ASTR 4080 -Week 2

Begin by asking: what is an inertial frame?

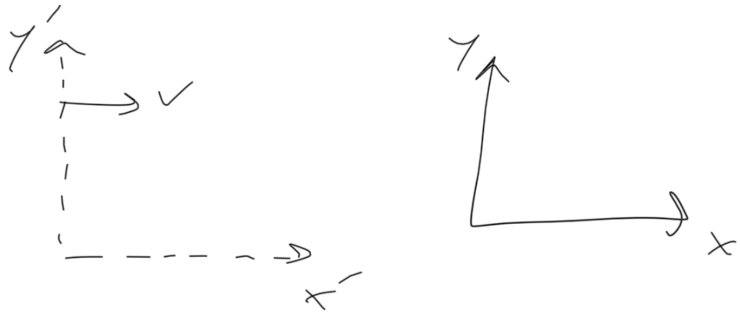
ANS: F=ma is true

$$F = ma = m\frac{d^2\vec{r}}{dt^2}, \text{ so if } \underline{r = r_0 - vt}$$

$$\frac{dr}{dt} = -v, \quad \boxed{\frac{d^2r}{dt^2} = 0}$$

$$\underline{F=0} \text{ if } \underline{r = r_0} \text{ or } \underline{r = r_0 - vt}$$

What is "special" about SR?



$t_0 = 0,$

$t' = t$

$x' = x$

$t_1 > t_0 \quad x' = 0 \rightarrow \quad x = x' + A$

$\uparrow$

$+\ \#$

$$x = vt \rightarrow \boxed{x = x' + vt}$$

Key difference in SR is that laws must be same, which means Maxwell's Eq. hold

↳ insight is that c must be same in both frames

@ $t=0$, lamp turns on, so BOTH frame see light sphere expand @ speed c

$$c^2t^2 = x^2 + y^2 + z^2$$
$$c^2t'^2 = x'^2 + y'^2 + z'^2$$

Galilean transform won't work

Postulate some factor: $x' = \gamma(v)(x - vt)$

$c^2t^2 = x^2 + \dots, \quad c^2t'^2 = x'^2 + \dots$

$$c^2t'^2 = \gamma^2(x - vt)^2 + \dots$$

☆ BC

$$c^2t'^2 - \gamma^2(x - vt)^2 = c^2t^2 - x^2 \rightarrow \begin{array}{l} y^2 + z^2 \\ y'^2 + z'^2 \end{array}$$

$$t'^2 = t^2 - x^2/c^2 + \frac{\gamma^2}{c^2}(x - vt)^2$$

$$= t^2\left[1 + \frac{v^2\gamma^2}{c^2}\right] - 2\frac{xv\gamma^2}{c^2}t + \frac{x^2}{c^2}\left[\gamma^2 -\right.$$

Complicated, @ this point guess γ so that equation can be factored

maybe $t' = \gamma t\, g(v, x, t)$

$$t'^2 = \gamma^2\left[t^2\left[\frac{1}{\gamma^2} + \frac{v^2}{c^2}\right] - \frac{2xv}{c^2}t + \frac{x^2}{c^2}\left(1 - 1/\gamma^2\right)\right]$$

guess that $g(v, x, t) = t \pm \dots$

want $\frac{1}{\gamma^2} + \frac{v^2}{c^2} = 1$

if $\frac{1}{\gamma^2} + \frac{v^2}{c^2} = 1$, then

$$\gamma^2 = \frac{1}{1 - v^2/c^2} \rightarrow$$ Lorentz factor

subbing in:

$$t'^2 = \gamma^2\left[t^2 - 2\frac{vx}{c^2}t + \underbrace{\frac{x^2}{c^2}\left(1 - \left[1 - \frac{v^2}{c^2}\right]\right)}_{\frac{x^2v^2}{c^4}}\right]$$

exactly what needed

$$t'^2 = \gamma^2\left(t - \frac{vx}{c^2}\right)^2$$

$$\boxed{t' = \gamma\left(t - \frac{vx}{c^2}\right)}$$

$$\boxed{x' = \gamma(x - vt)}$$

For $v \ll c$, $v/c \rightarrow 0$ & $\gamma = 1$: Galilean transform restored

* implies neither time or space fundamental, just relative to observers

However, the space-time is fundamental

$$\Delta s^2 = -c^2 \Delta t^2 + \Delta l^2 = \underbrace{-c^2 \Delta t'^2 + \Delta l'^2}_{\Delta s'^2}$$

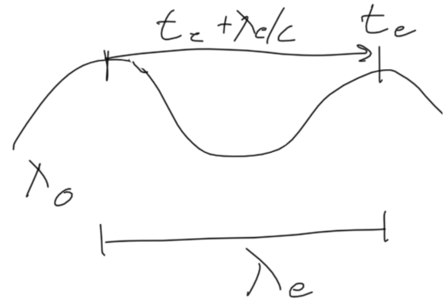
SLIDES

null geodesic: $c^2 dt^2 = a(t)^2 dr^2$

so $c \dfrac{dt}{a(t)} = dr$

can integrate this to get the distance b/t us & a galaxy @ 2 times, w/ $\Delta t \sim 0$ so distance $r_1 = r_2$

consider a wave of light:



want to compute wavelength λ_0 @ t_0 (when observed)

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^r dr = r = c\int_{t_e+\lambda_e/c}^{t_0+\lambda_0/c} \frac{dt}{a(t)}$$

→ can't integrate over $a(t)$, b/c we don't know its functional form

Subtract $\int_{t_e+\lambda_e/c}^{t_e} \frac{dt}{a(t)}$ from each side

$$\int_{t_e}^{t_e+\lambda_e/c} \frac{dt}{a(t)} = \int_{t_0}^{t_0+\lambda_0/c} \frac{dt}{a(t)}$$

now Δt very short, $a(t) \sim$ const.

$$\frac{1}{a(t_e)} \int_{t_e}^{t_e+\lambda_e/c} dt = \frac{1}{a(t_o)} \int_{t_o}^{t_o+\lambda_o/c} dt$$

$$\frac{\lambda_e}{\cancel{c}\, a(t_e)} = \frac{\lambda_o}{\cancel{c}\, a(t_o)}$$

$$z \equiv \frac{\lambda_o - \lambda_e}{\lambda_e} \quad \& \quad a(t_o) \equiv 1$$

$$\boxed{a(t_e) = \frac{\lambda_e}{\lambda_o} = \frac{1}{1+z}}$$

$$= \frac{\lambda_o}{\lambda_e} - 1 \qquad \text{or} \qquad \boxed{1+z = \frac{1}{a(t_e)}}$$

☆ → @ $z = 1$, universe was $\frac{1}{2}$ current size

→ holds true regardless of how universe expands