Recall the scattering eqn. for atoms
to interact w/ light

$$\Gamma = n_e \sigma_e c \quad (\text{per sec})$$

As CMB photons travel, they can sti
get scattered by intervening e^-'s

The prob. a photon is scattered is
expressed by optical depth

$$\tau_* = \int_{t_*}^{t_0} \Gamma(t) \, dt$$

$$= c\sigma_e \int_{t_*}^{t_0} n_e(t) \, dt$$

$$= 0.066 \pm 0.0016$$

$\underline{\qquad\qquad}$
from Planck

$1/15$ γ's get
scattered out
of l.o.s.

Can estimate t_* knowing this.

Assume everything is ionized $H + e^-$

$$n_e = n_p = n_{bary,0} \, a^{-3}$$

define $\Gamma_0 = c \sigma_e \, n_{bary,0} = 0.0023 H_0$

$$\tau_* = \Gamma_0 \int_{t_*}^{t_0} \frac{dt}{a(t)^3}$$

change to a from t, $\quad \frac{d}{dt} a(t) = \dot{a} \quad$ so

$$dt = \frac{da}{\dot{a}}$$

$$\tau_* = \Gamma_0 \int_{a(t_*)}^{1} \frac{da}{\dot{a} \, a^3} = \Gamma_0 \int_{a(t_*)}^{1} \frac{da}{H(a) \, a^4}$$

change from a to z :

$$= \Gamma_0 \int_0^{z_*} \frac{(1+z)^2 \, dz}{H(z)}$$

$a = (1+z)^{-1}$

$da = \frac{-dz}{(1+z)^2}$

$$= \Gamma_0 \int_0^{z_*} \frac{(1+z)^2 \, dz}{H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_\Lambda}}$$

(flat, matter + Λ dominated)

$$\tau_* = \frac{2}{3 \Omega_{m,0}} \frac{\Gamma_0}{H_0} \left(\left[\Omega_{m,0}(1+z_*)^3 + \Omega_{\Lambda,0} \right]^{1/2} - 1 \right)$$

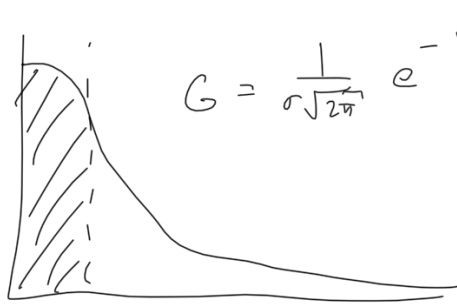$$= 0.00485 \left(\left[0.31 (1+z_*)^3 + 0.69 \right]^{1/2} - 1 \right)$$

$$\boxed{z_* = 7.8 \pm 1.3} \quad , \quad t_* = 650 \, Myr$$

Obs. universe : $d_p(t_o) \approx 14\ Gpc$

total mass inside is $M = \rho_{m,o}\ \frac{4\pi}{3}\ d_p(t_o)^3$

$$\approx 4.3 \times 10^{23} M_\odot$$

☆ There are 4.3×10^9 regions $\sim / 10^{14} M_\odot$,

so the $\underline{1^{ST}}$ $10^{14} M_\odot$ region to collapse

is a statistical outlier : $\frac{1}{4.3 \times 10^9}$ chance

$G = \frac{1}{\sigma \sqrt{2\pi}}\ e^{-x^2/2\sigma^2}$ $p = erf\left(\frac{n/\sigma}{\sqrt{2}}\right)$

$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} d$

$1\sigma\ \ 68\%$ $(2\sigma : 95\%)$

$\quad\quad\quad\quad (3\sigma : 99.7\%)$

$\boxed{2.3 \times 10^{-10} \rightarrow 6.2\sigma}$

$\delta_{coll} = 6.2 \rightarrow 1 + z_{coll} \approx \delta_{rm}(1 + z_{rm}) = \delta_{co}$

$\boxed{z = 5.2}$

$T_{gas} \approx 2.3 \times 10^7 K$

$\bar{\rho}_{gas} \approx 1.5 \times 10^{-24}\ kg\ m^{-3}$

$\hookrightarrow \boxed{t_{cool} \approx 42\ Gyr}$

Hierarchical Fragmentation

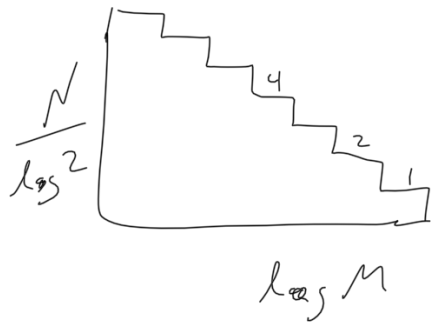- if 1% inefficient, start w/ 100 cores
  each w/ mass M

  1 M, 198 $M/2$

  1 m, 2 $M/2$, 392 $M/4$

  1 m, 2 $M/2$, 4$M/4$, 776 $M/8$

after n steps, have $1M$, $2 M/2$, ..., $2^n m$

log-spaced histogram: $\log M/2^n = \log M$
$$- n \log 2$$

$\frac{N}{\log 2}$ ... 4, 2, 1 ... $\log M$

$$\frac{dN}{d\log m} \propto \frac{1}{M}$$

Equivalent to $\chi(m) \propto M^{-2}$
(close to obs. $M^{-2.3}$)