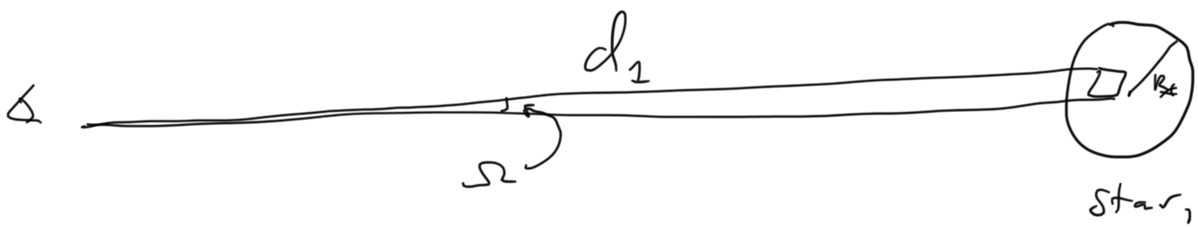
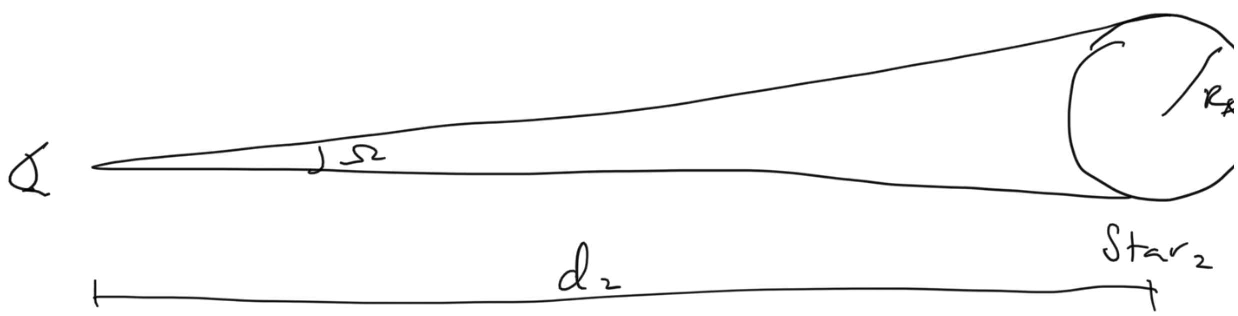
ASTR 4080 - Week 1

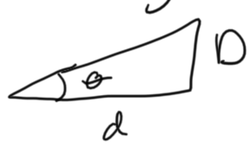
Surface brightness independent of distan

↳ $\Sigma = \dfrac{f}{\Omega}$ $\dfrac{\text{flux (E/area/time)}}{\text{Angular area (steradian or radian squared)}}$





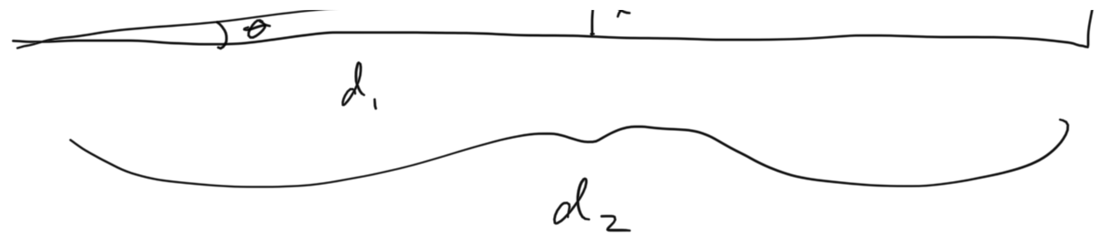
Consider angle to be 1D instead of

2D : 

$\tan\theta = \dfrac{D}{d} = \dfrac{\text{tangential di... on s...}}{\text{distance to sta...}}$

if $D << d$, then $\tan\theta \approx \theta$

if $R_1 = R_2$ + thus $d_1 < d_2$ if $\Omega_1 = \Omega$



$$\Omega \approx \theta^2 = \left(\frac{R_2}{d_2}\right)^2$$

By similar triangles, $\theta^2 = \left(\frac{x}{d_1}\right)^2$

where x is a fraction of R_1 & $\frac{d_1}{d_2}$

thus $x = \frac{d_1}{d_2} R_2 \rightarrow$ confirming θ sa

What about fluxes?

Total flux $F = \frac{L}{4\pi d^2} = \frac{\int \Sigma \, dA}{4\pi d^2}$

$$F_2 = \frac{\Sigma A_*}{4\pi d_2^2} = \frac{\Sigma \pi R^2}{4\pi d_2^2}$$ equal!

$$F_1 = \frac{\Sigma A_* \left(\frac{x}{R}\right)^2}{4\pi d_1^2} = \frac{\Sigma \pi R^2 \left(\frac{\cancel{d_1}}{d_2}\right)^2}{4\pi \cancel{d_1^2}} = \frac{\Sigma \pi}{4\pi d}$$

☆ Textbook & "Key concepts" derives this slightly differently, but same logic is used

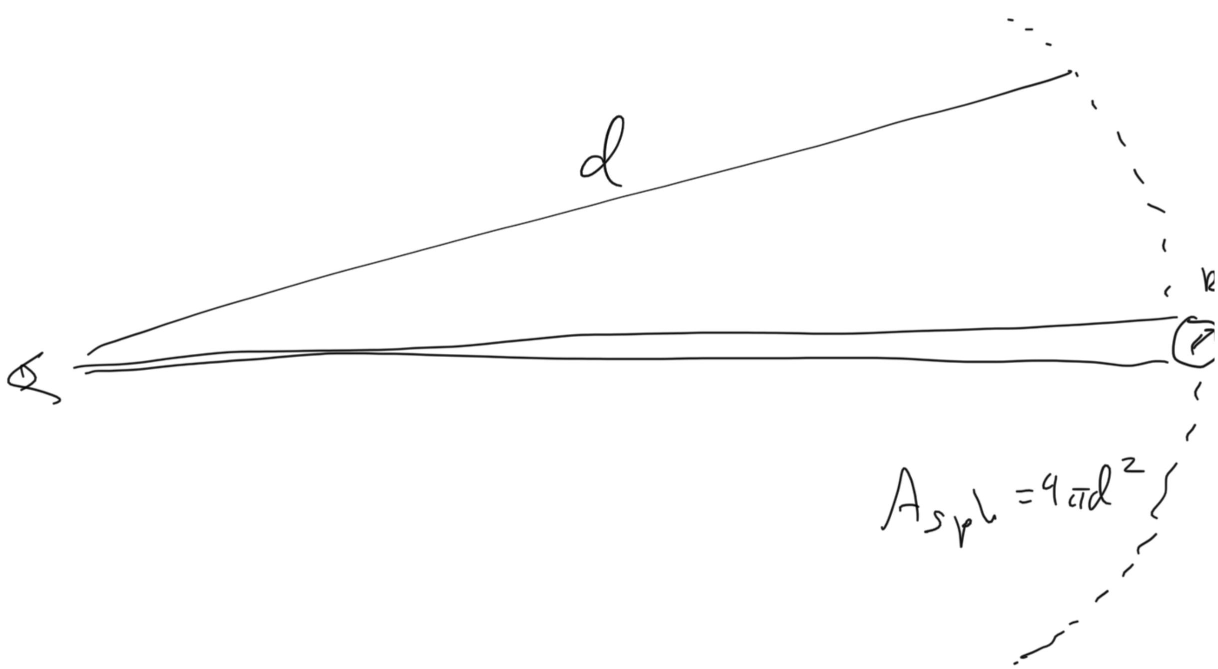
BACK TO SLI

SKIP

Total flux of star $f = \frac{L_*}{4\pi d^2}$

Angular area (if $d \gg R_*$, then $\theta \approx \frac{R_*}{d}$)

is $\Omega = \frac{\pi R_*^2}{4\pi d^2}$ $\quad \frac{\text{Area on proj. sphere of star}}{\text{Area of proj. sphere}}$



So, the SB is

$$\Sigma_* = \frac{f}{\Omega} = \frac{L_*}{\cancel{4\pi d^2}} \frac{\cancel{4\pi d^2}}{\pi R_*^2}$$

$$= \frac{L_*}{\pi R_*^2} \quad \text{indep. of } d$$

Imagine $r < R_*$, keep Ω fixed & move star away until $r = R_*$

→ more lum. enters Ω ($\propto d^2$)

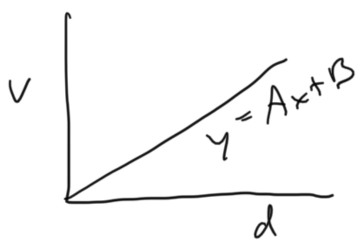
TO HERE → but its flux drops ($\propto d^{-2}$)

Redshift (z) & Hubble's law ($z = \frac{H_o}{c}$

Recession velocity ↑ w/distance

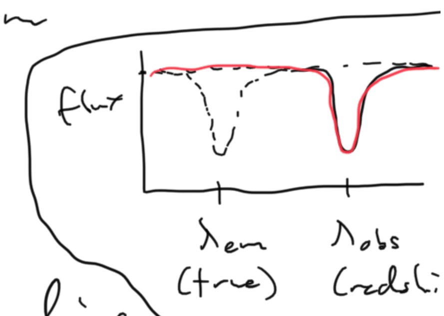
$$v = H_o d$$

↳ Hubble's constant (prop. constant)



Redshift $z \equiv \dfrac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \dfrac{v}{c}$

Thus $\boxed{cz = H_o d}$



Implies universe is expanding

- use current distance b/t 2 objects as reference, define a unitless scale factor $\boxed{a(t) = \dfrac{d(t)}{d(t_o)}}$ $\dfrac{\text{distance past}}{\text{distance now}}$

How does $a(t)$ relate to Hubble's law?

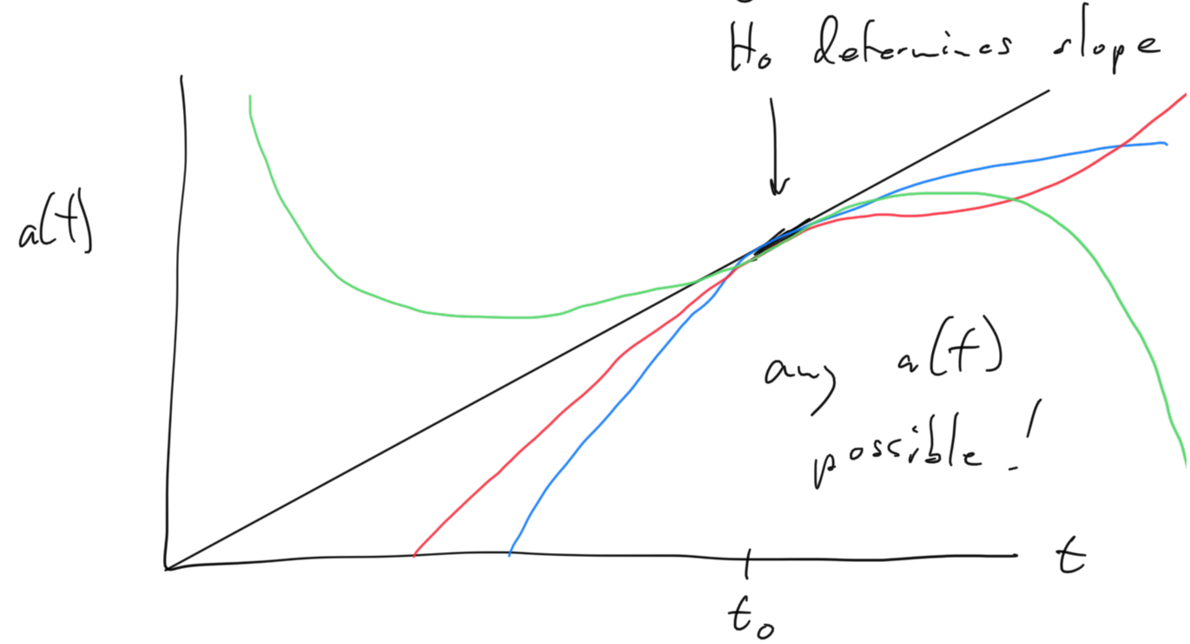
$a \rightarrow$ scaled distance (unitless

law $\rightarrow$ $v = H_0 d$, vel. is derivative of position

$$\frac{d}{dt} a(t) = \dot{a} = \frac{\dot{d}}{d(t_0)} = \frac{v}{d(t_0)} = \frac{H_0 d}{d(t_0)} = H_0 a$$

$$H_0 = \frac{\dot{a}}{a}$$ $\rightarrow$ at least as long as $d(t)$ is "local"

Measuring H_0 tells us how fast universe is expanding NOW



If expansion constant, can infer

age of universe

$\hookrightarrow \dfrac{d}{v} = \dfrac{d}{H_0 d} = H_0^{-1}$: Hubble time

(If not const. expansion, $t_{age} \neq H_0^{-1}$, but Hubble time useful unit)

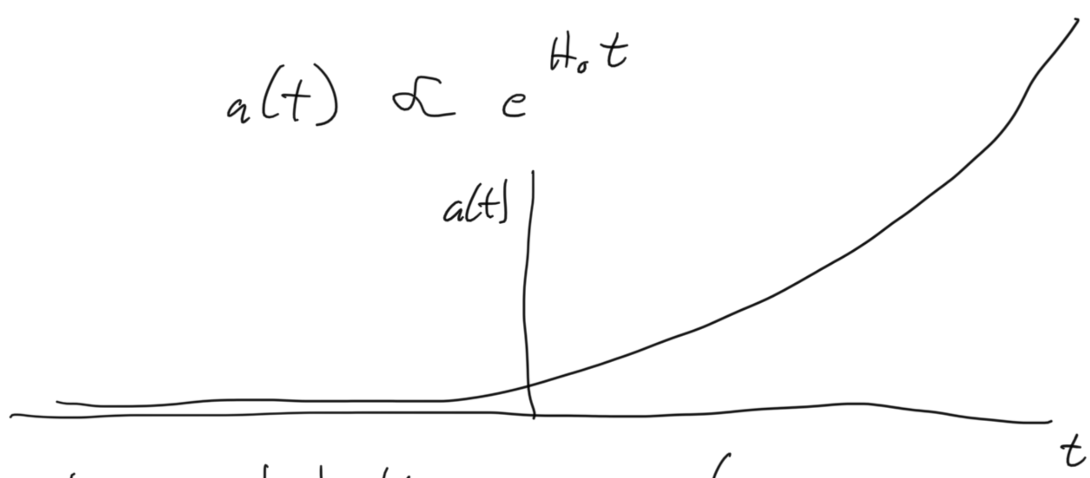
$H(t)$ [the slope] isn't constant, but what if it were?

$$\frac{\dot{a}}{a} = H_0 \quad \text{always}$$

$$\frac{da}{a} = H_0\,dt \rightarrow \int \frac{da}{a} = H_0 t$$

$$\ln a + \text{const} = H_0 t$$

$$a(t) \propto e^{H_0 t}$$



All times look the same (measure same H_0) - no beginning to time

Steady-state universe : create matter to keep density of universe constant

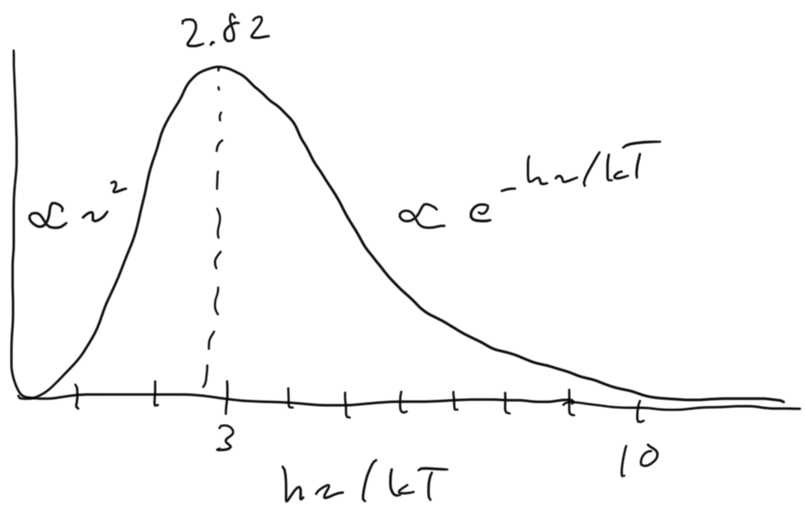
$\Rightarrow$ need 1 H atom per $1 cm^3$ per γ

☆ Why might this be problematic to a physicist?

SLIDE

Blackbody Radiation

$$\varepsilon(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3\,d\nu}{e^{h\nu/kT}-1}$$



Integrate over ν

$$\varepsilon = \alpha T^4, \quad \alpha = \frac{\pi^2 k^4}{15\hbar^3 c^3}$$

CMB: leftover radiation (BB) from the big bang

$T_0 = 2.73\,K$

As universe expands, $T \downarrow$ b/c photo
λ tied to expansion of space

Thermo: $dQ = dE + PdV = 0 \rightarrow$ 1st la

E conserv

so $\boxed{\frac{dE}{dt} = -P(t)\,\frac{dV}{dt}}$

PdV is th
work done
the system

differentiate

$$E = \varepsilon V = \alpha T^4 V$$

$$P = \frac{1}{3}\varepsilon = \frac{\alpha}{3}T^4$$

pressure of a
photon
gas

$$\frac{dE}{dt} = \alpha\left(4T^3\frac{dT}{dt}V + T^4\frac{dV}{dt}\right)$$

$$= -\frac{\alpha}{3}T^4\frac{dV}{dt} \quad \left(= -P\frac{dV}{dt} \text{ from 1st law}\right)$$

$$\cancel{4\alpha T^3}\,\frac{dT}{dt}V = -\frac{\cancel{4\alpha}}{3}T^4\frac{dV}{dt}$$

$$\frac{1}{T}\frac{dT}{dt} = -\frac{1}{3}\frac{1}{V}\frac{dV}{dt}$$

Since $V \propto a^3$, $\frac{dV}{dt} \propto 3a^2\frac{da}{dt}$

$$\frac{1}{V}\frac{dV}{dt} \propto \frac{3a^2}{a^3}\frac{da}{dt} = \frac{3}{a}\frac{d}{d}$$

$$\frac{1}{T}\frac{dT}{dt} \propto -\frac{1}{a}\frac{da}{dt}$$

$d(\ln x) = \frac{1}{x}dx$, so $\frac{d}{dt}(\ln T) \propto -\frac{d}{dt}(\ln a$

$$\ln T \propto -\ln a \propto \ln a^{-1}$$

$$T(t) \propto a(t)^{-1}$$

$T(t_0) = T_0$

$a(t_0) = 1$

$$\boxed{T(t) = T_0\, a(t)^{-1}}$$

What should T_0 be?

- In stars, we see that they're composed of ~75% H, ~25% He, trace elements @ ↑ mass
- Means that nucleosynthesis has to shut off before more elements burned, $T \sim 3000\,K$

↳ this must happen when

$$a(t_{decouple}) \sim \frac{a(t_0)}{1000} = 10^{-3}$$

$$= (10^{-3})^{-1}$$

$$3000K = T_0 (10^{\text{[illegible]}})$$

$$\underline{T_0 \approx 3K}$$