ASTR 4080 - Week 10

$1-O = -k (c/H/a/R_0)^2$
$1-O \lesssim 0.005$

Flatness Problem

Friedmann Eq. can also be written

as $$\frac{H^2}{H_0^2} = \sum \Omega_i(a) + \frac{1-\Omega_0}{a^2} \quad (5.81$$

$$\frac{1-\Omega_0}{a^2} \rightarrow 0$$

$$= \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} \quad \text{(observed}$$

$$1 - \Omega(t) = \frac{(1-\Omega_0)\, a^2}{\Omega_{r,0} + a\,\Omega_{m,0}}$$

as $a\downarrow$, $1-\Omega(t)$ also $\downarrow$ from its small value $1-\Omega_0$

$a_{rm} \sim 3\times10^{-4}$: $|1-\Omega_{rm}| < 2\times10^{-6}$

$a_{nucl} \sim 3.6\times10^{-9}$: $|1-\Omega_{nucl}| < 7\times10^{-16}$

- extrapolate back to $t_p \sim 5\times10^{-44}$ s,

$a_p \sim 2\times10^{-32}$: $|1-\Omega_p| < 2\times10^{-62}$

☆ Seems very improbable, BUT how do you calculate probabilities? (Kind of thing Hawking's last paper addressed)

NEXT SLIDE

Horizon Problem

CMB comes from the surface of last scattering @ t_{ls}

$$d_{hor}(t_{ls}) = a(t_{ls})\, c \int_0^{t_{ls}} \frac{dt}{a(t)}$$

$a = 0 \rightarrow a_{ls}$ is first rad. dominated

$$a \propto t^{1/2}$$

and then matter-dom. : $a \propto t^{2/3}$

Do the integral & get $d_{hor}(t_{ls}) = 2.24\, c\, t_{ls}$

$$= \underline{0.251 \text{ Mpc}}$$

How big is that on the sky?

Ask how to compute

$$\theta_{hor} = \frac{d_{hor}(t_{ls})}{d_A} = \frac{0.251 \text{ Mpc}}{12.8 \text{ Mpc}} \approx \boxed{1.1^\circ}$$

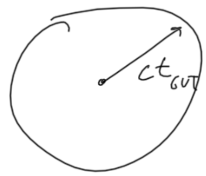
41,253 deg² of sky, so ~ 40k acausal regions

Next Slide

Monopole Problem

Expect 1 per causally-connected volume

$$V_{GUT} = 2ct_{GUT}, \quad t_{GUT} \sim 10^{-36}\,\mathrm{s}$$



$$n_{monopoles}(t_{GUT}) \simeq \frac{1}{V} \sim 10^{82}\,\mathrm{m}$$

$$\varepsilon_M(t_{GUT}) \sim m_M c^2 n_M$$

$$\sim E_{GUT}\, n_M \sim 10^{9}\ \mathrm{TeV\,m}$$

$$\varepsilon_\gamma(t_{GUT}) \sim 10^{104}\ \mathrm{TeV\,m^{-3}}$$

Monopoles would have dominated evolution of universe after 10^{-16} s

$$\boxed{\text{Obs.}: \ \Omega_{mono,0} < 5\times10^{-16}}$$

Properties of inflation

☆ HERE

Accel. Eq.: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$

need $\ddot{a} > 0$, so $P < -\frac{\varepsilon}{3}$

$P = w\varepsilon$, so $\underline{w < -\frac{1}{3}}$

↳ cosmol. const. Λ_i can do it ($w = -1$

If dominant, then $\frac{\ddot{a}}{a} = \frac{\Lambda_i}{3}$ (>0)

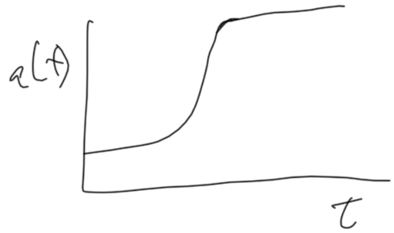
Friedmann Eq.: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3}$ = const. $= H_i^2$

$$\frac{da}{a} = \sqrt{\frac{\Lambda_i}{3}}\, dt = H_i\, dt$$

$$\boxed{a(t) \propto e^{H_i t}}$$

At earliest times, rad. dom. b/c $\varepsilon \propto a^{-4}$

so
$$a(t) = \begin{cases} a_i (t/t_i)^{1/2} & t < t \\ a_i e^{H_i(t-t_i)} & t_i < t < \\ a_i e^{H_i(t_f - t_i)} (t/t_f)^{1/2} & t > t \end{cases}$$



Space increases in size during inflation

by $$\frac{a(t_f)}{a(t_i)} = \frac{a_i\, e^{H_i(t_f - t_i)}}{a_i} = e^{H_i(t_f - \cdots}$$

$$= e^N \quad (N \equiv \#\ e\text{-folding}$$

Specific acceptable case:

$$t_i = t_{GUT} \sim 10^{-36}\,s \quad w/\ H \approx t_{GUT}^{-1}$$

$$t_f = (N+1)\, t_{GUT}$$

[Aside: $\varepsilon_{\Lambda,0} \approx 3.4 \times 10^{-3}$ TeV m^{-3}, while

$\varepsilon_i = 10^{105}$ TeV m^{-3}]

Flatness Problem Resolved

$$|1-\Omega| = \frac{c^2}{R_0^2 a(t)^2 H(t)^2} \propto e^{-2H_i t}$$

$a \propto e^{Ht}$ const

$$N = H_i(t_f - t_i)$$

$$|1-\Omega(t_f)| = e^{-2N} |1-\Omega(t_i)|$$

so even if $|1-\Omega(t_i)| \sim 1$,

$$|1-\Omega(t_f)| \sim e^{-2N}$$

Since $|1-\Omega_0| < 0.005$, can infer what N needs to be

$(a(t_f) \approx 2\times10^{-28}\sqrt{N+1}$

?

$$|1-\Omega(t_f)| < 2\times10^{-54}(N+1)$$

$N = 60$ $(e^{60} \sim 10^{26})$ $e^{2N} \sim 10^{52}$ =

Horizon Problem Resolved

$$d_{hor}(t) = a(t)\, c \int_0^t \frac{dt}{a(t)}$$

$$= a_i c \int_0^{t_i} \frac{dt}{a_i (t/t_i)^{1/2}} = 2 c t_i$$

Monopoles: $n_M(t_0) \sim 5\times10^{-16}\ \mathrm{Mpc}^{-3}$

$$d_{hor}(t_f) = a_i e^N c \left[2t_i + \int_{t_i}^{t_f} \frac{dt}{a_i e^{H_i(t-t_i)}} \right.$$

$$= e^N c \left[2t_i + H_i^{-1} \right]$$

$$t_i = 10^{-36}\,\mathrm{s} \rightarrow 2ct_i \approx 6\times10^{-28}\,\mathrm{m}$$

$$H_i^{-1} \sim t_i, \quad d_{hor}(t_f) = 2ct_i e^N + ct_i e^N$$

$$N = 60 \qquad = \boxed{15\,\mathrm{m}}$$

Current horizon (14 Gpc), $a_f \sim 2\times10^{-27}$

$$d_p(t_f) = a_f\, d_p(t_0) \sim \boxed{0.9\,\mathrm{m}} \quad < d_{hor}$$

$$d_p(t_i) = e^{-N} d_p(t_f) \sim \boxed{4\times10^{-29}\,\mathrm{m}} \quad \sim 6\times1$$

Physics of Inflation

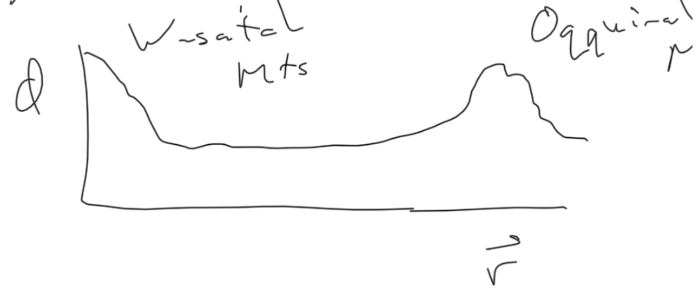
Universe has some indefinite period of exp. expansion, then stops somehow

Can imagine an "inflaton field" (a new field/particle like the Higgs)

↳ has a value @ every point in space

$$\phi(\vec{r}, t)$$

Elevation Ex:



$$V(\phi) = g\phi$$

Write $$\mathcal{E}_\phi = \frac{1}{2\hbar c^3}\dot{\phi}^2 + V(\phi)$$

$\phi \to E$ units, $V \to E$ density units

pressure: $$P = \frac{1}{2\hbar c^3}\dot{\phi}^2 - V(\phi)$$

Why? I don't know → need a DE E.O.S.-like behavior, & this choice can do it

If ϕ changes slowly w/ time, $\dot{\phi}$ is small & if it's $\ll \hbar c^3 V(\phi)$

then $\varepsilon_\phi \approx V(\phi)$ & $P_\phi \approx -V(\phi)$

so behaves like $w = -1$

Can use the fluid eq.: $\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P)$

w/ $\dot{\varepsilon}_\phi = \frac{1}{\hbar c^3}\dot{\phi}\ddot{\phi} + \frac{dV}{d\phi}\dot{\phi}$, so

$$\cancel{\dot{\phi}}\left(\frac{\ddot{\phi}}{\hbar c^3} + \frac{dV}{d\phi}\right) + 3H(t)\left[\frac{\dot{\phi}^{\cancel{2}}}{\hbar c^3}\right] = 0$$

$$\ddot{\phi} + 3H\dot{\phi} + \hbar c^3 \frac{dV}{d\phi} = 0$$

Falling body w/ air resistance: $\underbrace{m\ddot{x}}_{\text{accel.}} + \underbrace{k\dot{x}}_{\text{friction}} = \underbrace{mg}_{\text{grav. force}}$

Size of Hubble parameter imposes a "frictional" force

- speed is $\dot{x} = \frac{mg}{k}\left(1 - e^{-kt/m}\right)$
  - ↳ $t \to \infty$, reach "terminal velocity"
- similarly, $\dot{\phi} = -\frac{\hbar c^3}{3H}\frac{dV}{d\phi}$ (& $\ddot{\phi} = 0$)

We need $\dot{\varphi}^2 \ll \hbar c^3 V$ for φ to act like a cosmological constant:

$$\dot{\varphi}^2 = \frac{\hbar^2 c^6}{9H^2}\left(\frac{dV}{d\varphi}\right)^2 \ll \hbar c^3 V$$

$$\text{or} \quad \left(\frac{dV}{d\varphi}\right)^2 \ll \frac{9H^2 V}{\hbar c^3}$$

Friedmann Eq. for cosmol. const. (Ch. 5, Λ-

$$H = \left(\frac{8\pi G \varepsilon}{3c^2}\right)^{1/2} = \left(\frac{8\pi G V}{3c^2}\right)^{1/2}$$

Substitute in above, get

$$\left(\frac{dV}{d\varphi}\right)^2 \ll \frac{24\pi G V^2}{\hbar c^5}$$

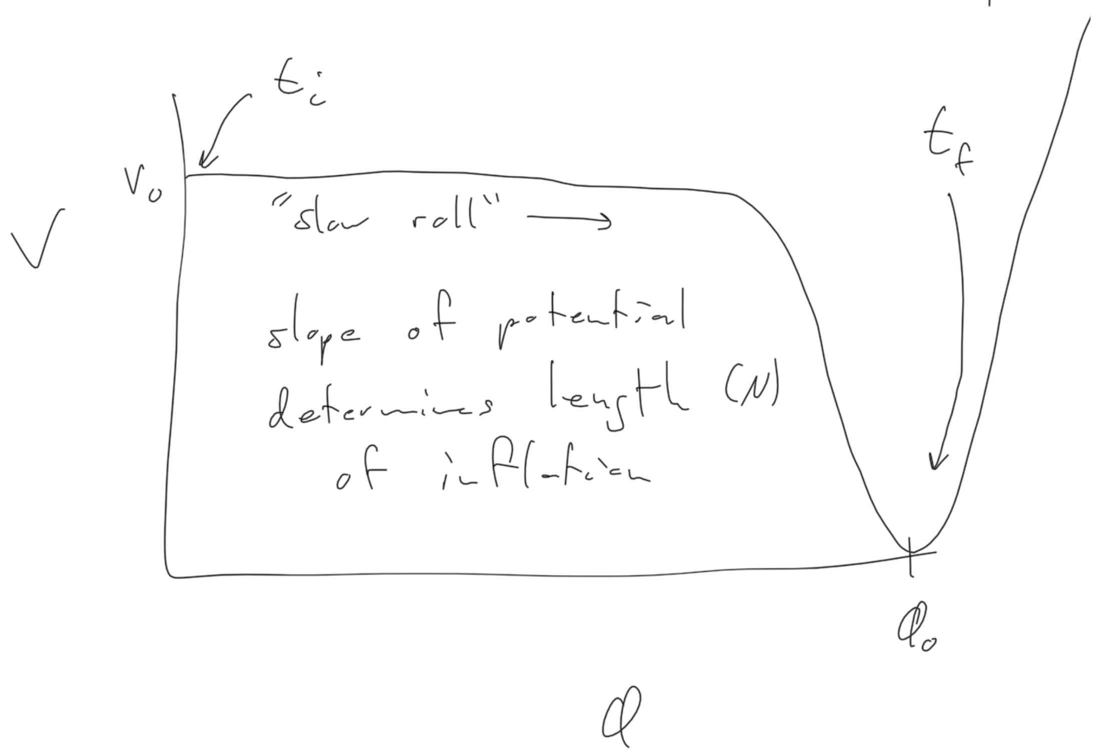
& given $E_p = M_p c^2 = \left(\frac{\hbar c}{G}\right)^{1/2} c^2$

$$\left(\frac{E_p}{V}\frac{dV}{d\varphi}\right)^2 \ll 1$$

Need TWO things

1) $\frac{dV}{d\varphi}$ small (pot. E changes slowly as φ ch

2) $V \simeq \varepsilon_\varphi$ big (to dominate tot E density)



- For some reason, the field φ starts out out-of-equilibrium, in a ↑E state

- "metastable false vacuum state"

$\varphi = 0$ @ start, it will roll down toward the min. in V (given the local slope)

↳ not dynamically important until its E. density $\varepsilon_\varphi \simeq V_0$ dominates over radia

$\varepsilon_r \sim \alpha T^4$, so $V_0 > \alpha T^4$

which happens @

$$T_i \simeq 2\times10^{28}\,K \left(\frac{V_0}{10^{105}\,\text{TeV}\,m^{-3}}\right)^{1/4}$$

$$t_i \sim 3\times10^{-36}\,K \left(\frac{V_0}{10^{105}}\right)^{-1/2}$$

N (length of inflation) depends on shape of $V(\varphi)$, w/ ↑ V_0, φ_0 giving ↑N

Eventually, $V \to 0$ + $\varphi \to \varphi_0$, but not w/o oversh + oscillations (damped by H (friction term

↳ can happen faster if φ coupled to other fields, like γs (so E in V converted t γ

☆ reheats universe after wiping out $\Sigma\varepsilon$ of other components

⟶ "latent heat" of phase trans.

$T \propto a^{-1}$, so if $a \propto e^{N}$,
then $T \propto e^{-N}$

$T_i(t_{GUT}) \sim 10^{28}K \rightarrow T_f(t_f) \sim e^{-65} T_{GUT}$
$\sim 0.6\,K$

☆ so need reheating for the CMB, however it works exactly (unknown)

Problem: flattens too much!
large fluctuations $\frac{\delta\varepsilon}{\varepsilon} \sim 1$ are reduced by $e^{-65} \sim 10^{-28} \rightarrow$ no structure

☆ Rely on quantum fluctuations in tiny ($d \sim 4\times10^{-29}\,m$) patch that becomes observable universe

Great explanatory power, just need a new field w/ fine-tuned properties!
$\rightarrow$ Which fine-tuning do you prefer??