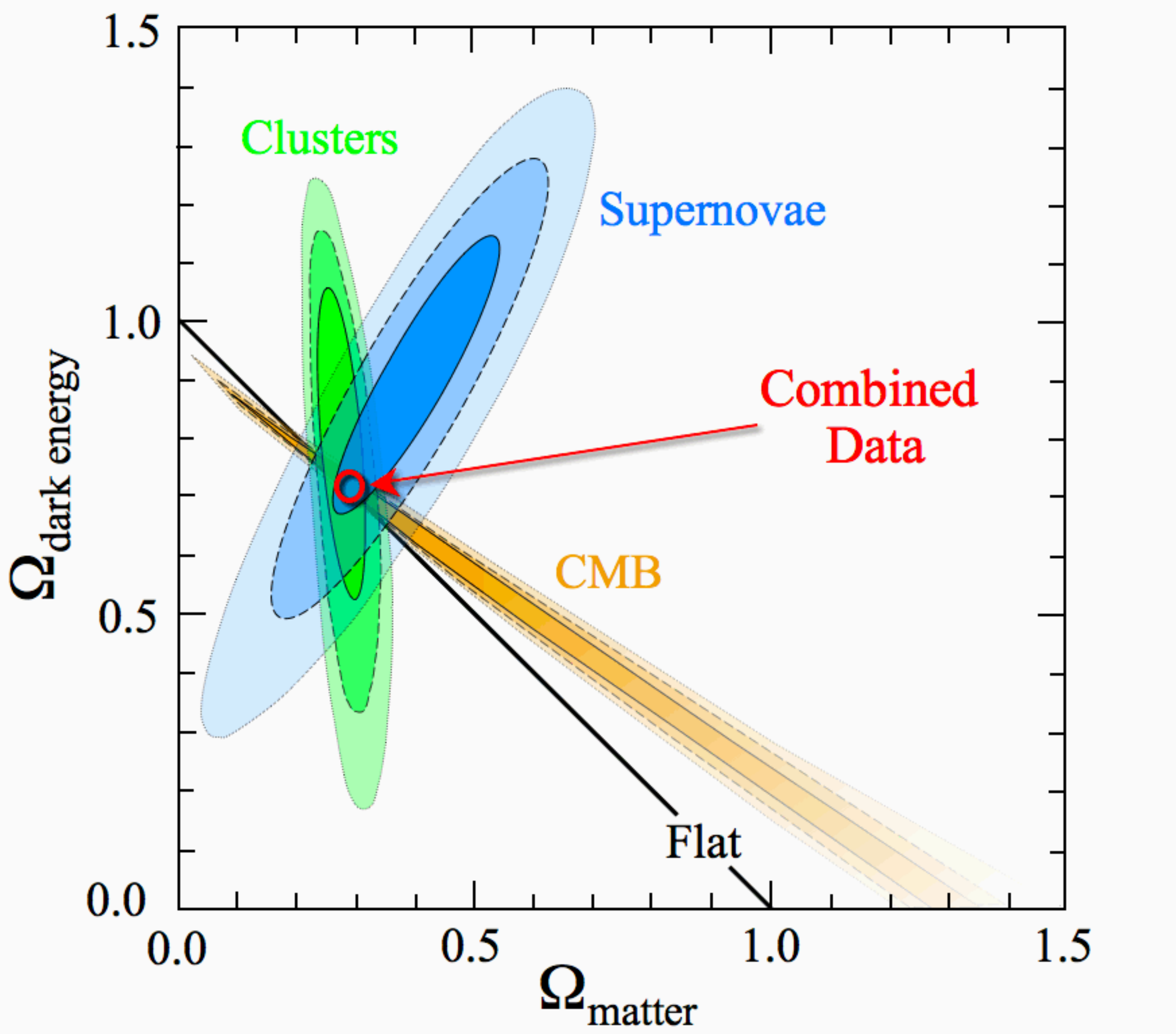
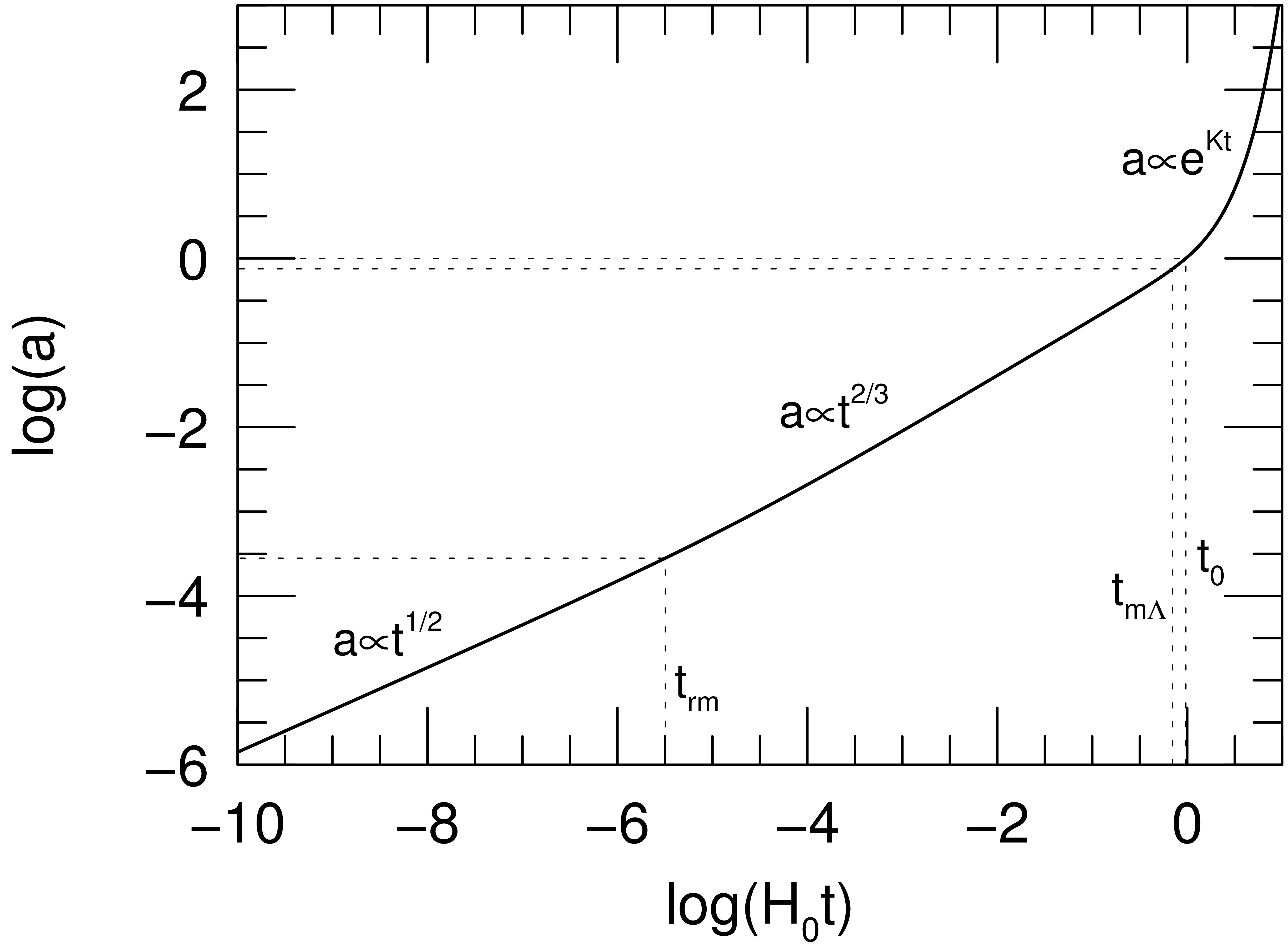
Grand Summary

The Concordance: 1998-present



Theory

Benchmark Model



$$\kappa = 0$$
$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$
$$\Omega_r = \Omega_\gamma + \Omega_\nu$$
$$\Omega_m = \Omega_{bary} + \Omega_{dm}$$

$\Omega_{m,0}$	=	0.31
$\Omega_{bary,0}$	=	0.048
$\Omega_{dm,0}$	=	0.262

$\Omega_{r,0} = 9.0 \times 10^{-5}$
$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$
$\Omega_{\nu,0} = 3.65 \times 10^{-5}$

$$\Omega_{\Lambda,0} \approx 0.69$$

Rad. – Matter :	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 50$ kyr
Matter – Λ :	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now :	$a_0 = 1$	$t_0 = 13.7$ Gyr

Friedmann Eq:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{Kc^2}{R_0^2 a^2}$$

Fluid Eq:

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

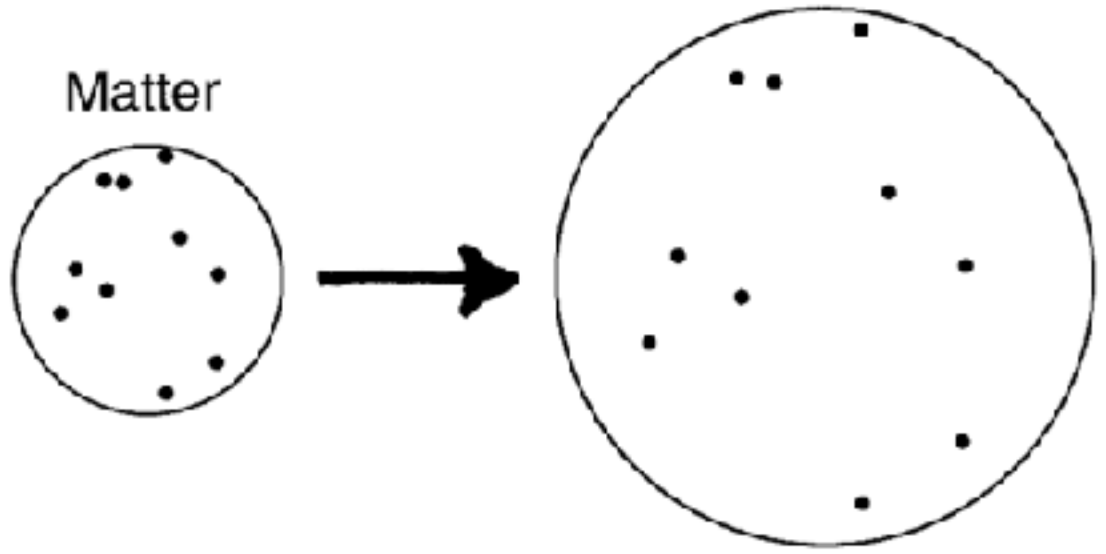
Eq. of State: $P = w\varepsilon$

$$a = \frac{1}{1+z}$$

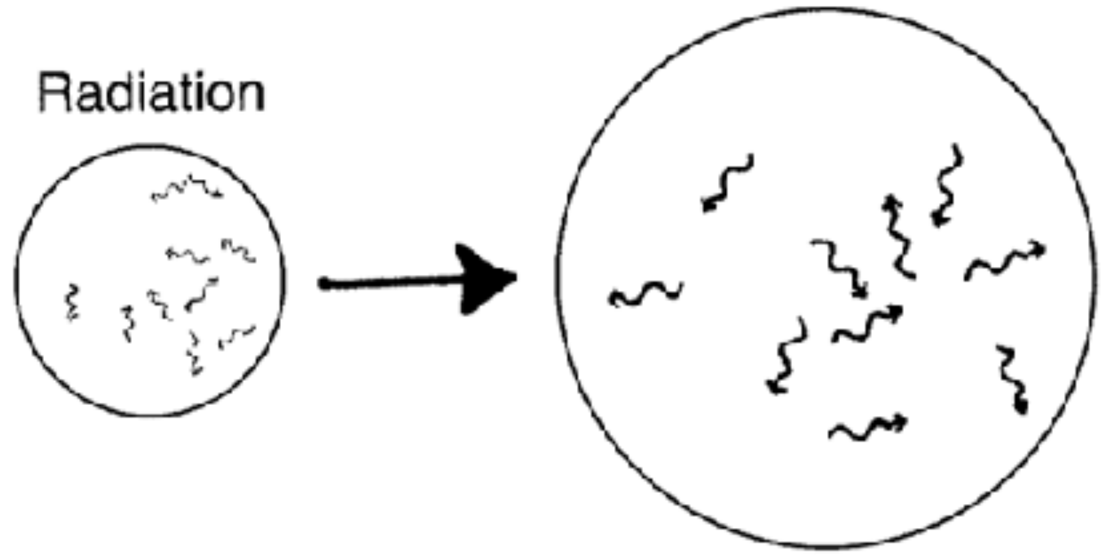
$$\varepsilon = \sum_i \varepsilon_i$$

$$P = \sum_i w_i \varepsilon_i$$

Matter



Radiation



$$w = 0 \qquad \varepsilon_m = \varepsilon_{m,0}/a^3$$

$$w = \frac{1}{3} \qquad \varepsilon_r = \varepsilon_{r,0}/a^4$$

$$w = -1 \qquad \varepsilon_\Lambda = \varepsilon_{\Lambda,0}$$

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$H_0^{-1}\dot{a} = \left[\Omega_{r,0}\,a^{-2} + \Omega_{m,0}\,a^{-1} + \Omega_{\Lambda,0}\,a^{2} + (1-\Omega_0)\right]^{1/2}$$

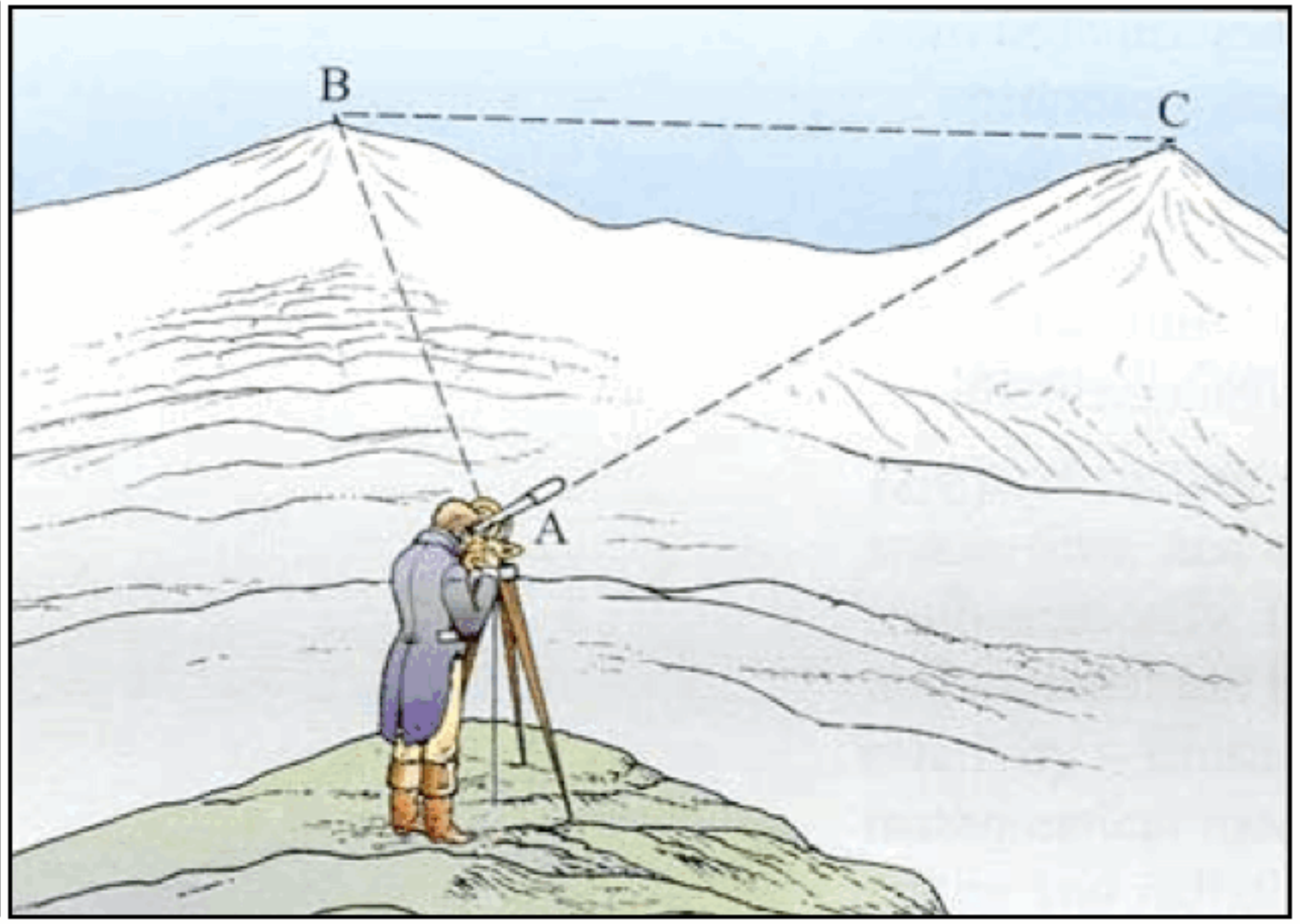
$$d_p(t_e) = \frac{1}{1+z}\,d_p(t_0)$$

$$t = \frac{1}{H_0}\int_0^a \frac{da}{E(a)}$$

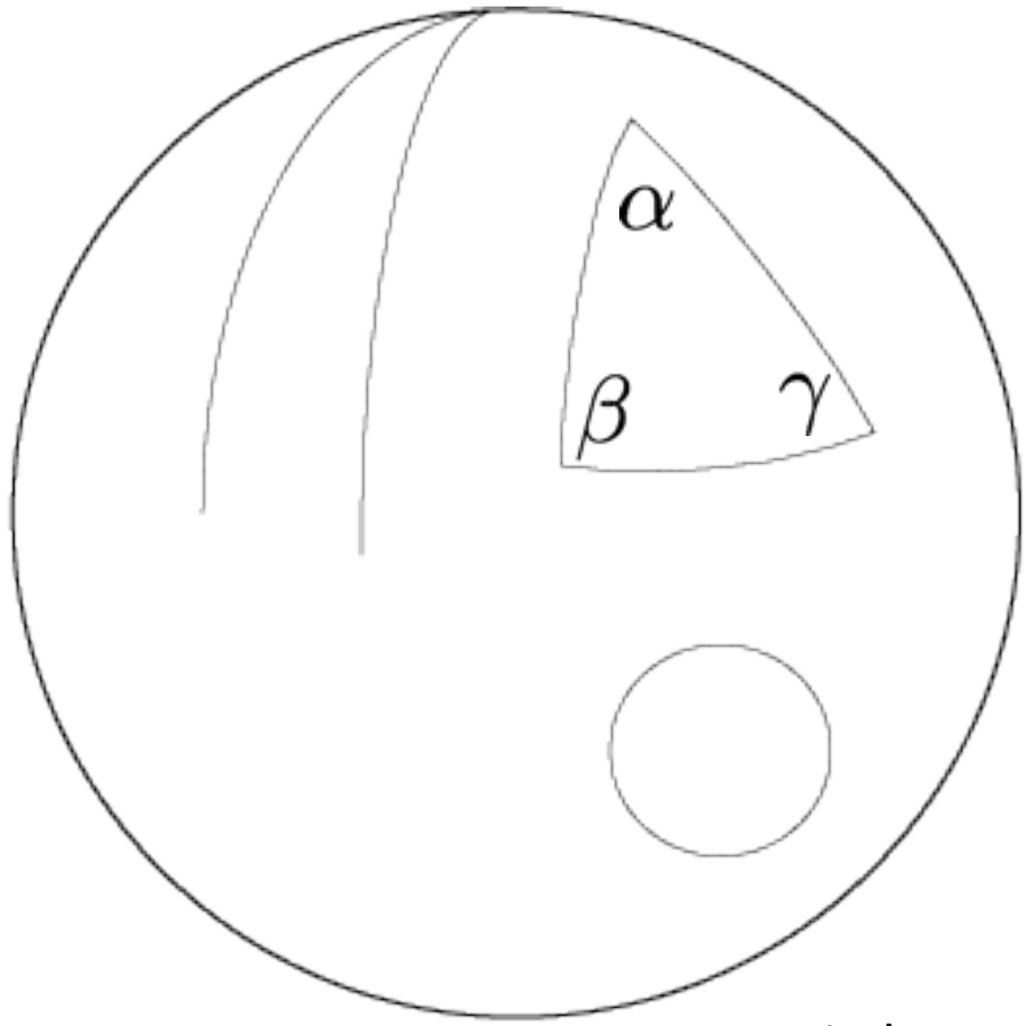
Curvature

How can we measure the curvature of spacetime?

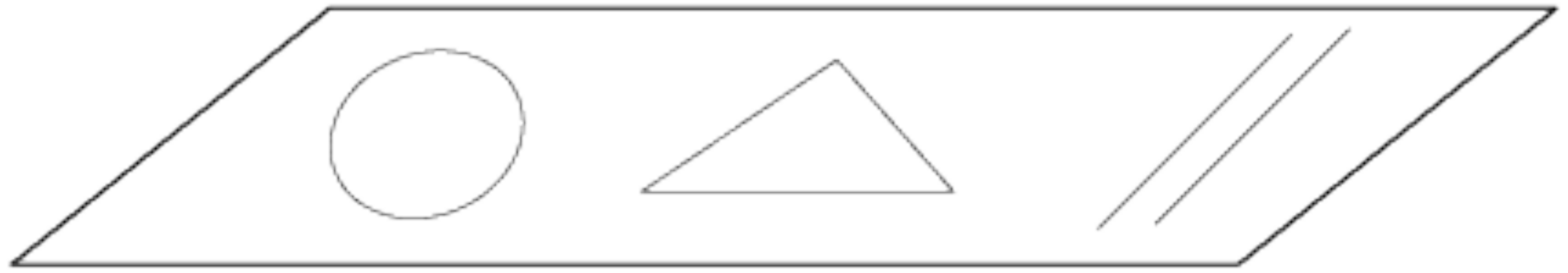
Carl Friedrich Gauss
1777 - 1855



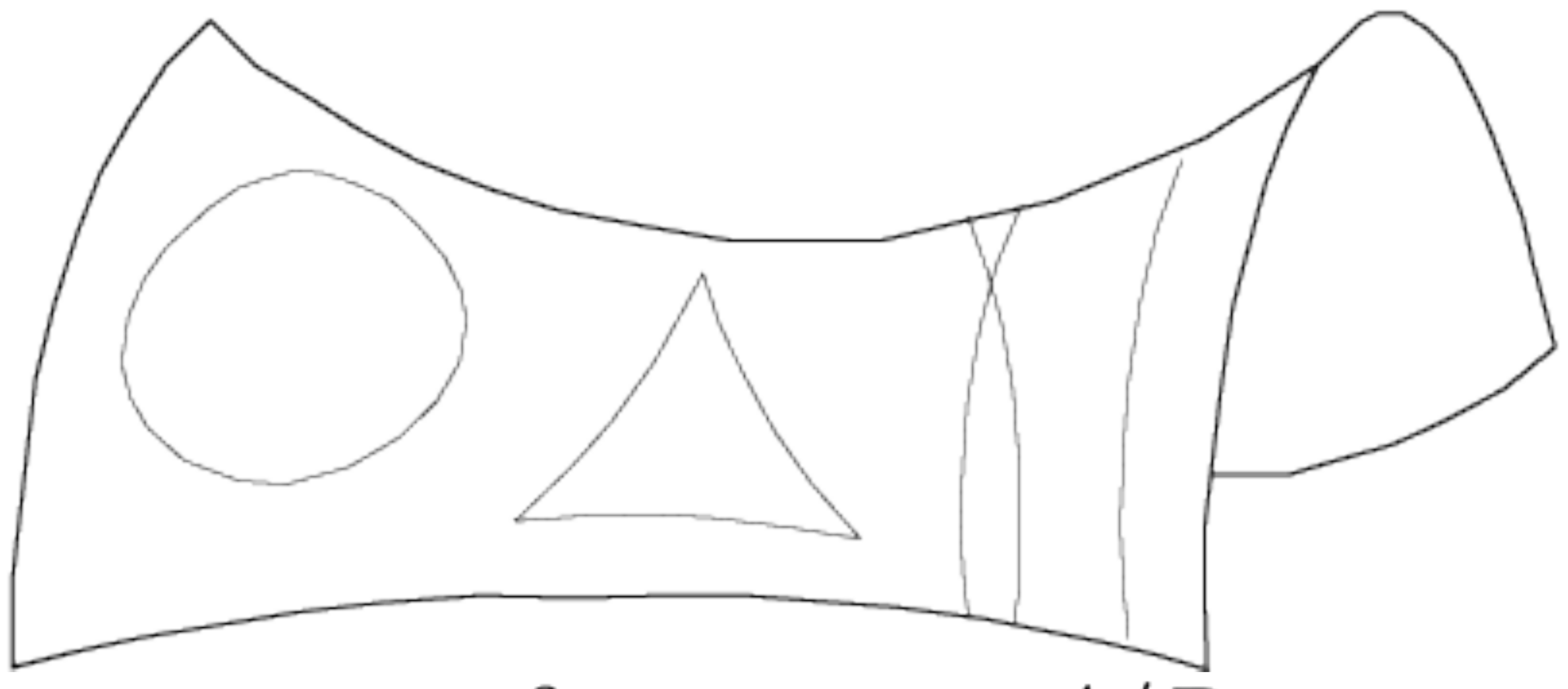
Gauss finds 180 degrees in large survey triangles:
Space is not (grossly) non-Euclidean



$$\alpha + \beta + \gamma = \pi + A/R$$



$$\alpha + \beta + \gamma = \pi$$

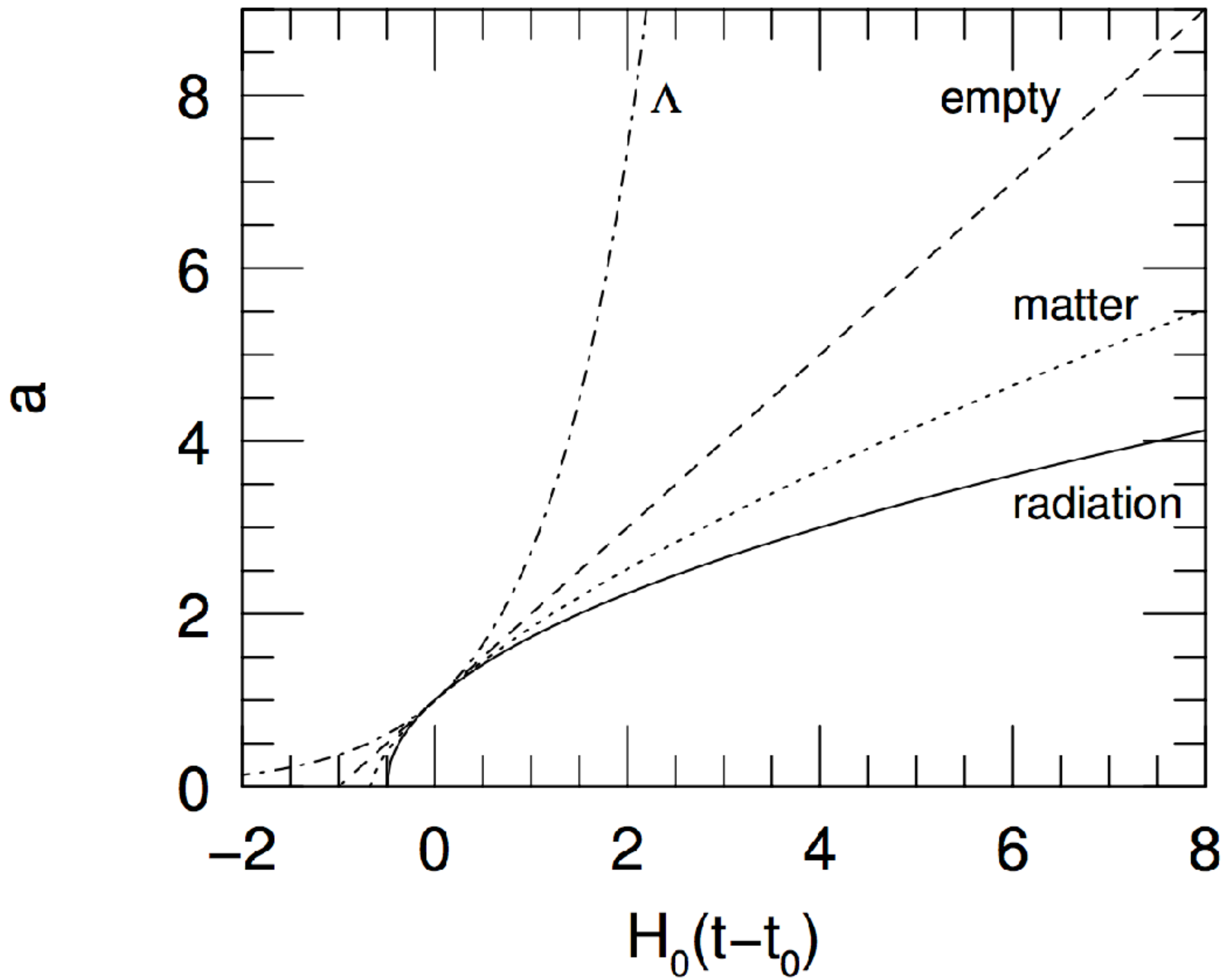


$$\alpha + \beta + \gamma = \pi - A/R$$

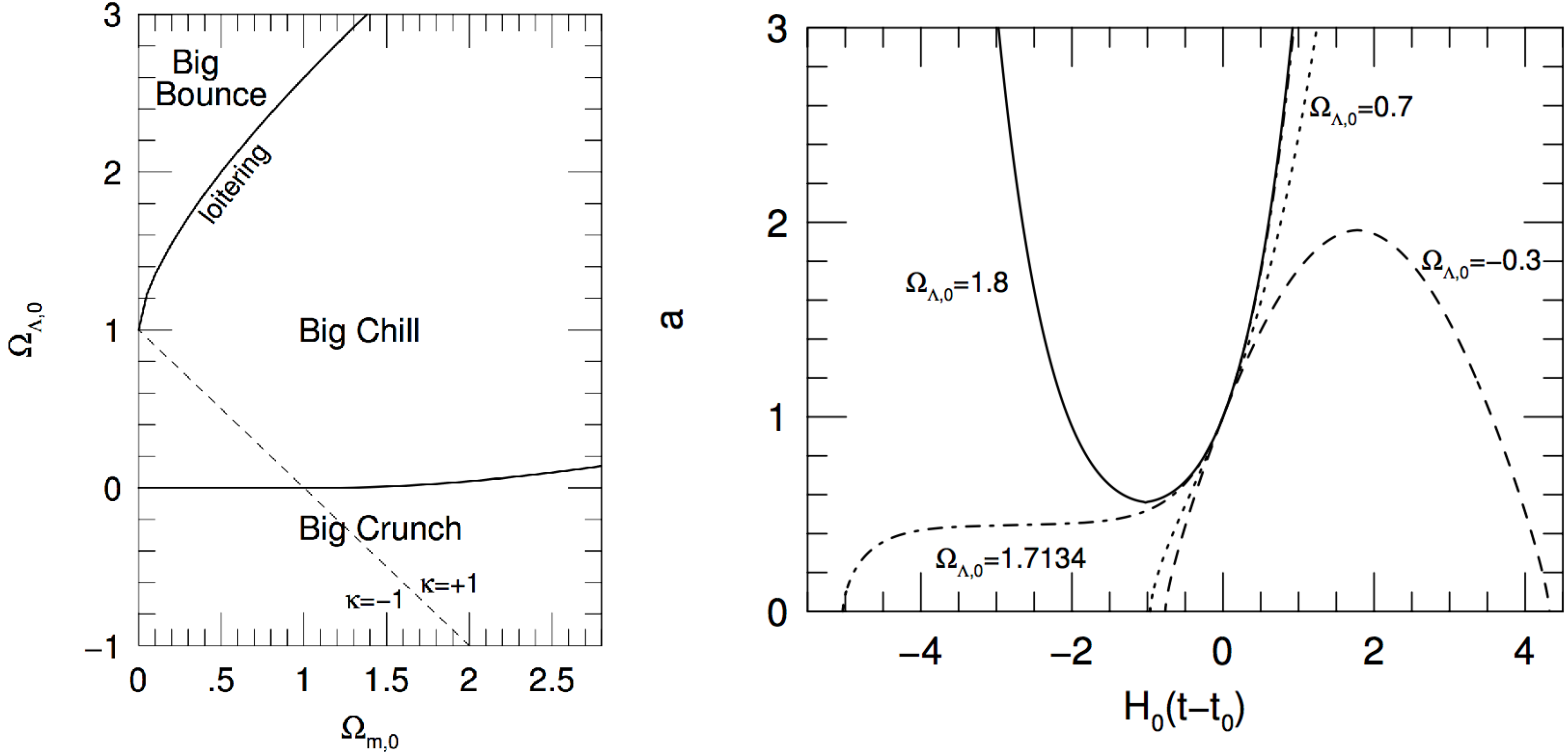
A = area of triangle R = Radius of Curvature

Only possible geometries that are homogeneous/isotropic

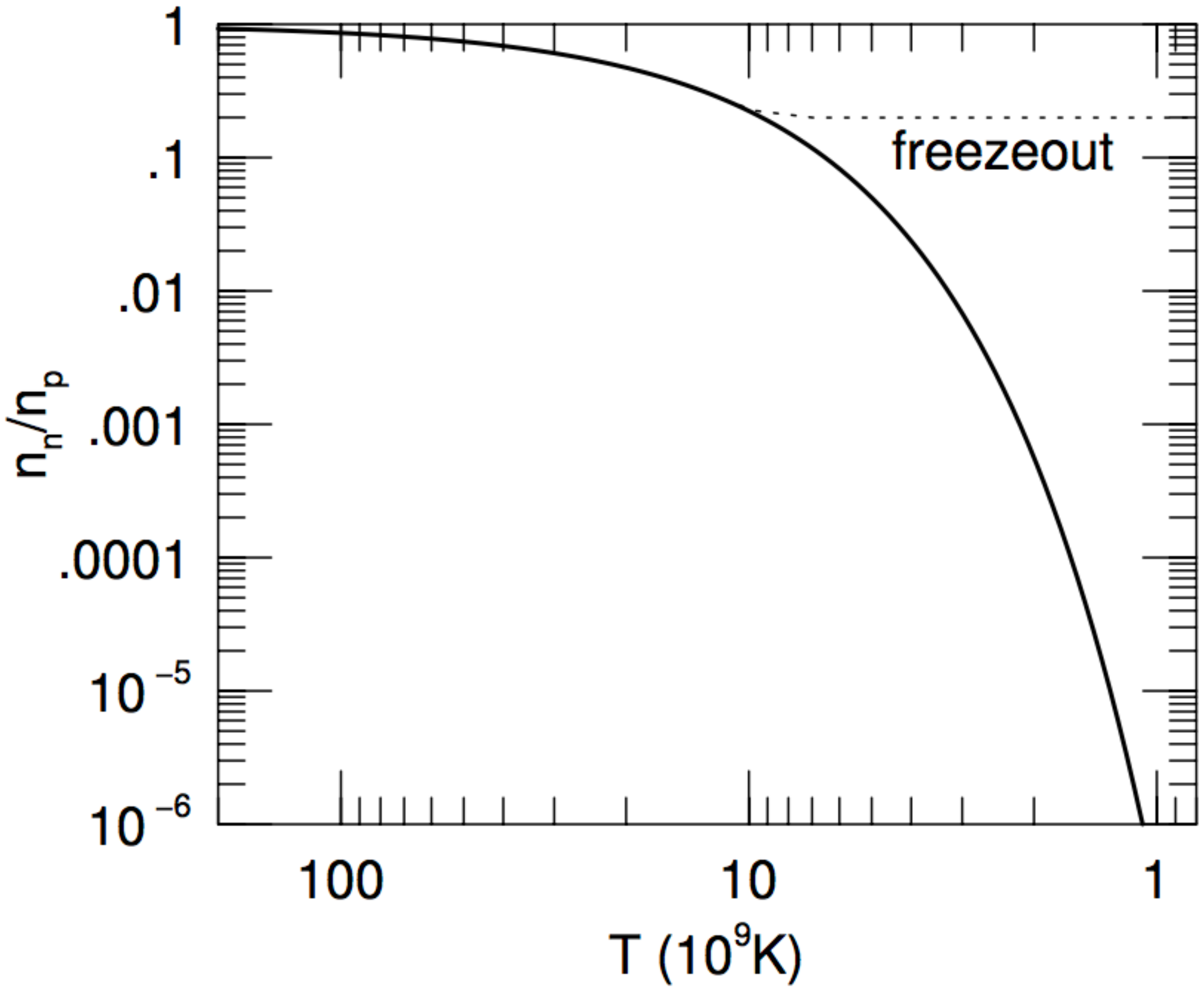
Only 1 Constituent in a Flat Spacetime



Matter + Lambda + Curvature



neutron-proton ratio

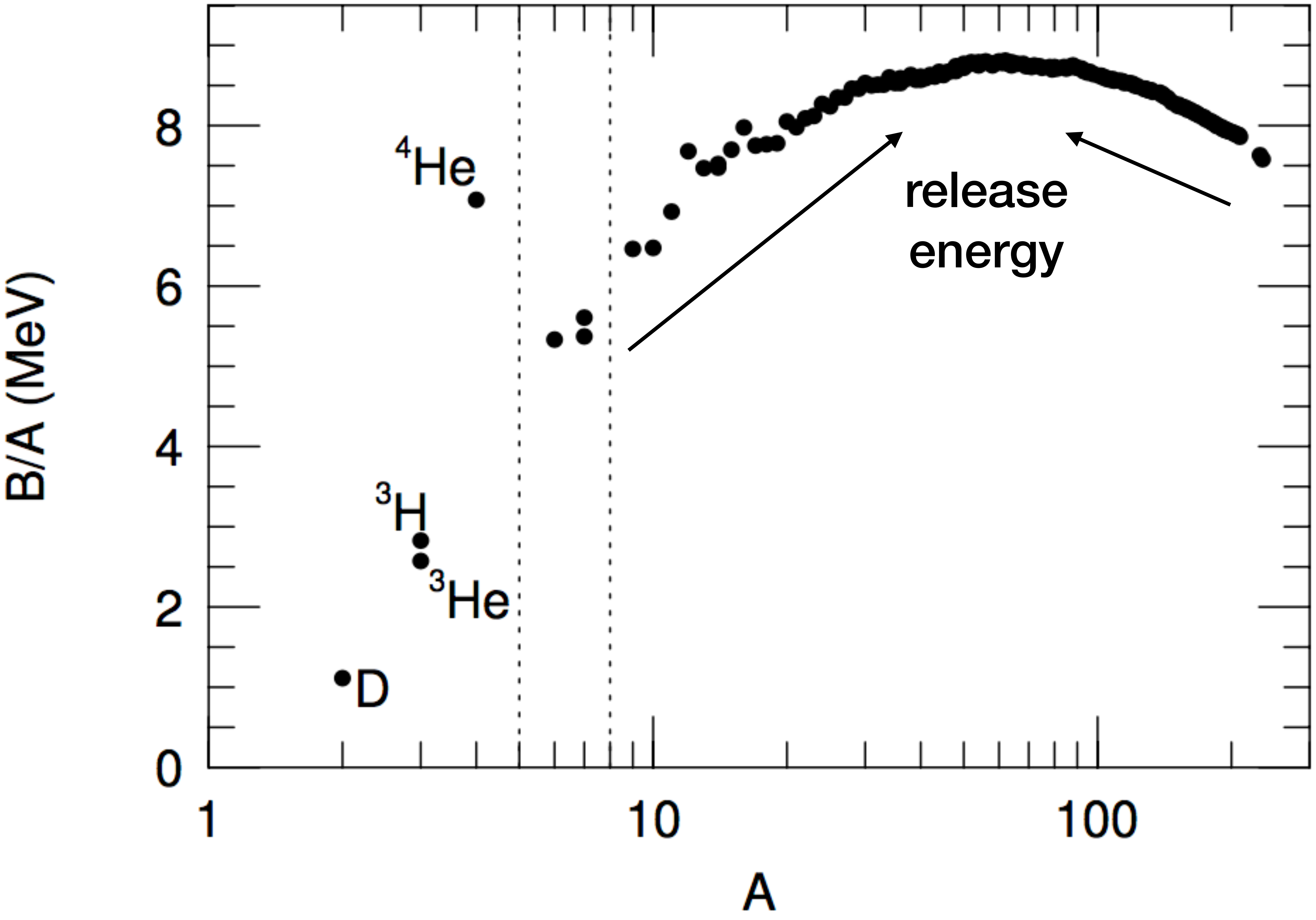


$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2} \right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT} \right)$$

$$\frac{n_n}{n_p} = \exp\left(-\frac{(m_n - m_p)c^2}{kT} \right)$$

$$\Gamma = n_\nu c \sigma_w$$

Nuclear Binding Energy



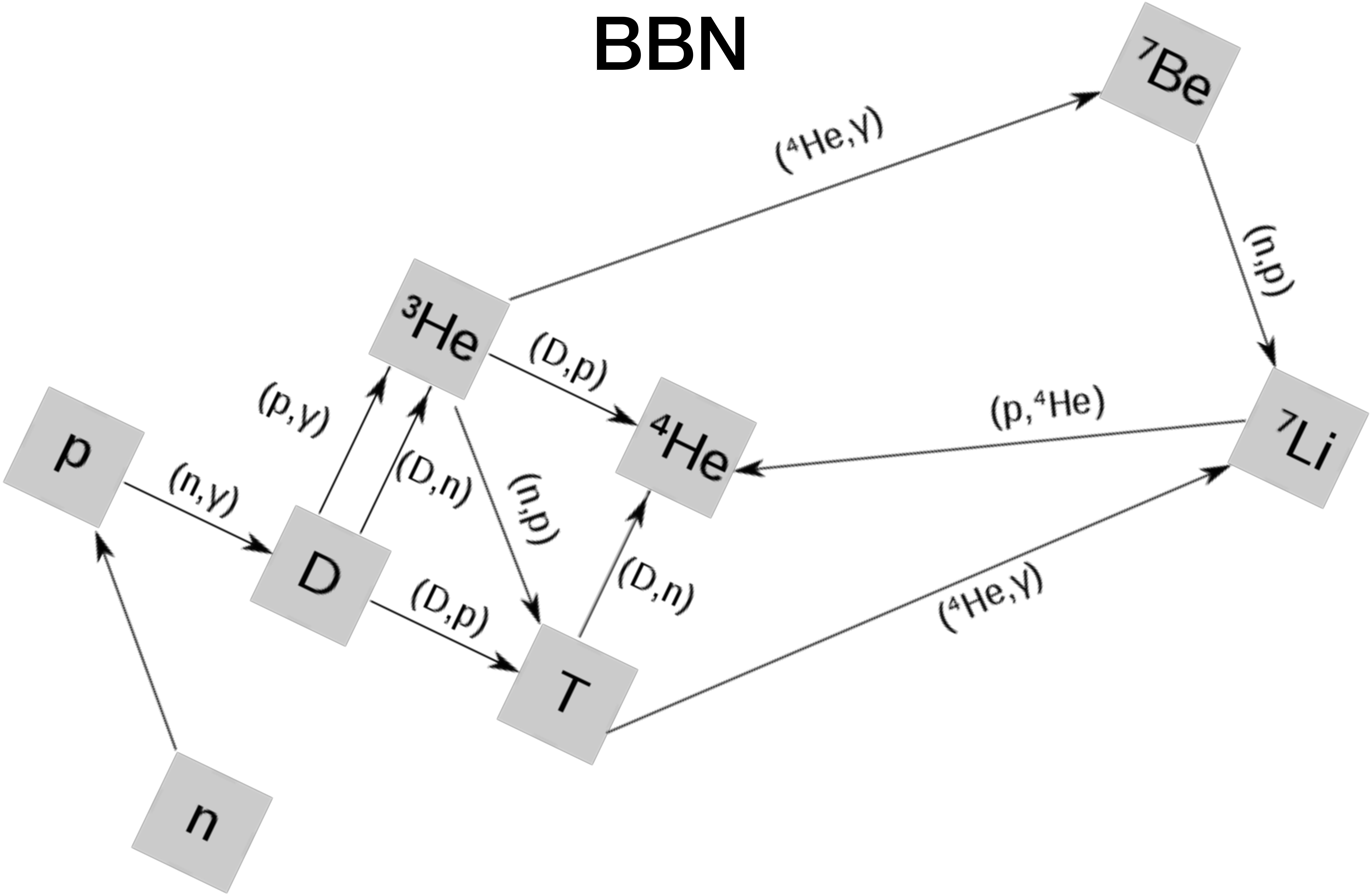
$$p + n \rightleftharpoons D + 2.22 \text{ MeV}$$

expect nucleosynthesis to result in all atoms becoming iron

does not happen - why not?

$$Y_p \equiv \frac{\rho(^4\text{He})}{\rho_{\text{bary}}}$$

BBN



Recombination $H + \gamma \rightleftharpoons p + e^-$

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{\exp([E - \mu_x]/kT) \pm 1}$$

(minus for bosons, plus for fermions)

g —> 2 (for non-nucleons, g_H=4)
chemical potential of photons = 0

$$\mu_H = \mu_p + \mu_e$$

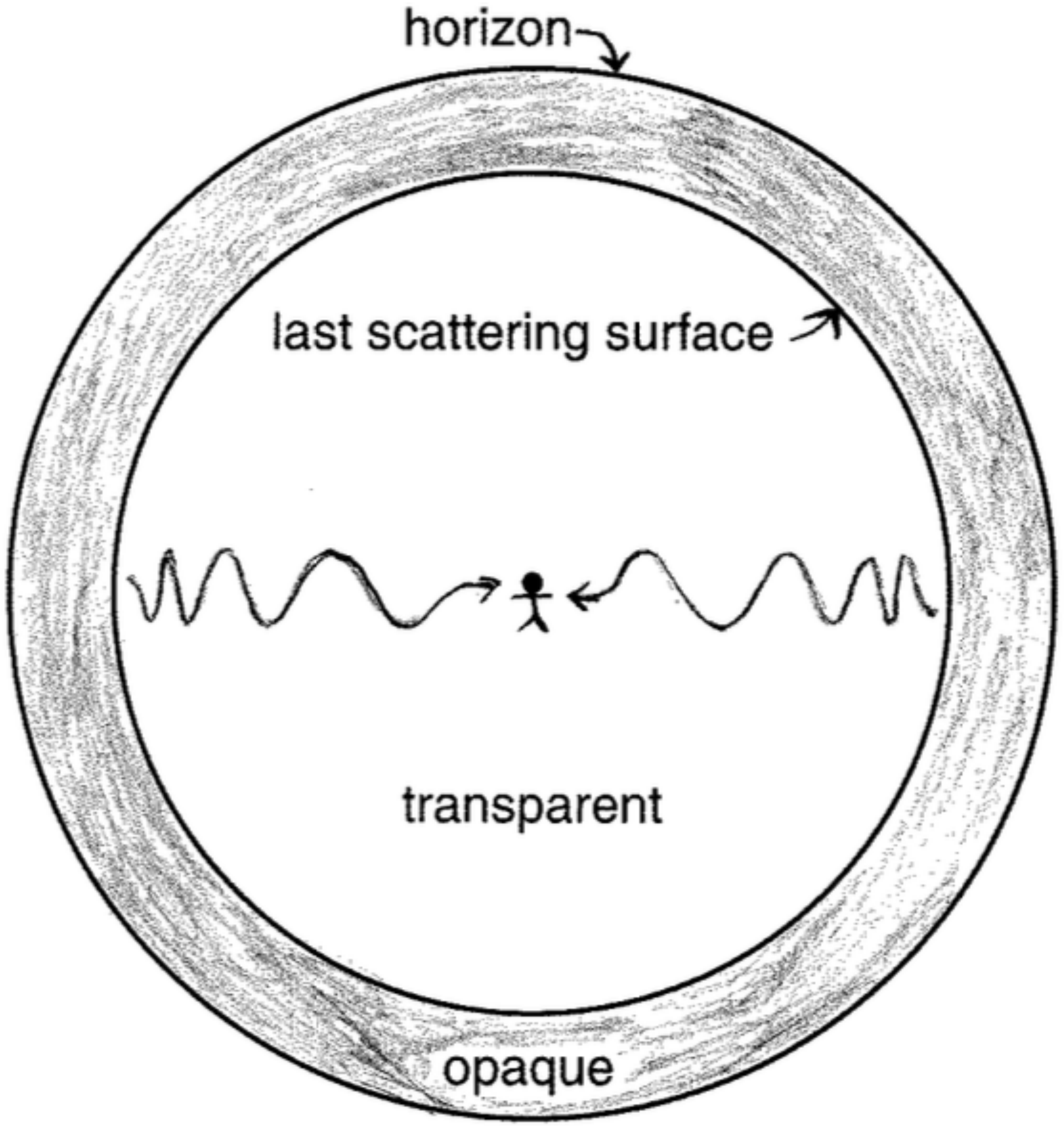
$$n_\gamma = \frac{2.4041}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3$$

$$n_x = g_x \left(\frac{m_x kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{kT}\right)$$

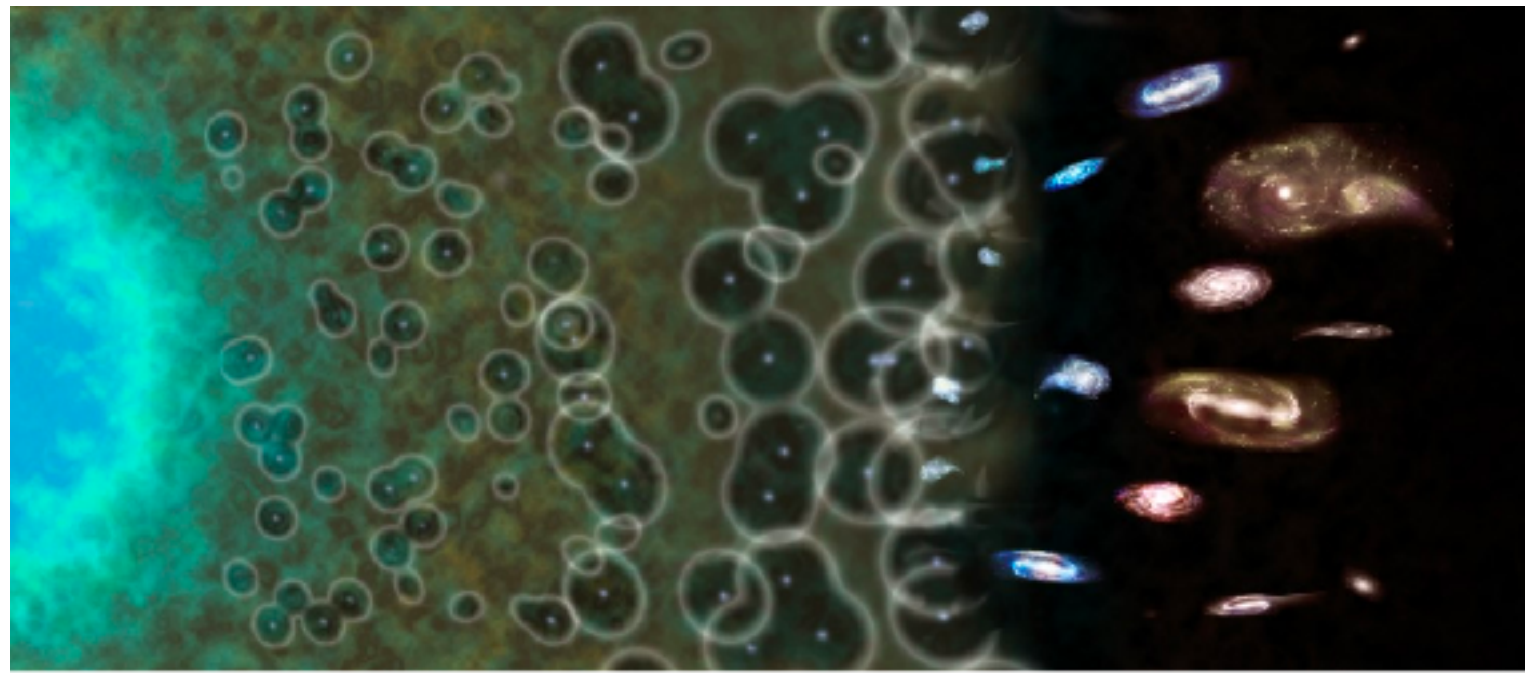
$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e}\right)^{3/2} \left(\frac{kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{[m_p + m_e - m_H]c^2}{kT}\right) = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{kT}\right)$$

Saha Equation

Surface of Last Scattering



Reionization



1/15 γs get scattered out of l.o.s.

$$\tau_* = \int_{t_*}^{t_0} \Gamma(t)\, dt$$

$$\Gamma = n_e \sigma_e c$$

0.066 ± 0.0016 from Planck

(flat, matter + Λ dominated)

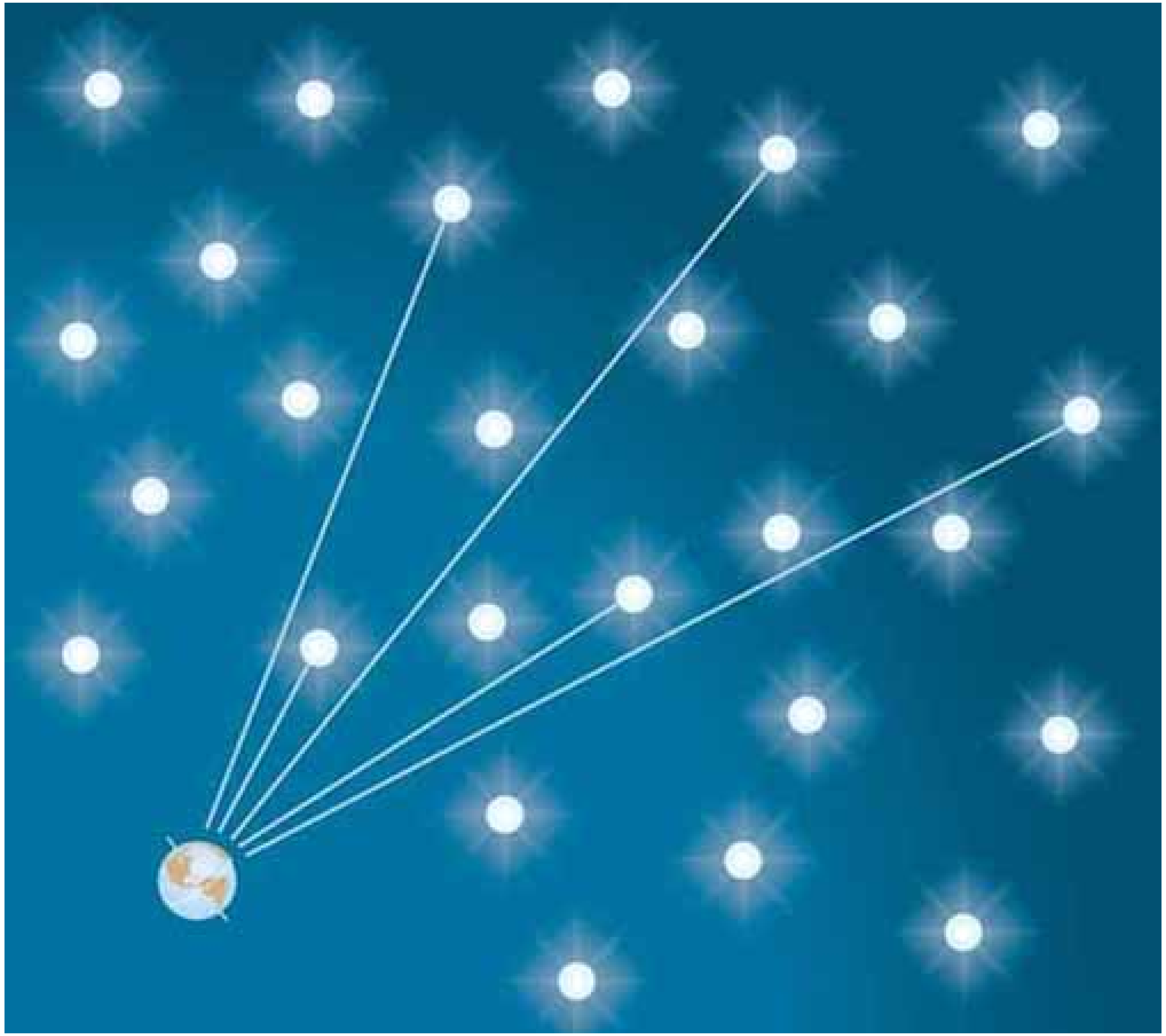
$$\tau_* = \frac{2}{3\Omega_{m,0}} \frac{\Gamma_0}{H_0} \left(\left[\Omega_{m,0} (1+z_*)^3 + \Omega_{\Lambda,0} \right]^{1/2} - 1 \right)$$

$z_* = 7.8 \pm 1.3$, $t_* = 650\,\text{Myr}$

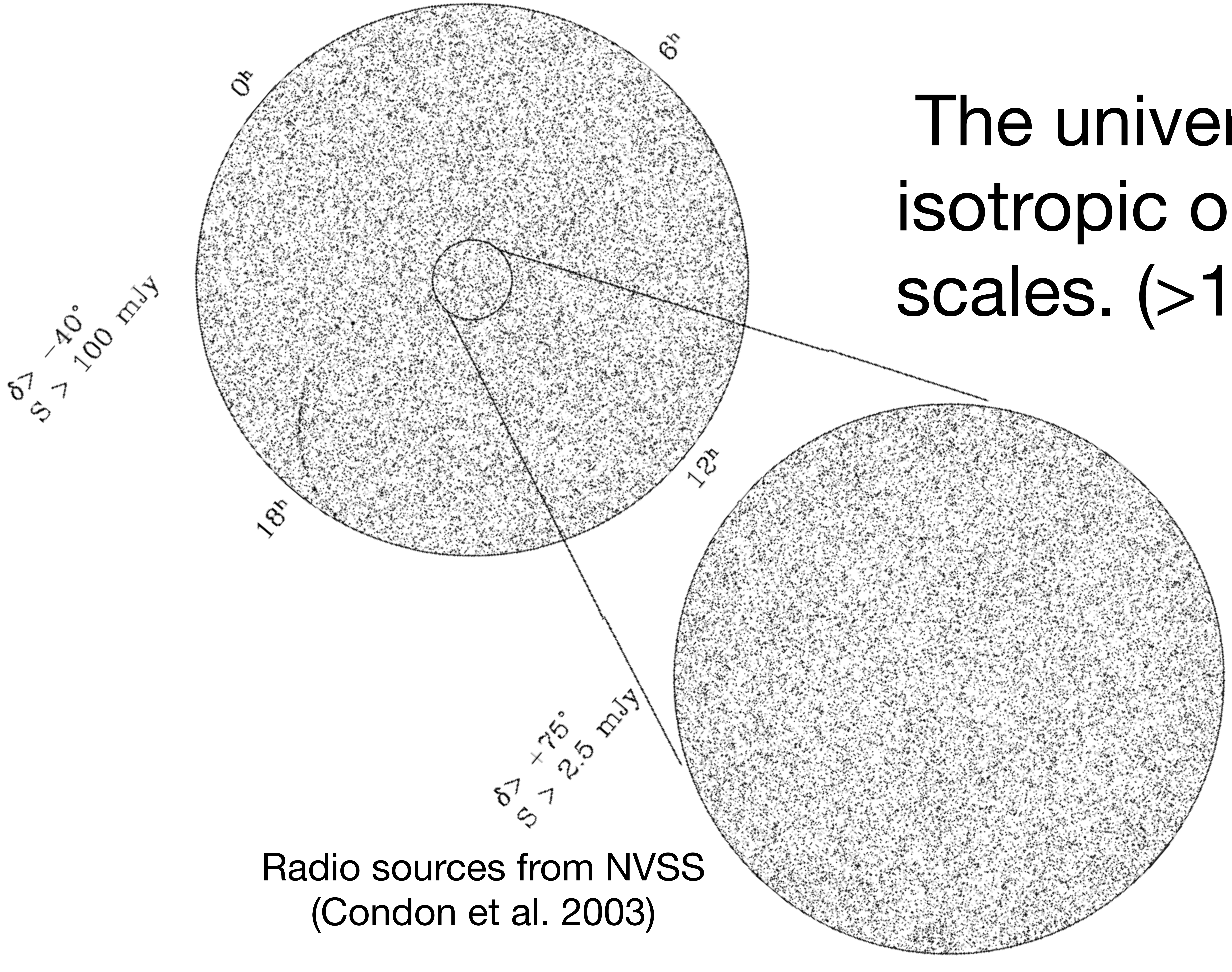
Observation

Olber's Paradox (1823)

Resolution?



Cosmological Principle



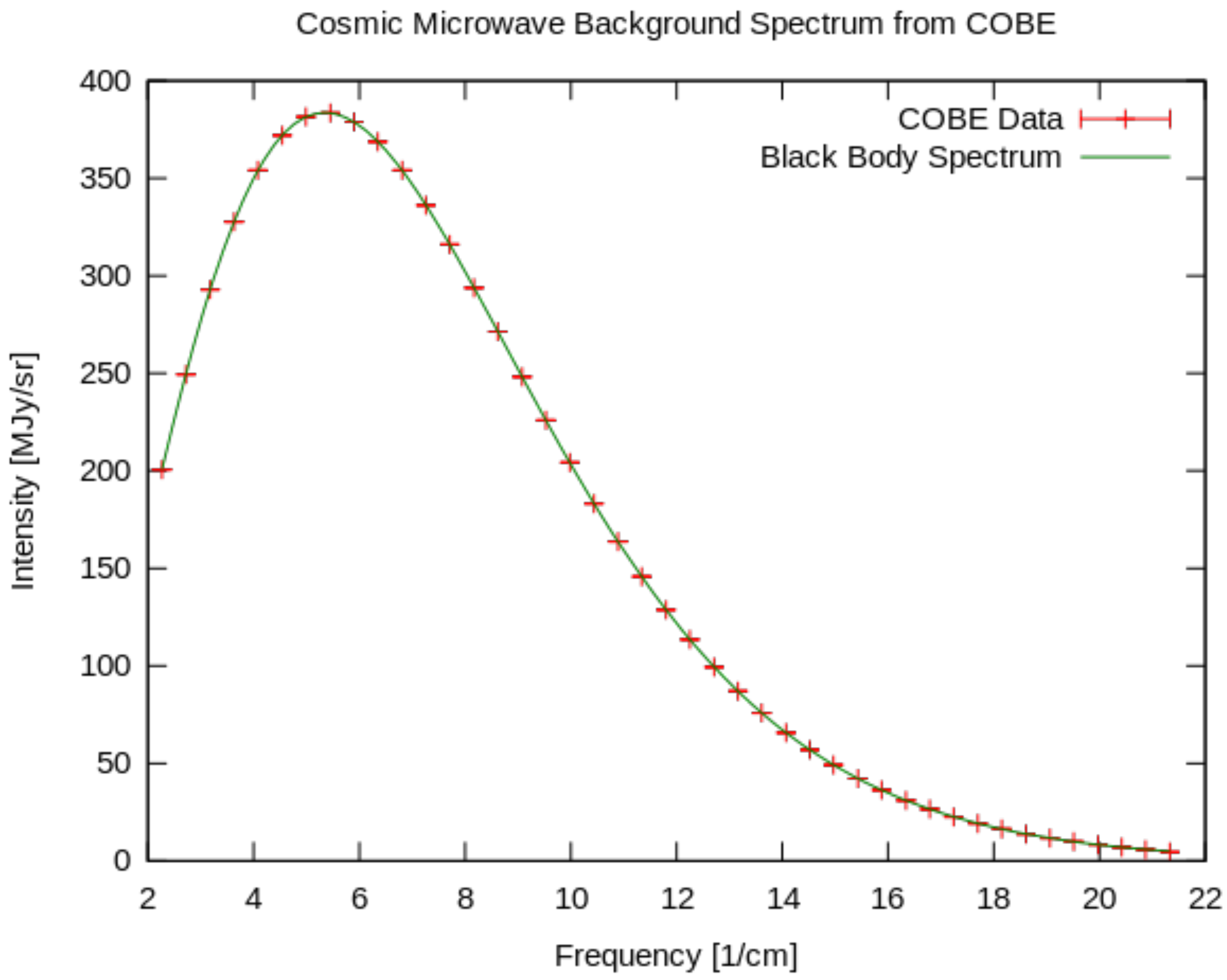
Radio sources from NVSS
(Condon et al. 2003)

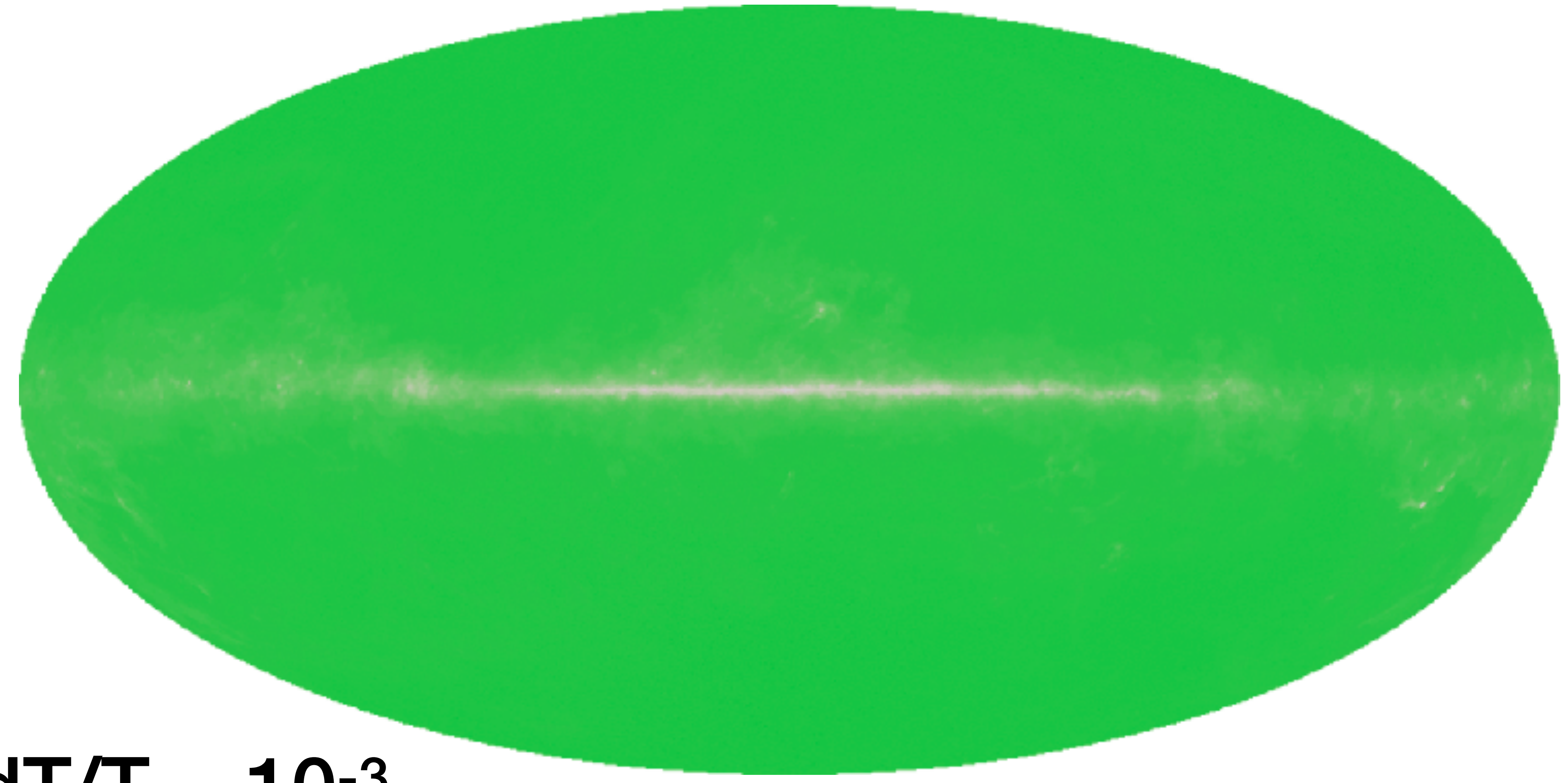
The universe is isotropic on very large scales. (>100Mpc).

Copernican Principle
=> homogeneous & isotropic

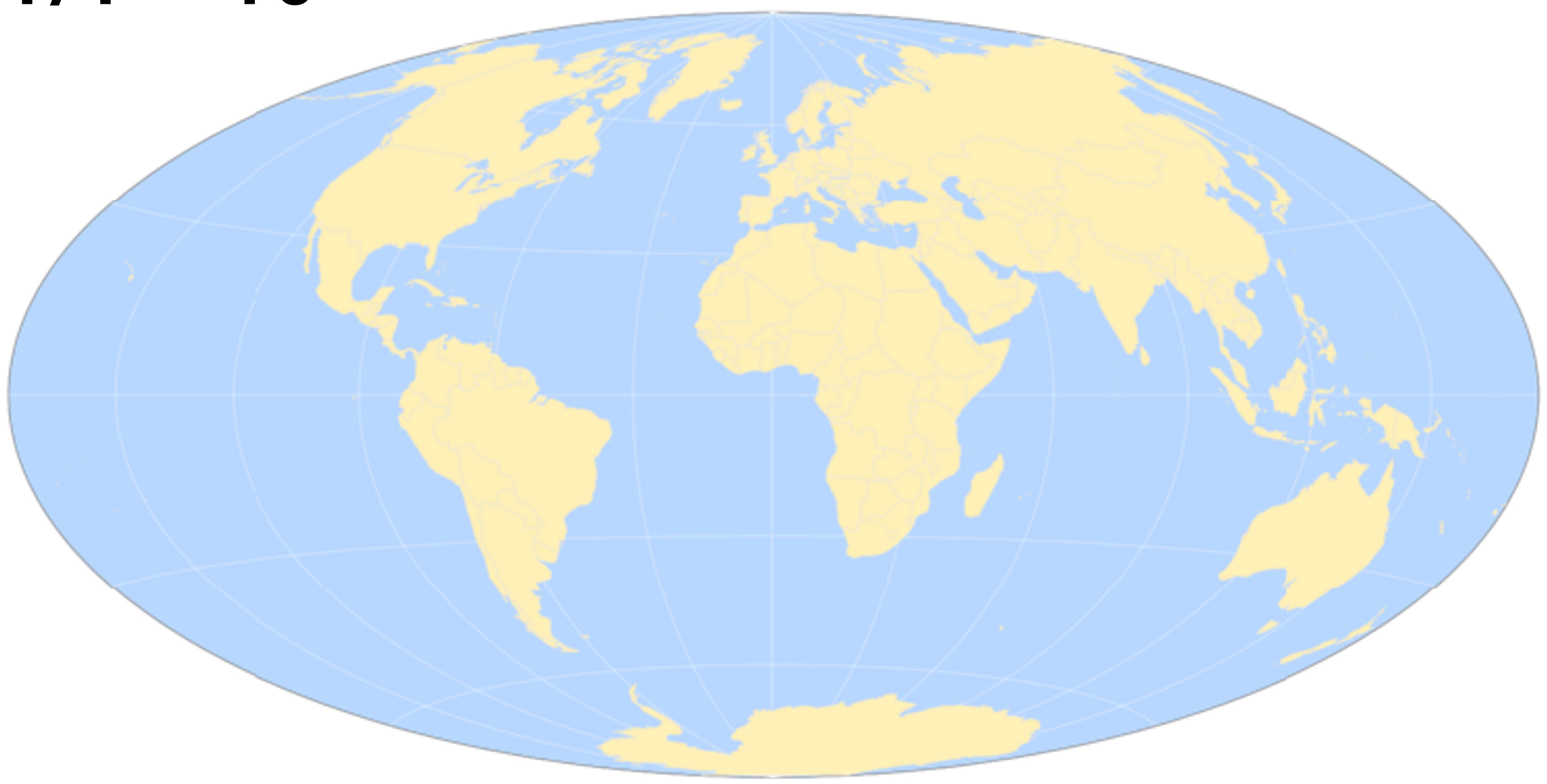
(Cosmological Principle)

Near perfect BB everywhere on the sky





$dT/T \sim 10^{-3}$



Getting distances to the nebulae

$$v_p(t_0) \equiv H_0 d_p(t_0) \rightarrow d_H(t_0) \equiv c/H_0$$

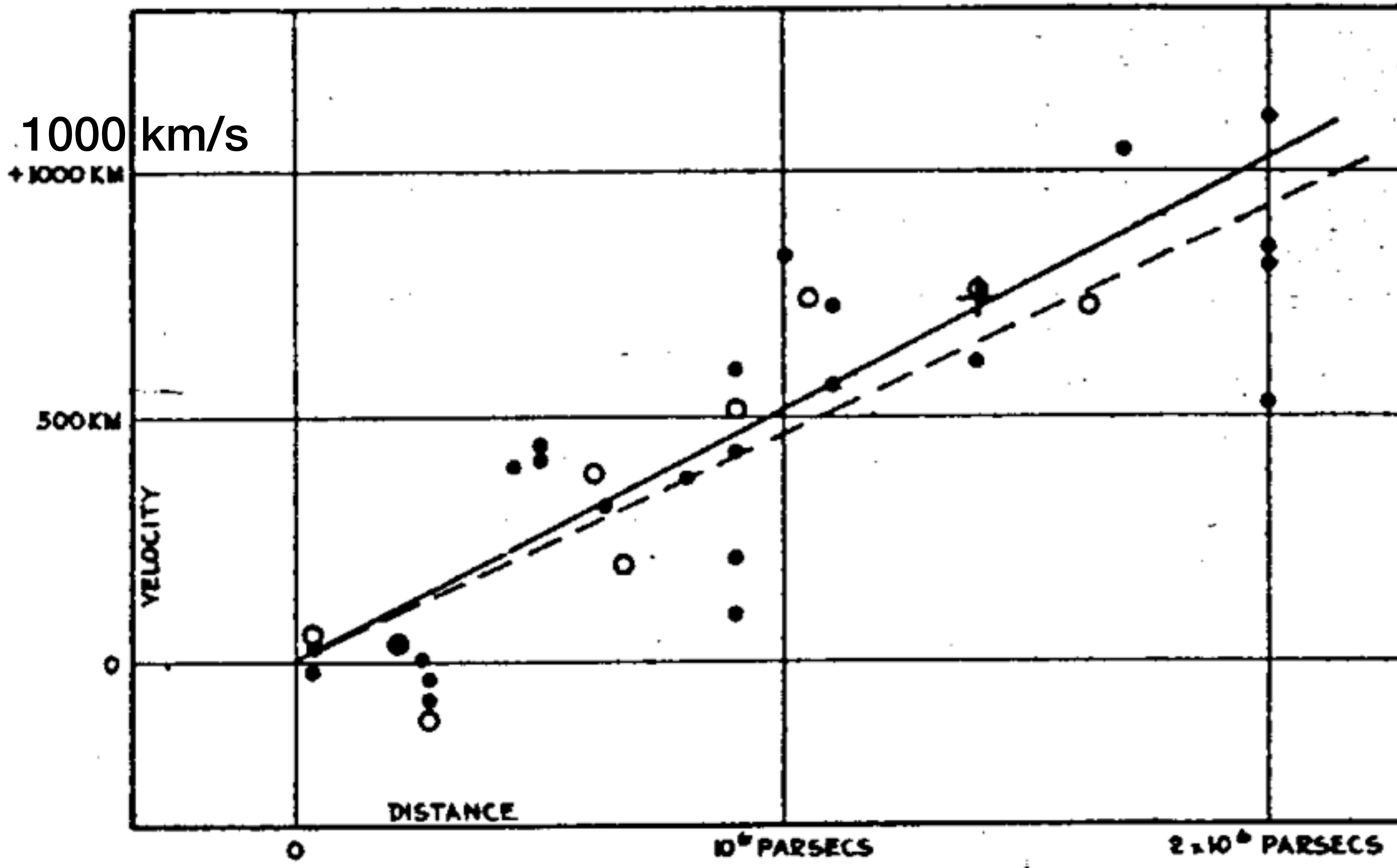
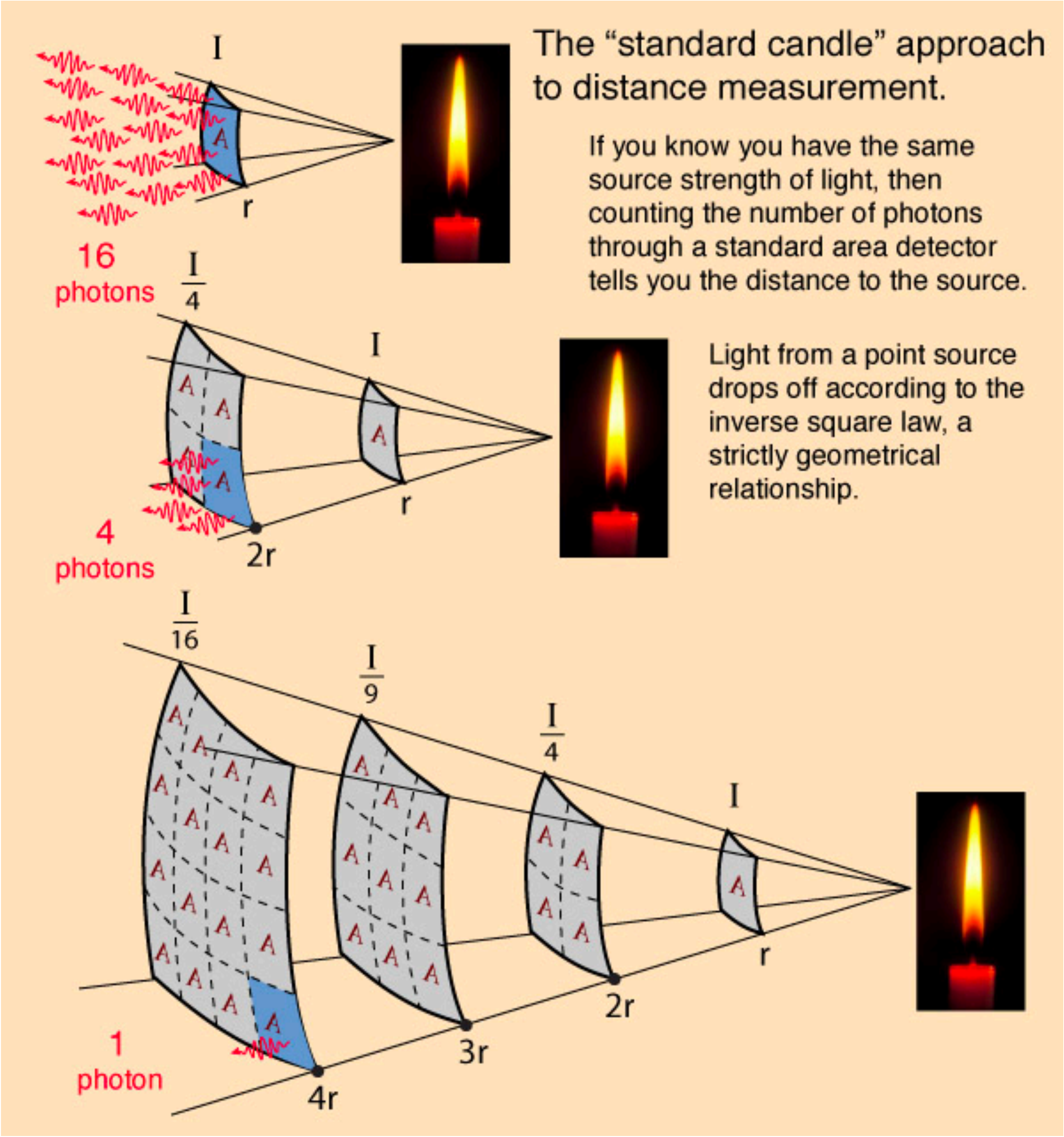


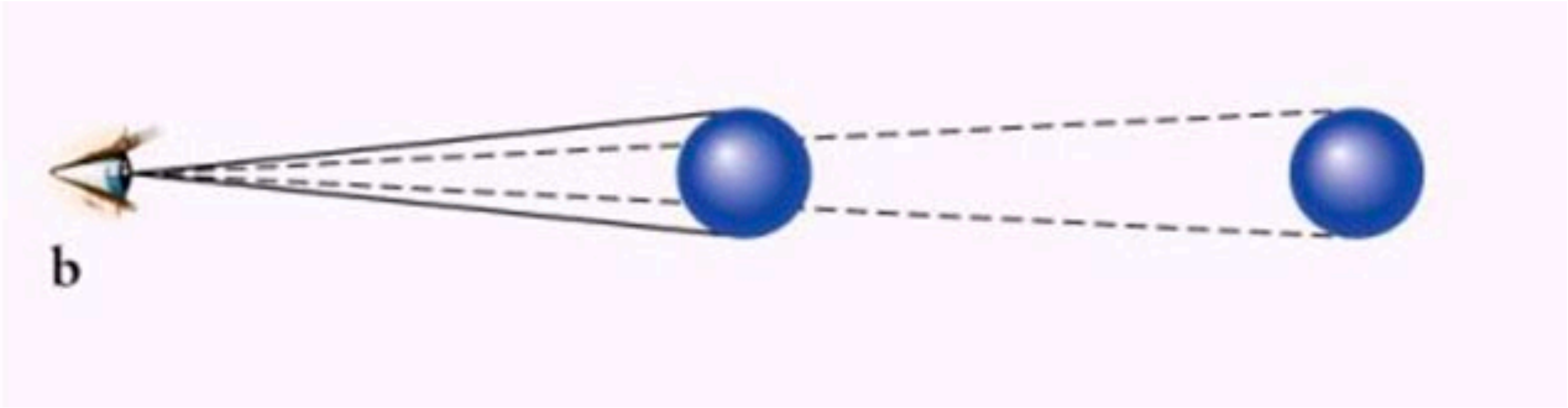
FIGURE 1

Practical Distance Measures

Luminosity Distance



Angular Diameter Distance



Practical Distance Measures

$$d_L = \sqrt{\frac{L}{4\pi f}}$$

$$d_A = \frac{D}{\theta}$$

in flat, static universe,

$$d_L = d_A = d_p$$

$$d_p \rightarrow ds^2 = -c^2 dt^2 + dr^2 + r^2 d\Omega^2$$

$$ds^2 = -c^2 dt^2 + a^2 \left[dr^2 + S_k(r)^2 d\Omega^2 \right]$$

$$S_k \begin{cases} R_0 \sin r/R_0 & k = +1 \\ r & k = 0 \\ R_0 \sinh r/R_0 & k = -1 \end{cases}$$

$$\frac{S_k(r)}{1+z} = d_A$$

$$d_L = S_k(r)(1+z)$$

$$k = 0, \quad d_A = \frac{d_p(t_0)}{1+z} = \frac{d_L}{(1+z)^2}$$

Practical Distance Measures

can define

$$q_0 = -\frac{\ddot{a} a}{\dot{a}^2}\bigg|_{t=t_0} = -\frac{\ddot{a}}{aH^2}\bigg|_{t=t_0}$$

$$a(t) = 1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2$$

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right)z^2\right] + \frac{c H_0}{2}\frac{z^2}{H_0} = \frac{cz}{H_0}\left[1 - \frac{1+q_0}{2} z\right]$$

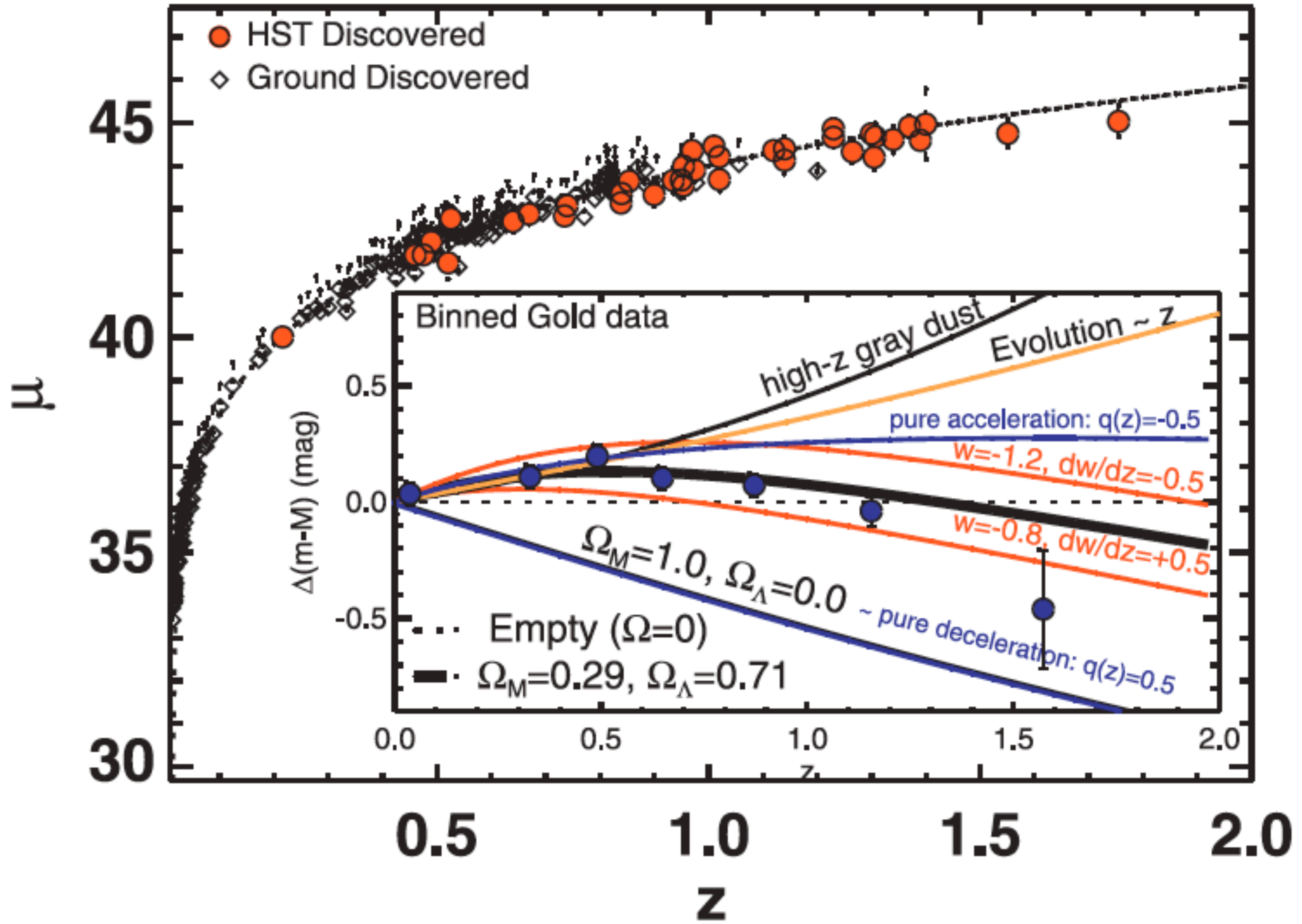
$$q_0 = -\frac{\ddot{a}}{aH^2}\bigg|_{t=t_0} = \frac{1}{2}\sum \Omega_{i,0}(1+3w_i)$$

$$d_L \cong \frac{cz}{H_0}\left(1 + \frac{1-q_0}{2} z\right)$$

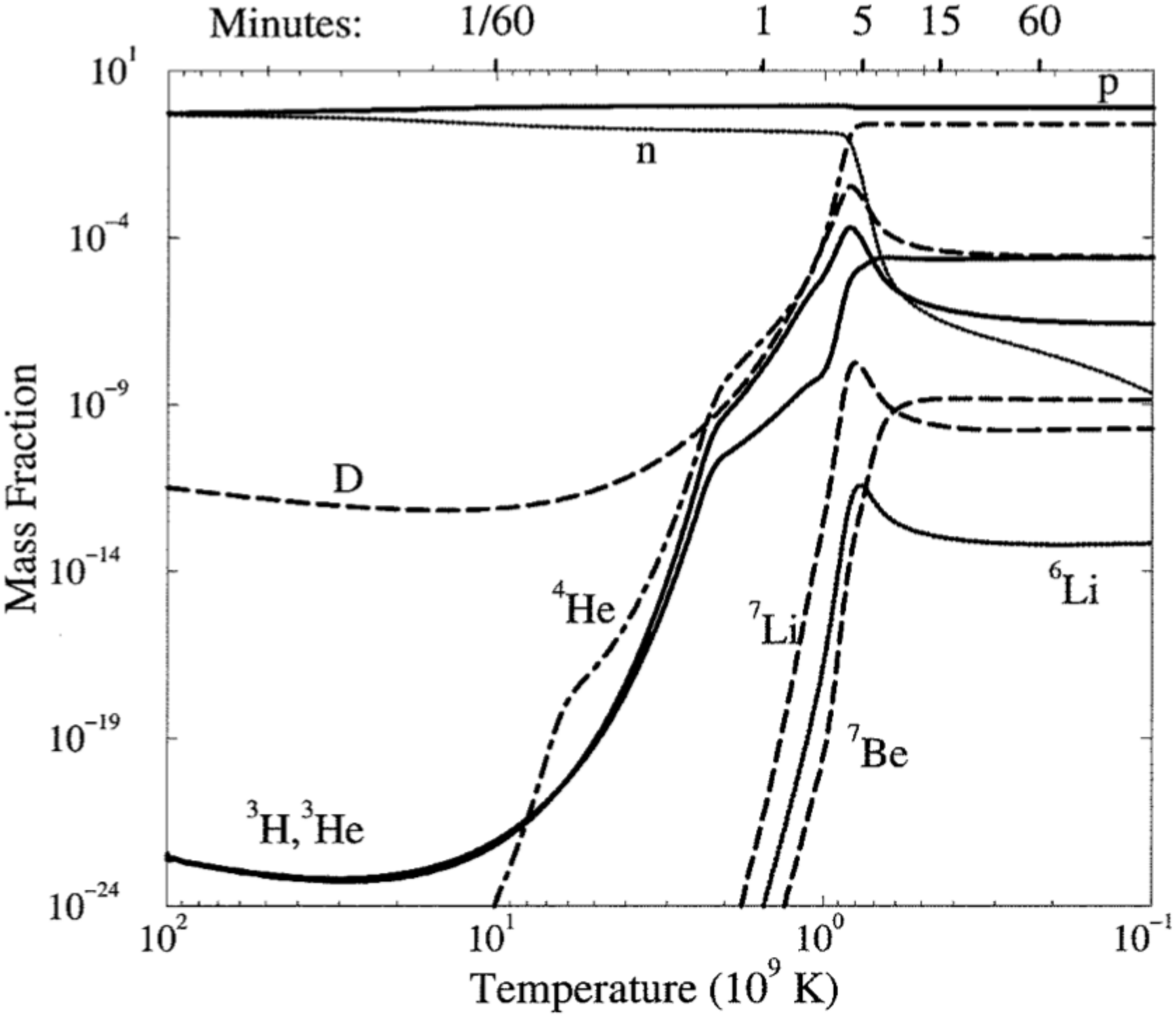
For the general case (rad, matter, Λ), set

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$$

$$d_A = \frac{cz}{H_0}\left(1 - \frac{3+q_0}{2} z\right)$$


HST Discovered
Ground Discovered
45
40
35
30
μ
0.5
1.0
1.5
2.0
z
Binned Gold data
high-z gray dust
Evolution ~ z
pure acceleration: q(z)=-0.5
w=-1.2, dw/dz=-0.5
w=-0.8, dw/dz=+0.5
Ω_M=1.0, Ω_Λ=0.0
~ pure deceleration: q(z)=0.5
Empty (Ω=0)
Ω_M=0.29, Ω_Λ=0.71
Δ(m-M) (mag)
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
z

Abundances from Nucleosynthesis



Creation process depends on relative abundances at any given time, so have to calculate computationally

Nucleosynthesis doesn't run to completion like in stars — rapidly dropping temperature cuts it off and "freezes" abundance pattern

Exact yields depend most on baryon-to-photon ratio: η (determines temperature of nucleosynthesis)

Baryonic Matter

$$\Omega_{*,0} \lesssim 0.005$$

$$M_{\mathrm{gas},0} \approx 10 \times M_{*,0}$$

By the time of the Big Bang and thereafter, normal matter is the subdominant form of matter in the universe, with some other form of matter (non-baryonic dark matter) making up the majority of non-relativistic matter in the universe

$$\text{early universe measurements}$$

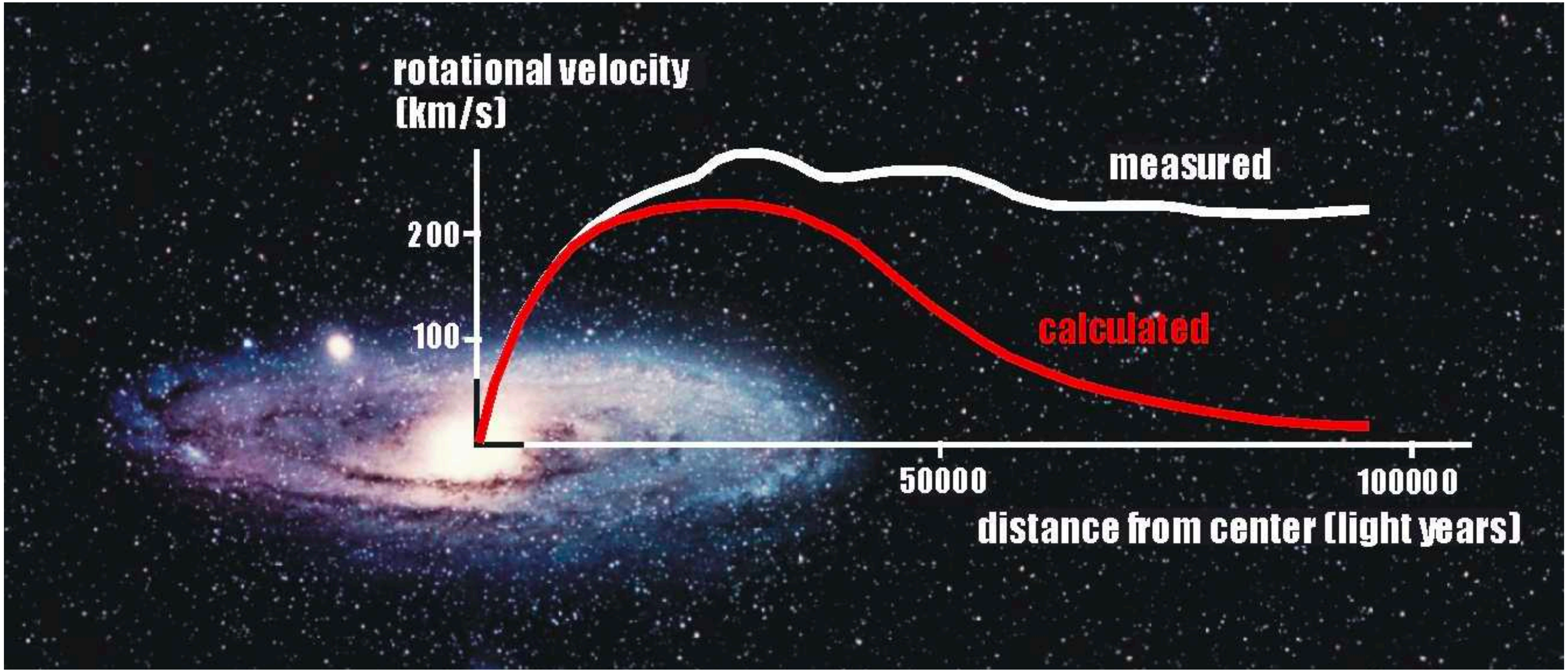
$$\Omega_{\mathrm{bary},0} = 0.048 \pm 0.003$$

Could be primordial black holes that were made before this time (i.e., not from stars).

$$\Omega_{m,0} = 0.31$$

$$\text{baryonic matter only } 15\%$$

Dark Matter

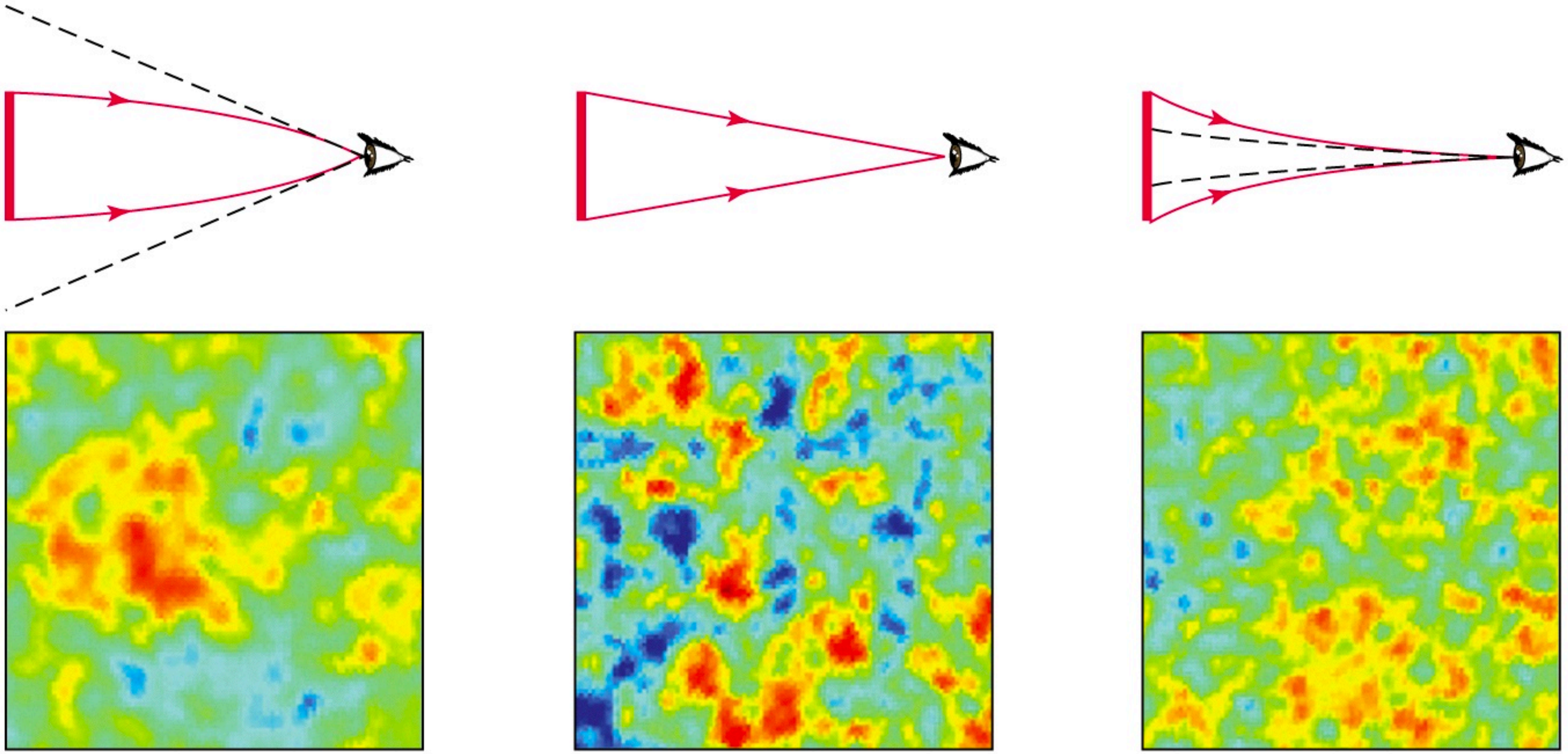


Galaxy Clusters:

$$M \approx \frac{\langle v^2 \rangle r_h}{\alpha G}$$

$$M(R) = \frac{v^2 R}{G} = 1.05 \times 10^{11}\ \mathrm{M}_\odot \left(\frac{v}{235\ \mathrm{km\ s}^{-1}}\right)^2 \left(\frac{R}{8.2\ \mathrm{kpc}}\right)$$

CMB provides a giant triangle of known size!

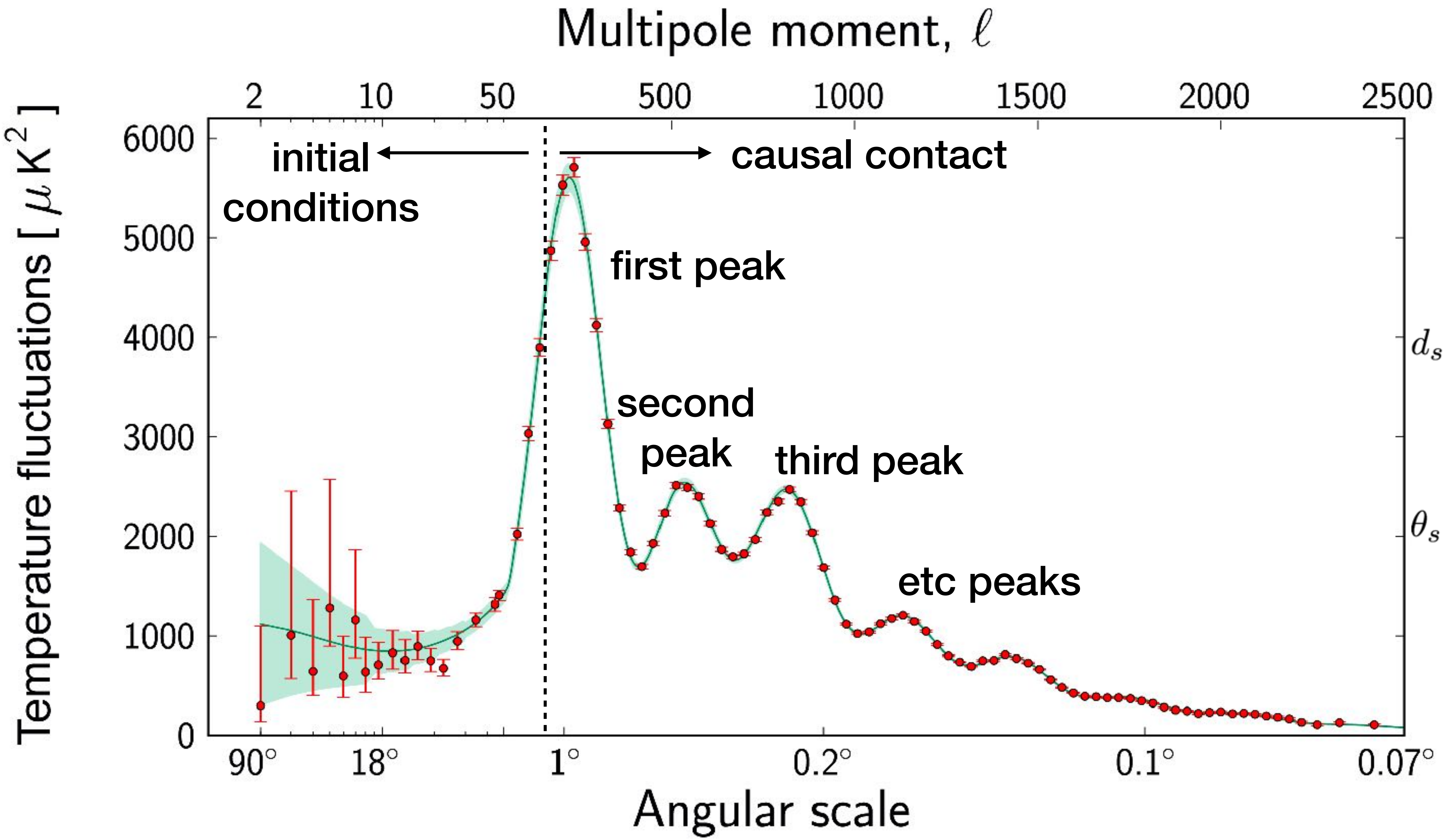


a If universe is closed, "hot spots" appear larger than actual size

b If universe is flat, "hot spots" appear actual size

c If universe is open, "hot spots" appear smaller than actual size

Acoustic peaks



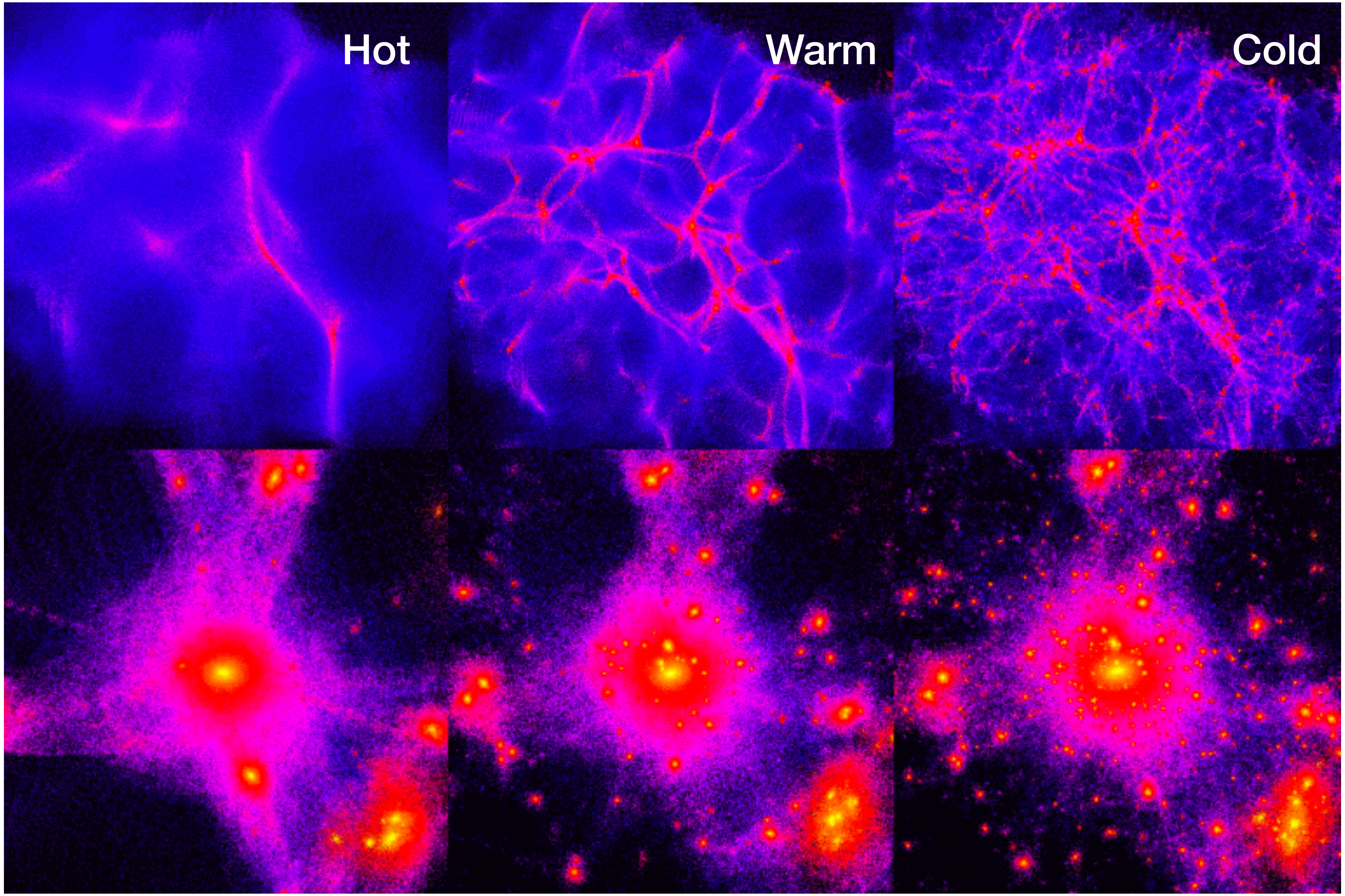
$$d_s(t_{\rm ls}) = a(t_{\rm ls}) \int_0^{t_{\rm ls}} \frac{c_s(t)dt}{a(t)}$$

$$d_s(t_{\rm ls}) \approx \frac{1}{\sqrt{3}} d_{\rm hor}(t_{\rm ls}) \approx 145 \text{ kpc}$$

$$\theta_s \approx \frac{d_s(t_{\rm ls})}{d_A} \approx \frac{145 \text{ kpc}}{12.8 \text{ Mpc}} \approx 0.7^\circ$$

size scale of a DM potential well where baryon collapse reaches turnaround due to its pressure

Temperature of the Dark Matter



velocity of particles compared to the speed of light

relativistic at time of collapse (like neutrinos): hot

non-relativistic at time of collapse (like WIMPs): cold

fast motions wipe out initial overdensities on small scales: "free-streaming"

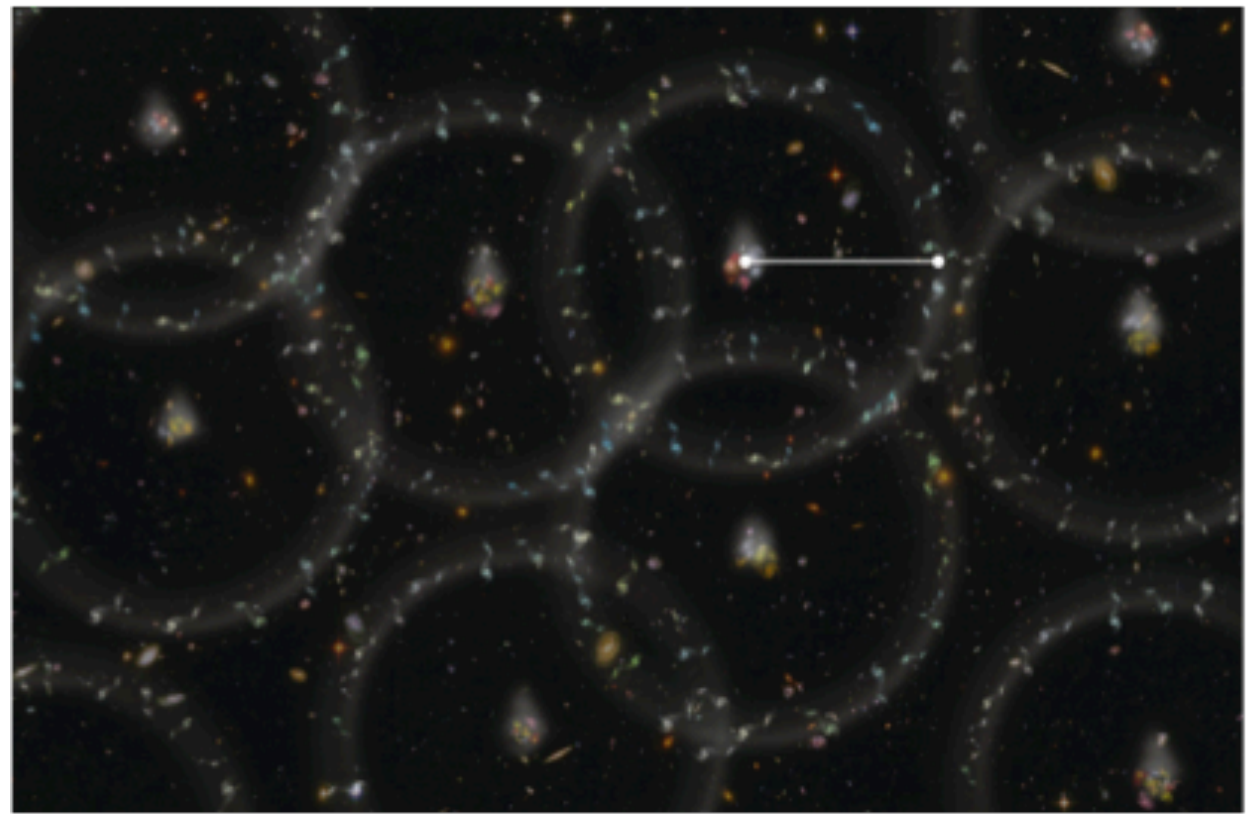
Baryon Acoustic Oscillations

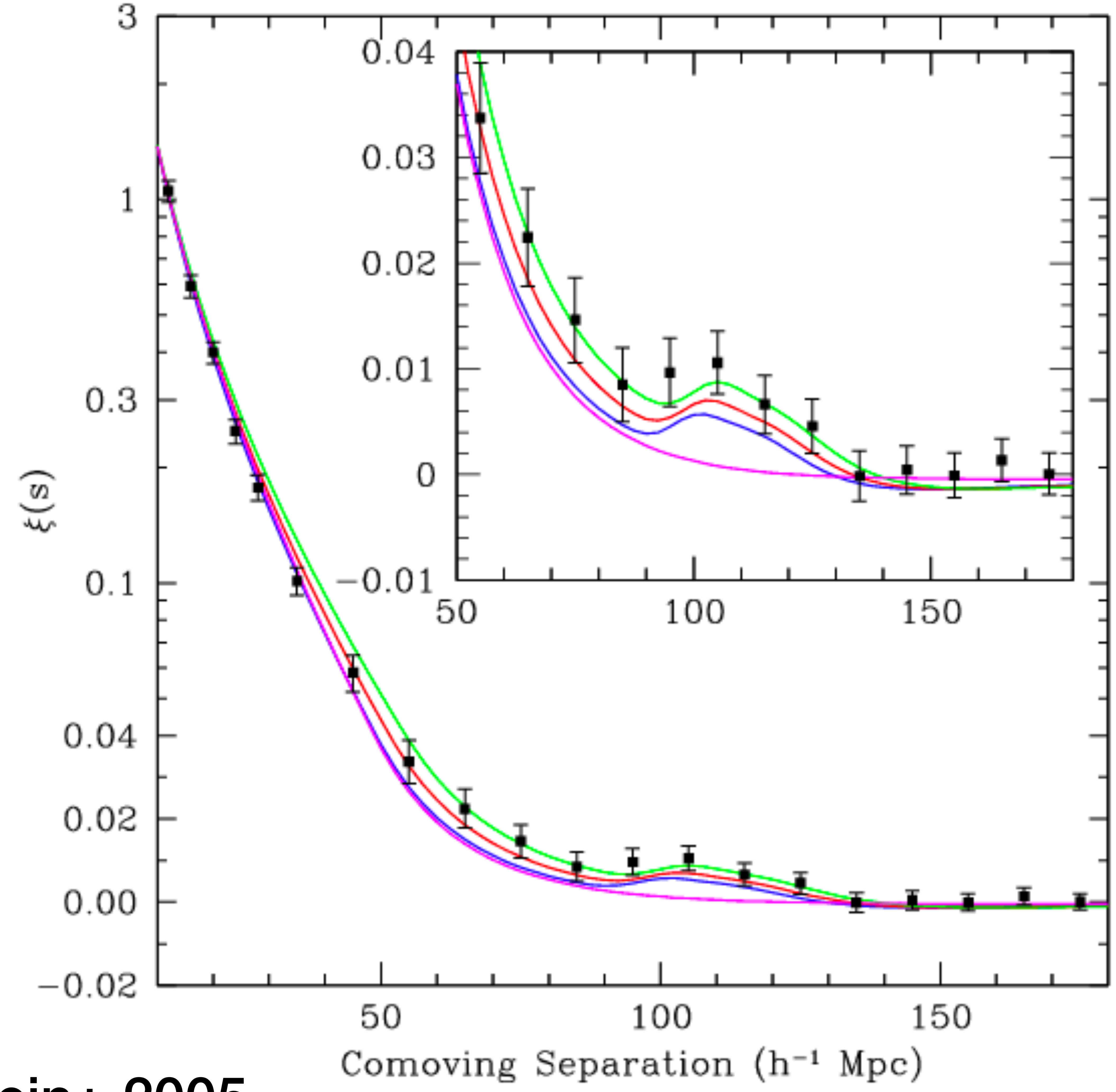
To measure, use galaxies to trace the signature of these oscillations

The number of galaxies should be correlated with each other on scales comparable to the sound horizon of the largest acoustic peaks (~150 Mpc comoving)

The number of galaxies within a given volume is

$$dN = n_{\mathrm{gal}}[1 + \xi(r)]dV$$





Eisenstein+ 2005

Fin