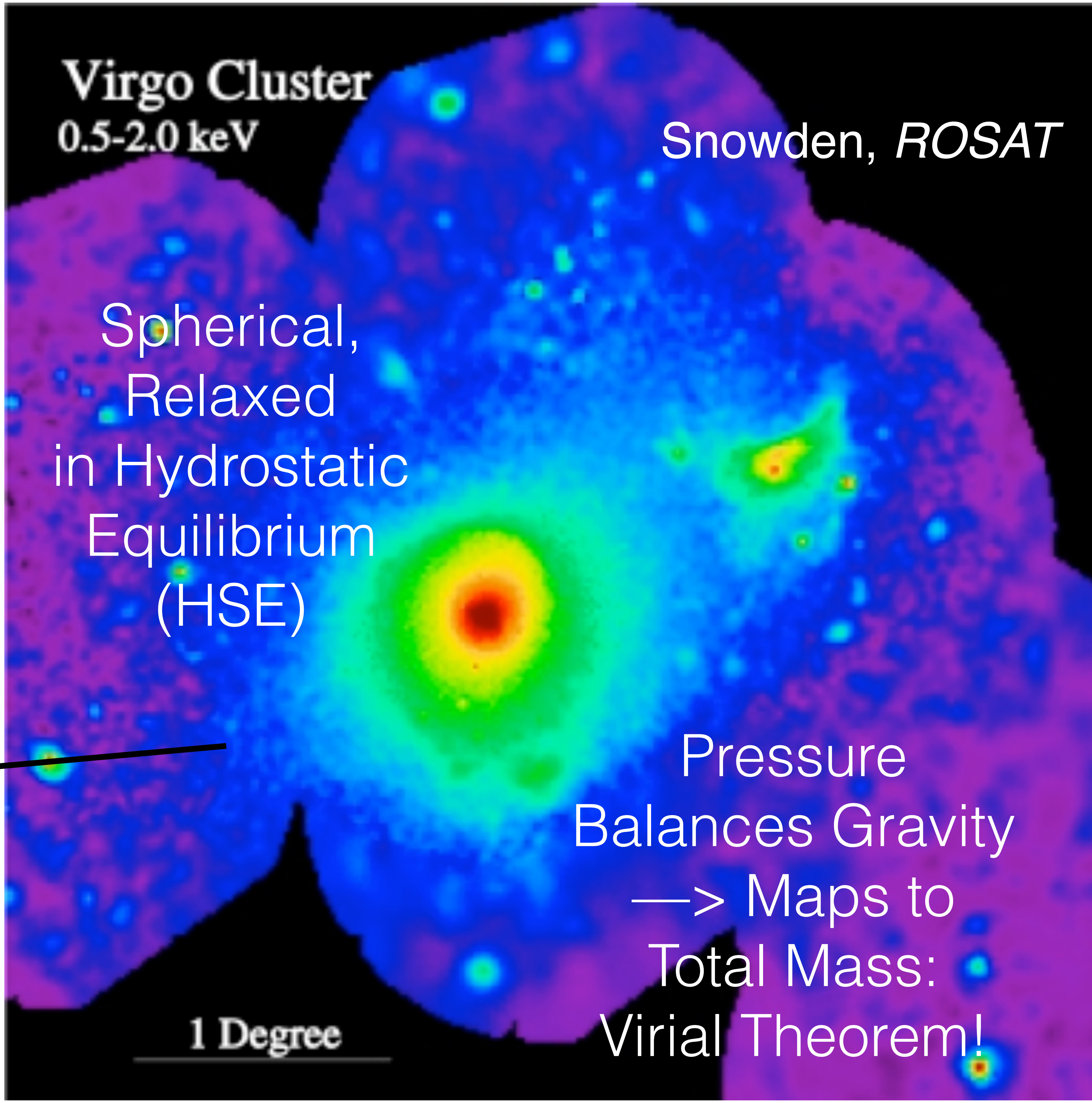
Dark Matter in Galaxies



$$M(R) = \frac{v^2 R}{G} = 1.05 \times 10^{11}\ \mathrm{M}_\odot \left(\frac{v}{235\ \mathrm{km\ s^{-1}}}\right)^2 \left(\frac{R}{8.2\ \mathrm{kpc}}\right)$$

Dark Matter in Galaxy Clusters





Snowden, *ROSAT*

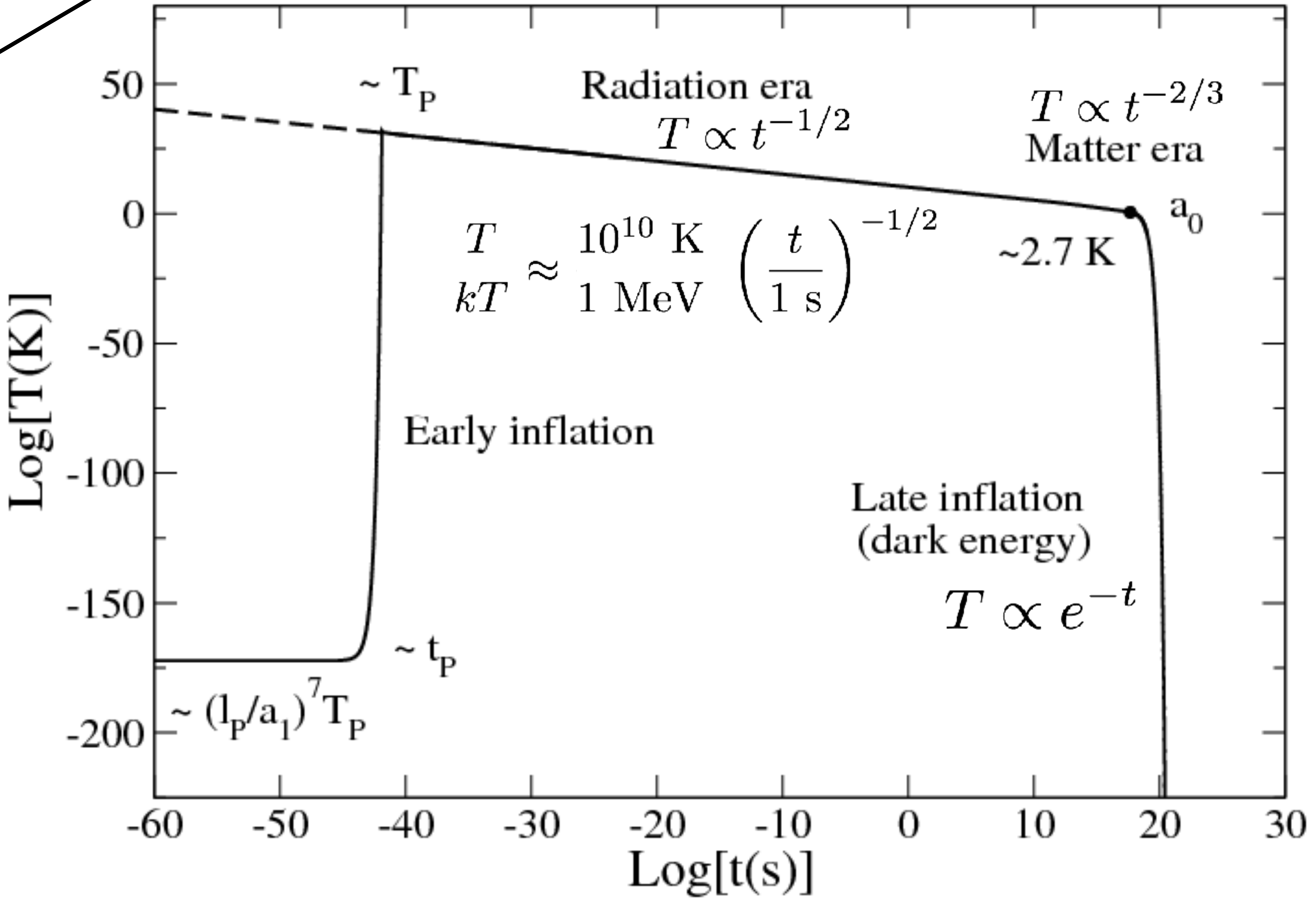
Spherical,
Relaxed
in Hydrostatic
Equilibrium
(HSE)

Pressure
Balances Gravity
—> Maps to
Total Mass:
Virial Theorem!

Inflation - quark soup - neutron capture - nucleosynthesis - recomb/decoup
kT: 150 MeV   10 MeV   0.07 MeV   3760/2970K

baryogenesis
photon-baryon ratio

Early Universe Timescales



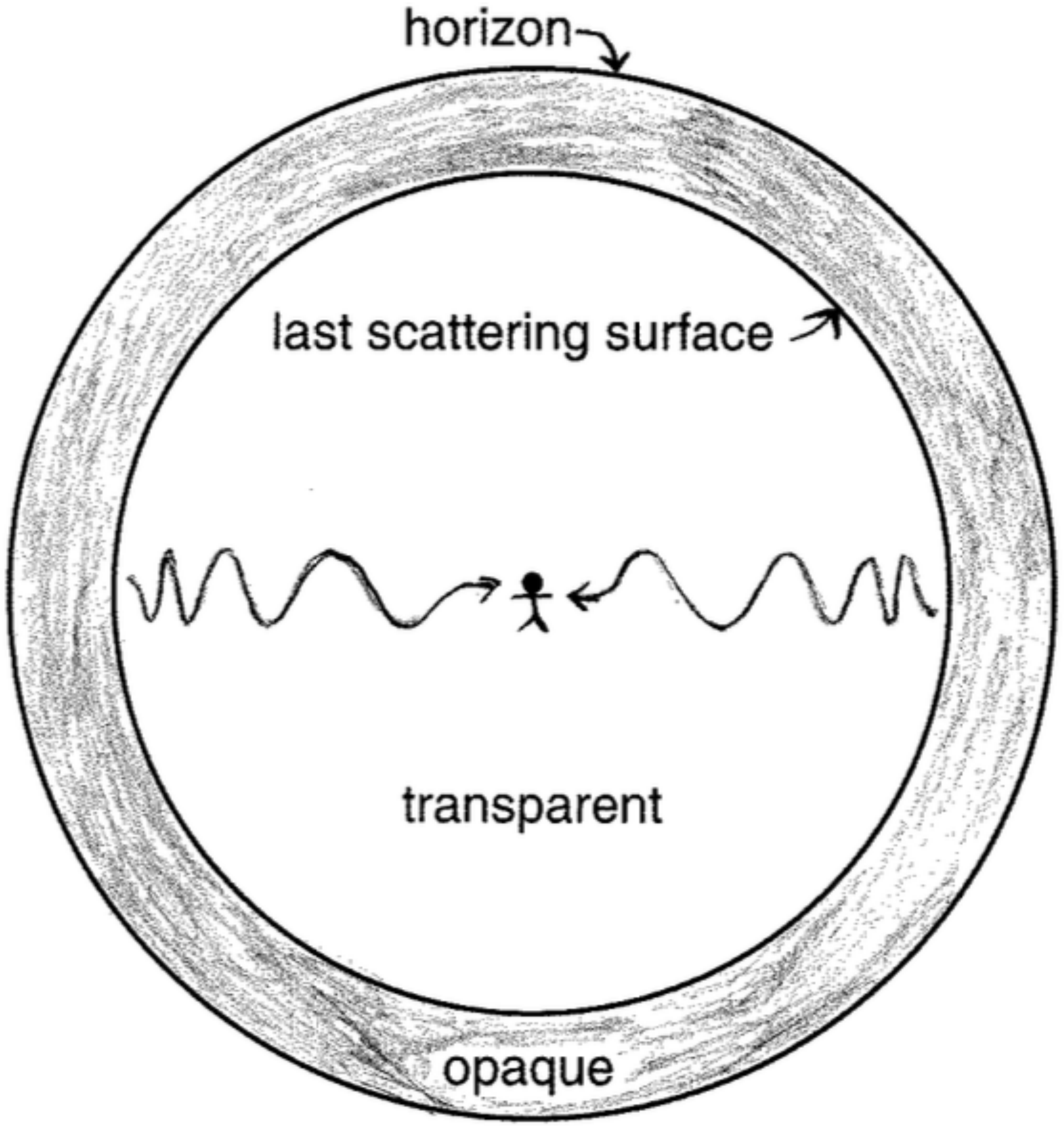
Surface of Last Scattering

Thomson cross-section
("size" of an electron)

$$\sigma_e = 6.65 \times 10^{-29} \text{ m}^2$$

mean free path of a photon

$$\lambda_{\text{mfp}} = \frac{1}{n_e \sigma_e}$$



Scattering interaction rate

$$\Gamma = \frac{c}{\lambda} = n_e \sigma_e c$$

When photons scatter off of electrons at the same rate as the expansion rate:

$$H(z) = \Gamma(z)$$

Photons and baryons will "decouple" from each other

Recombination $H + \gamma \rightleftharpoons p + e^-$

 # density of particles of type x with momenta b/t $p \to p + dp$

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{\exp([E - \mu_x]/kT) \pm 1}$$

(minus for bosons, plus for fermions)

g —> 2 (for non-nucleons, g_H=4)
chemical potential of photons = 0

$$\mu_H = \mu_p + \mu_e$$

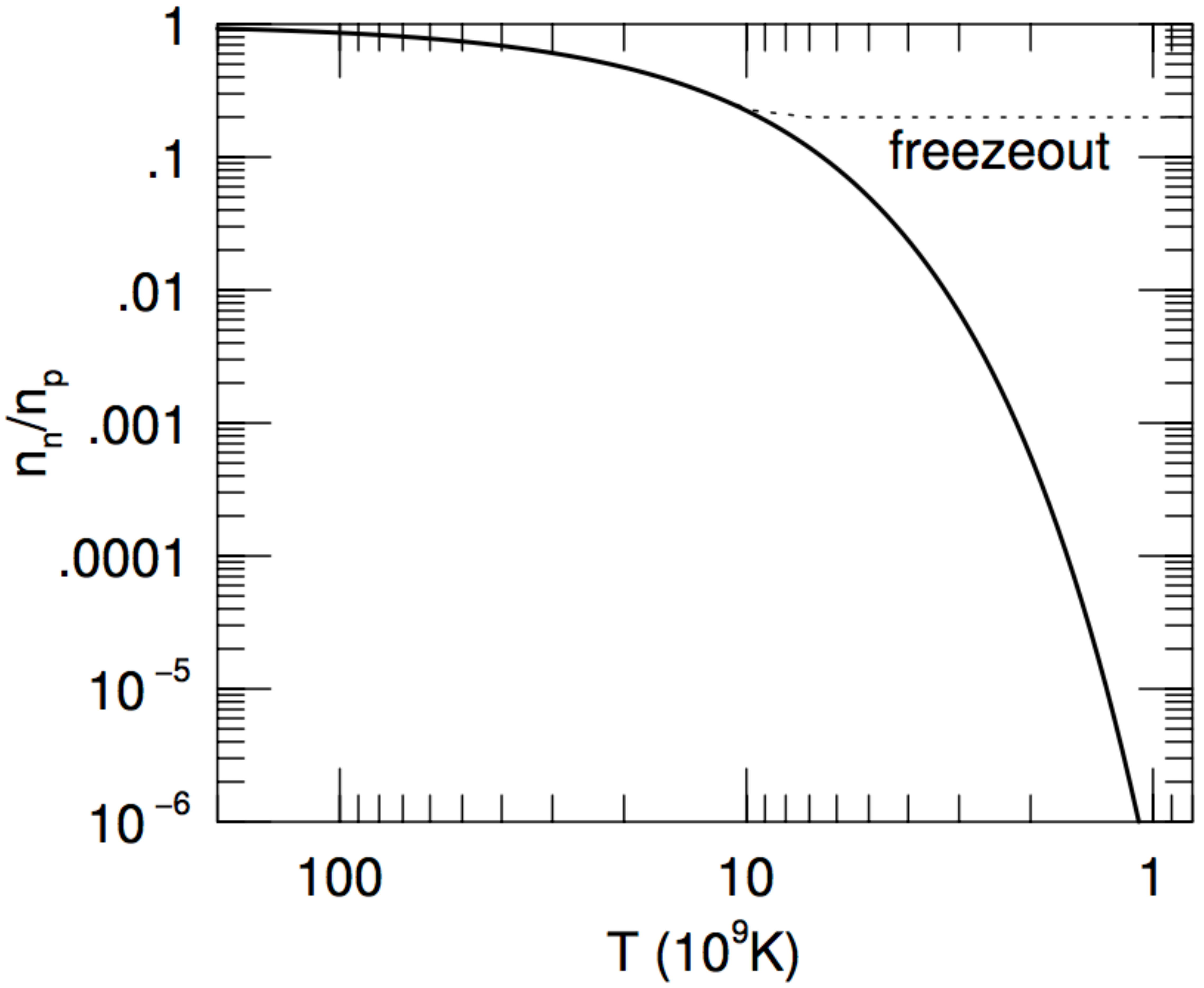
$$n_\gamma = \frac{2.4041}{\pi^2} \left(\frac{kT}{\hbar c} \right)^3$$

$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT} \right)$$

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e} \right)^{3/2} \left(\frac{kT}{2\pi\hbar^2} \right)^{-3/2} \exp\left(\frac{[m_p + m_e - m_H]c^2}{kT} \right) = \left(\frac{m_e kT}{2\pi\hbar^2} \right)^{-3/2} \exp\left(\frac{Q}{kT} \right)$$

Saha Equation

neutron-proton ratio

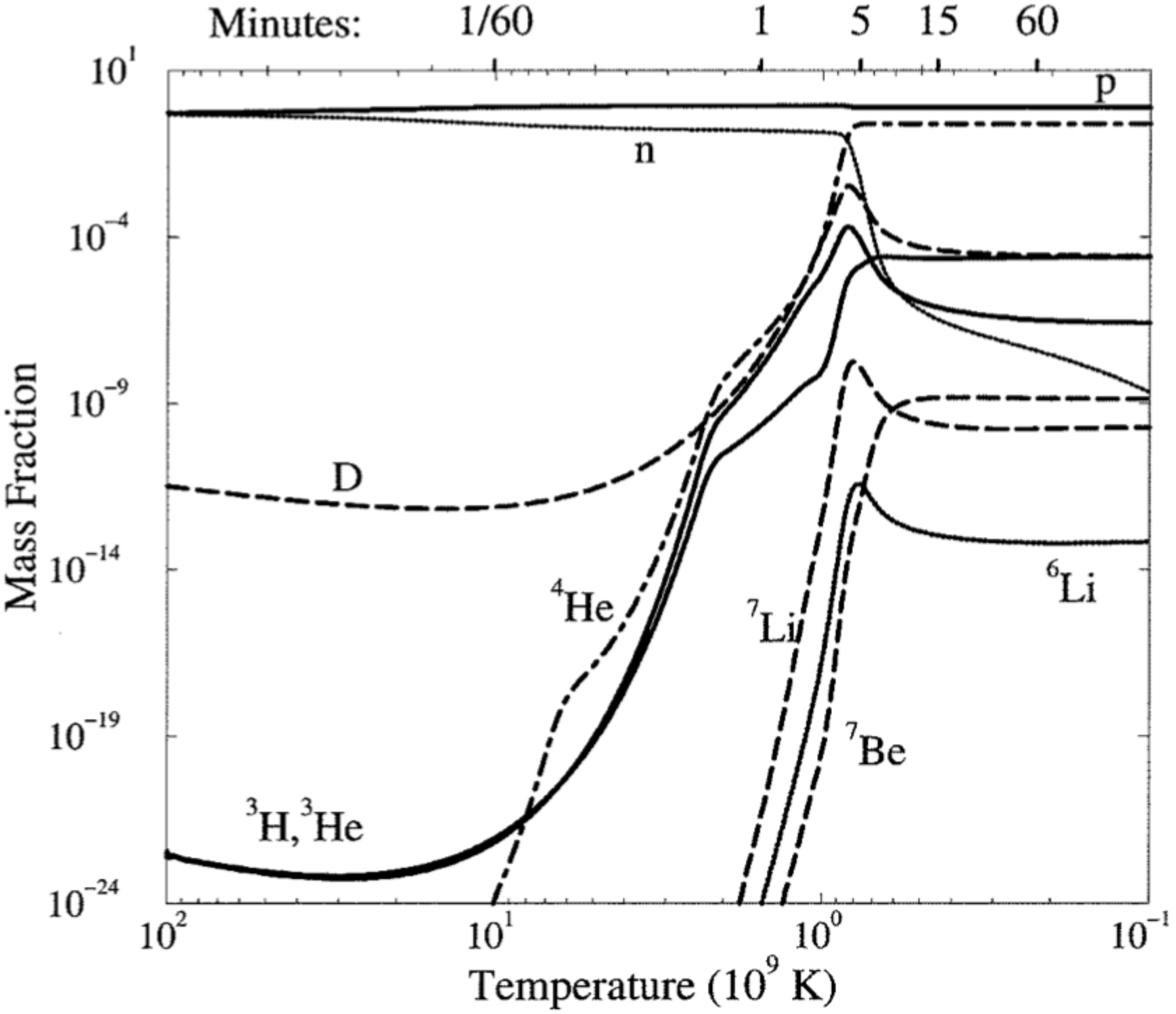


$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT}\right)$$

$$\frac{n_n}{n_p} = \exp\left(-\frac{(m_n - m_p)c^2}{kT}\right)$$

$$\Gamma = n_\nu c \sigma_w$$

Abundances from Nucleosynthesis

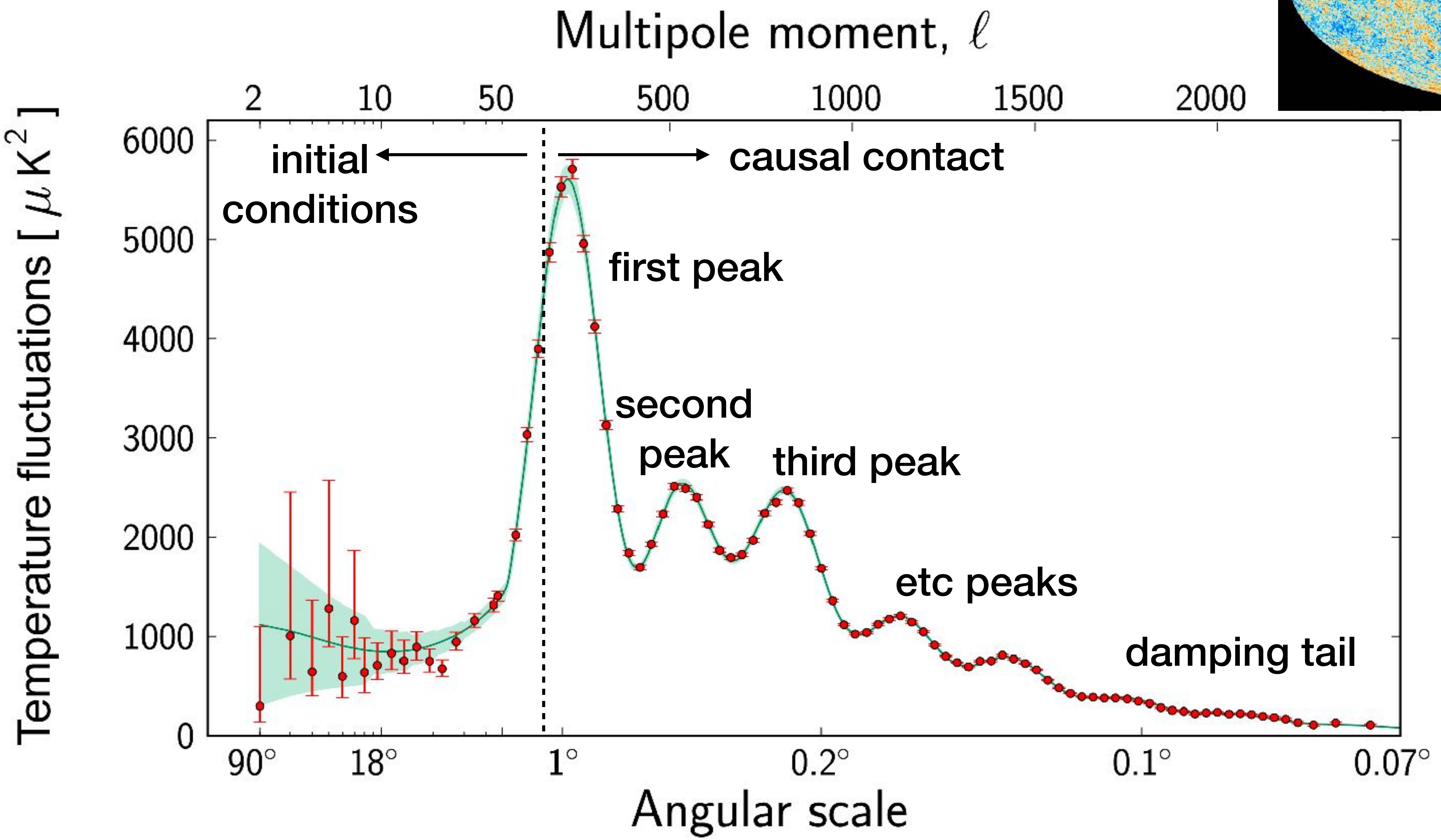


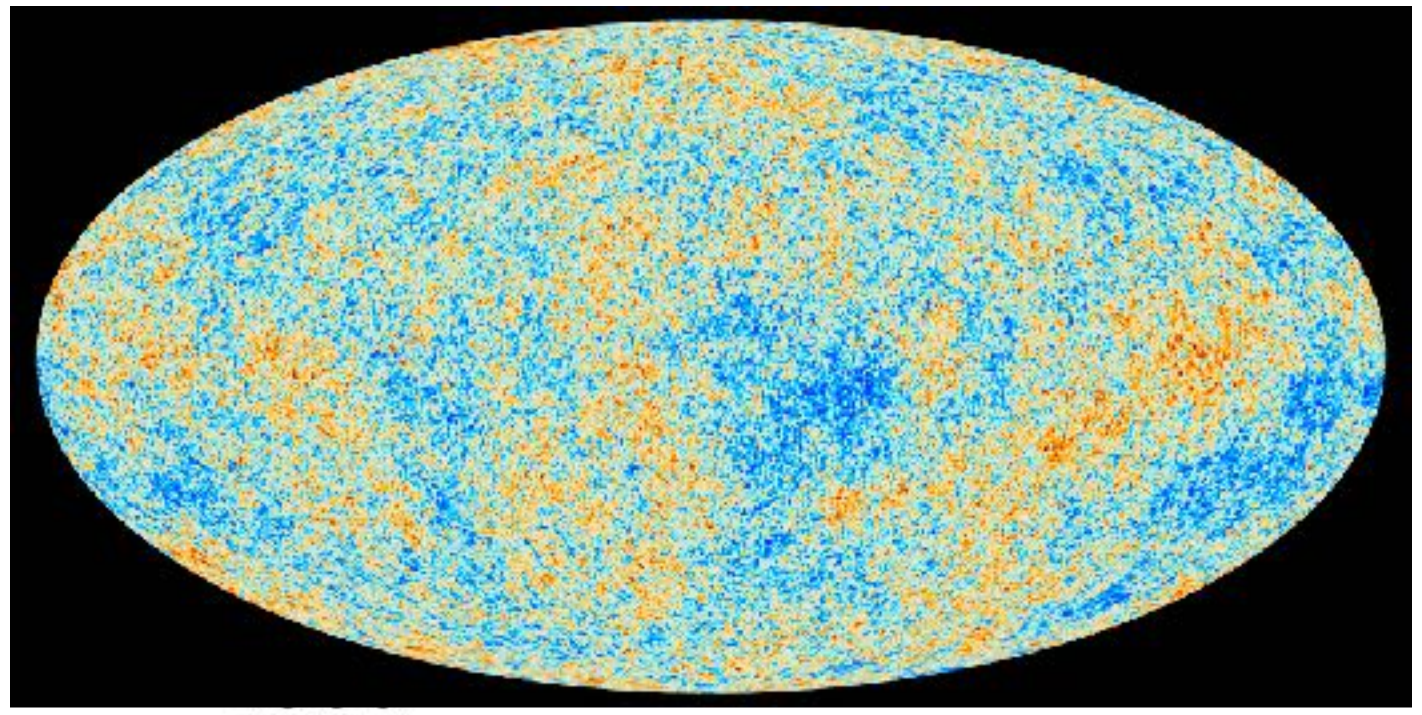
Creation process depends on relative abundances at any given time, so have to calculate computationally

Nucleosynthesis doesn't run to completion like in stars — rapidly dropping temperature cuts it off and "freezes" abundance pattern

Exact yields depend most on baryon-to-photon ratio: η
(determines temperature of nucleosynthesis)

Acoustic peaks





First peak:
spatially flat

Second peak:
existence of "dark baryons"

Third peak:
amount of dark matter

Damping tail:
photons can cross entire grav. fluct., wipes out signal

Baryonic Matter

$$\Omega_{*,0} \lesssim 0.005$$

$$M_{\text{gas},0} \approx 10 \times M_{*,0}$$

early universe measurements

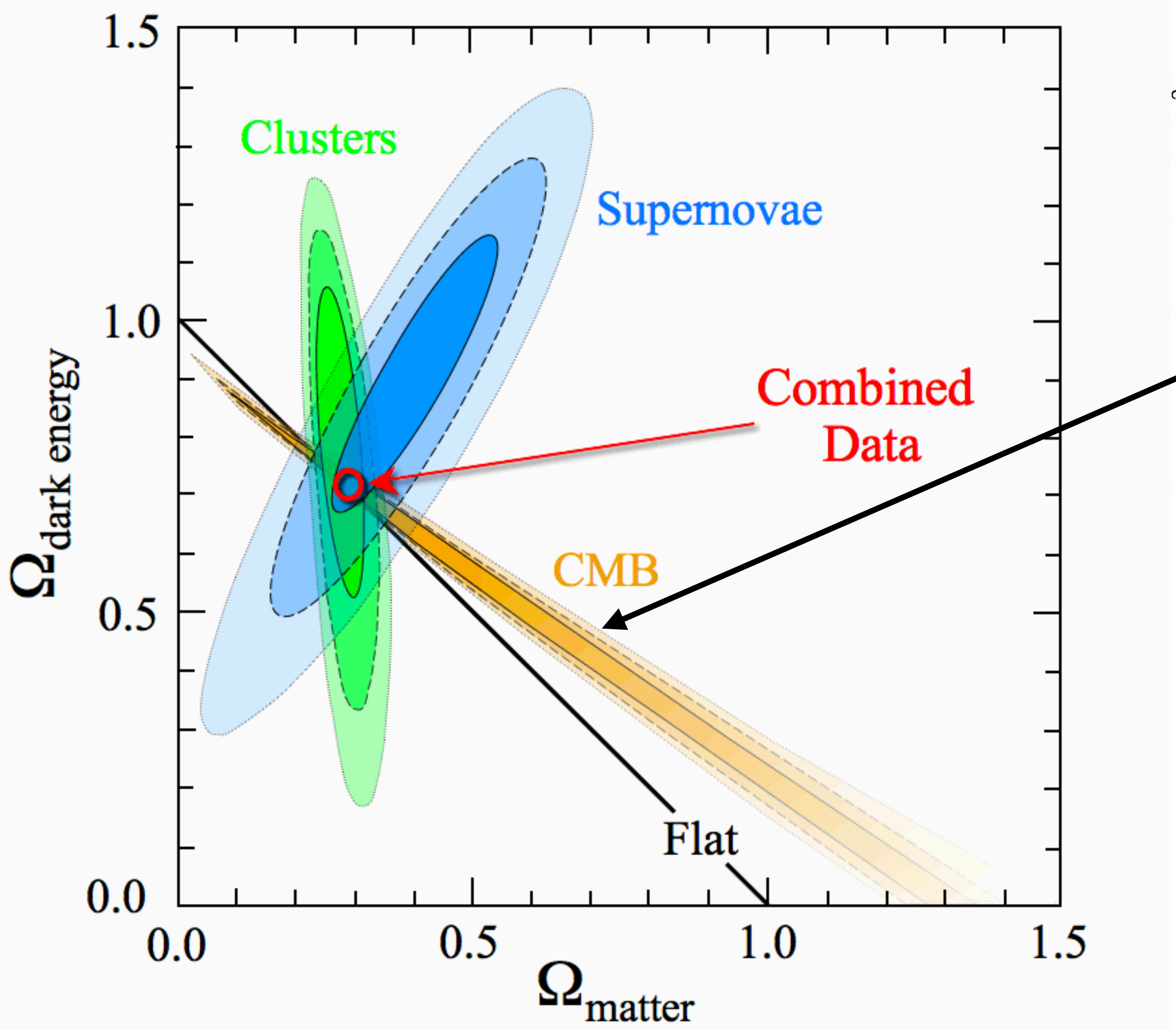
$$\Omega_{\text{bary},0} = 0.048 \pm 0.003$$

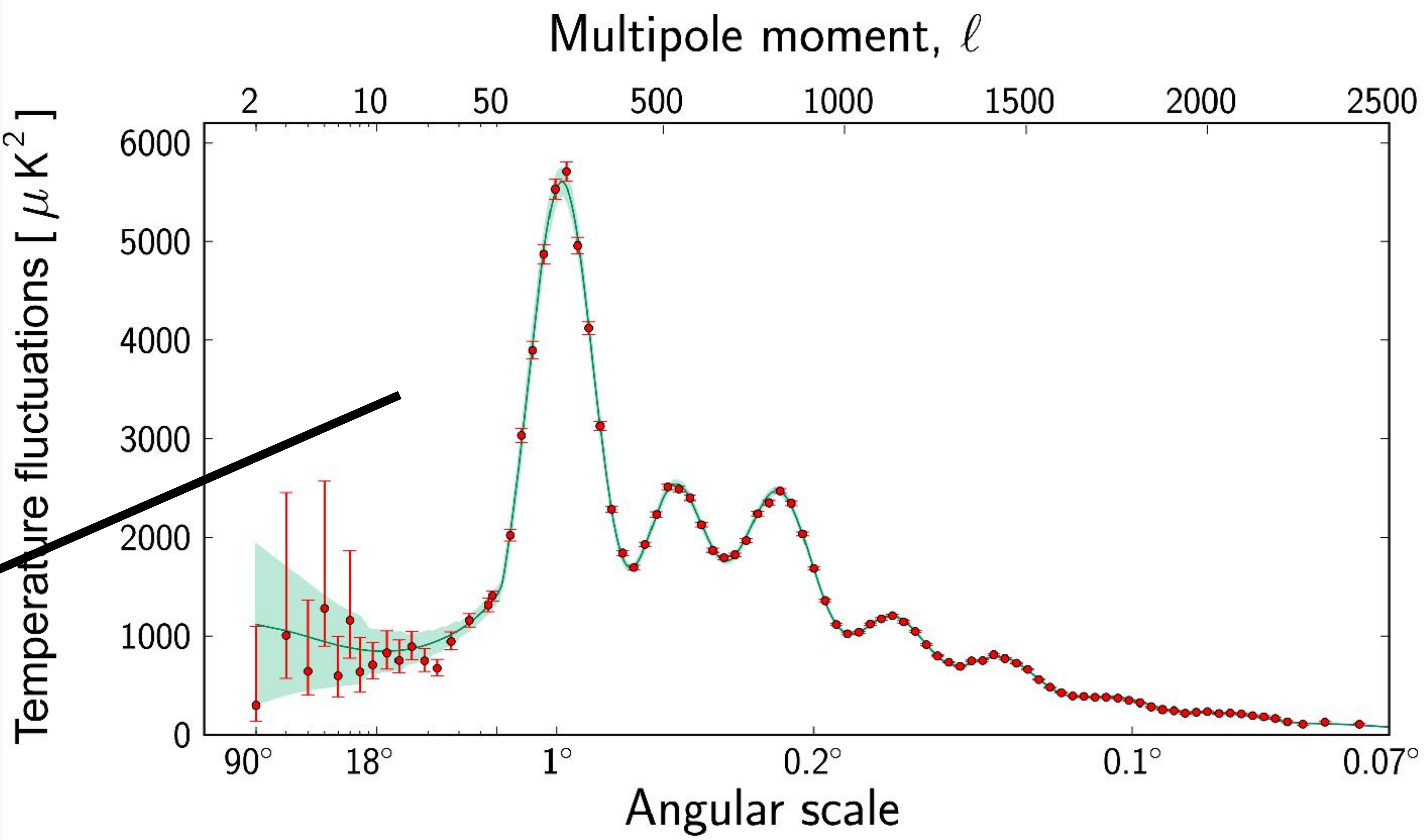
$$\Omega_{m,0} = 0.31$$

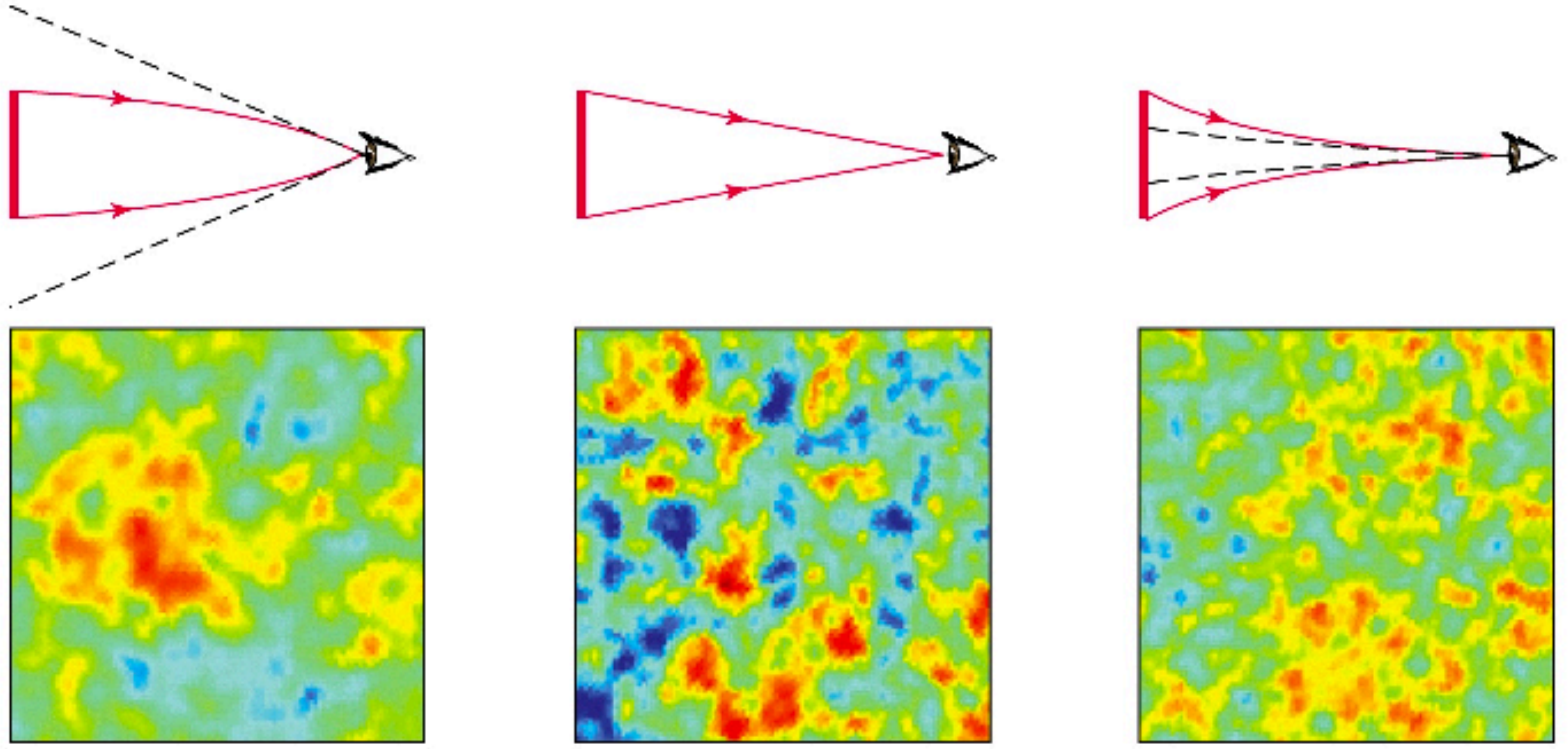
baryonic matter only 15%

By the time of the Big Bang and thereafter, normal matter is the subdominant form of matter in the universe, with some other form of matter (non-baryonic dark matter) making up the majority of non-relativistic matter in the universe

Could be primordial black holes that were made before this time (i.e., not from stars).





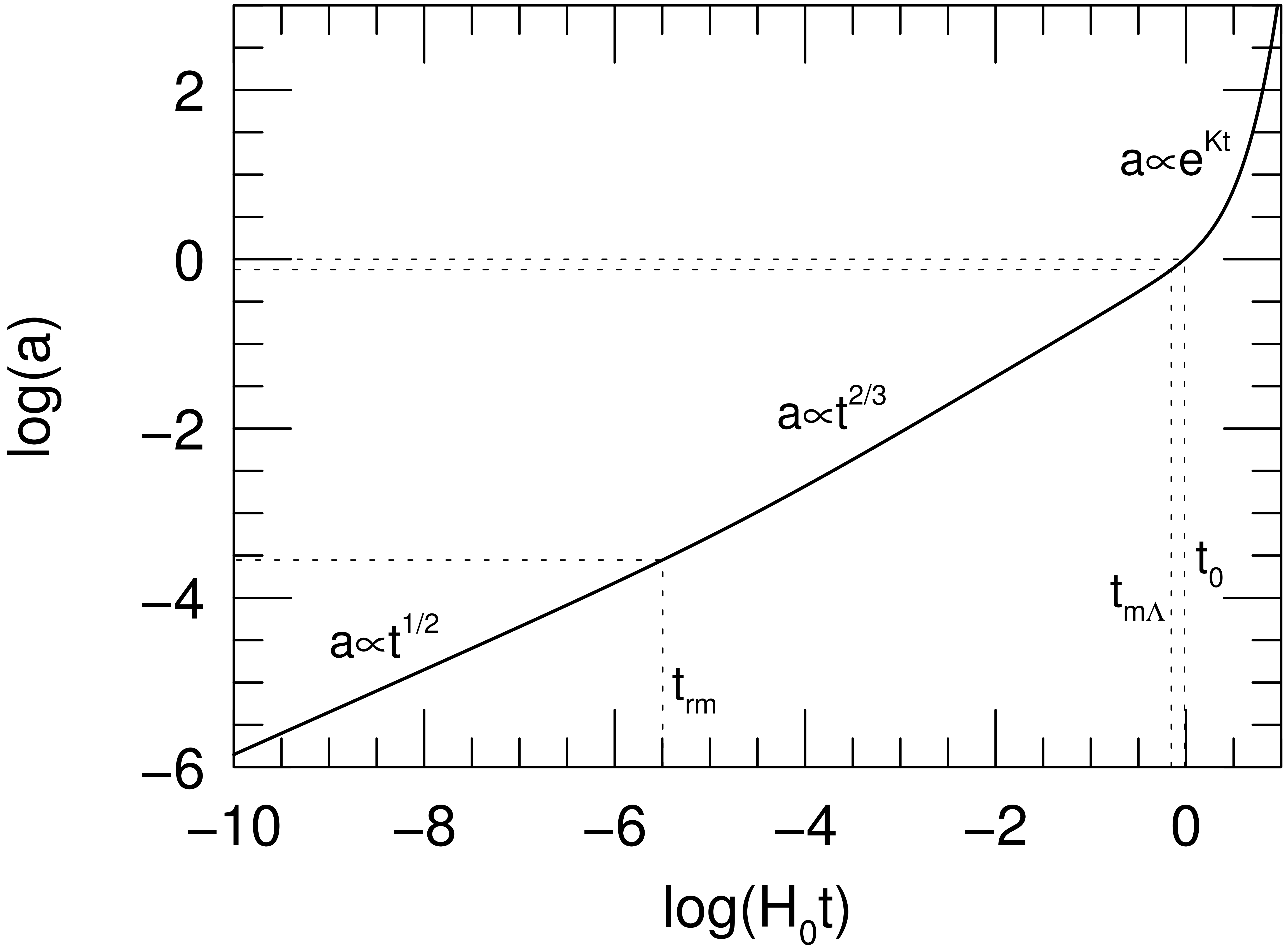


a If universe is closed, "hot spots" appear larger than actual size

b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

Benchmark Model



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