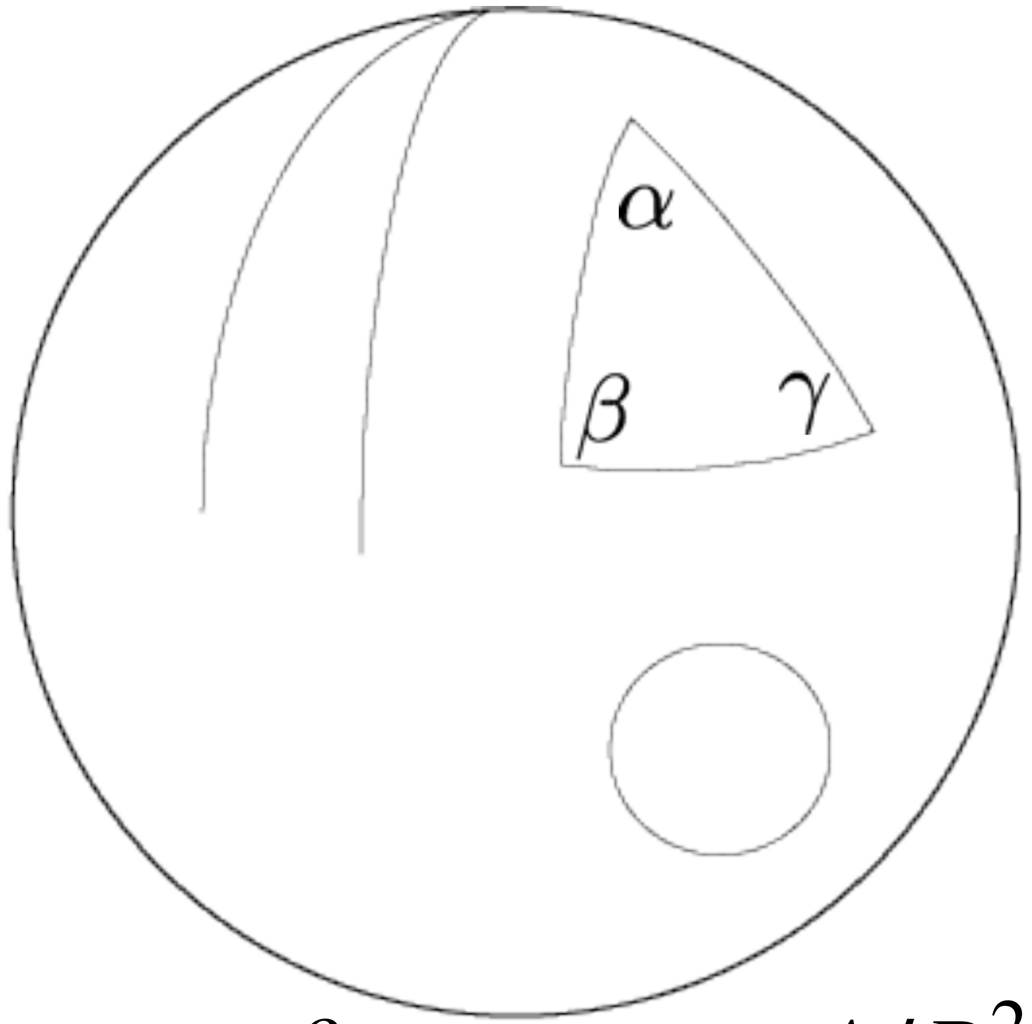
Week 3

Read thru Chapter 4
Also read the Key Concepts for those chapters

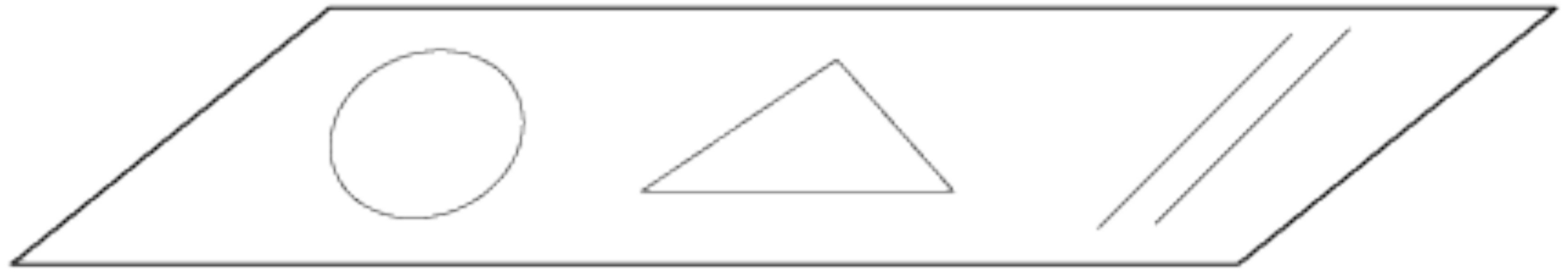
Today: The Friedmann Equation

HW 2 due Thursday

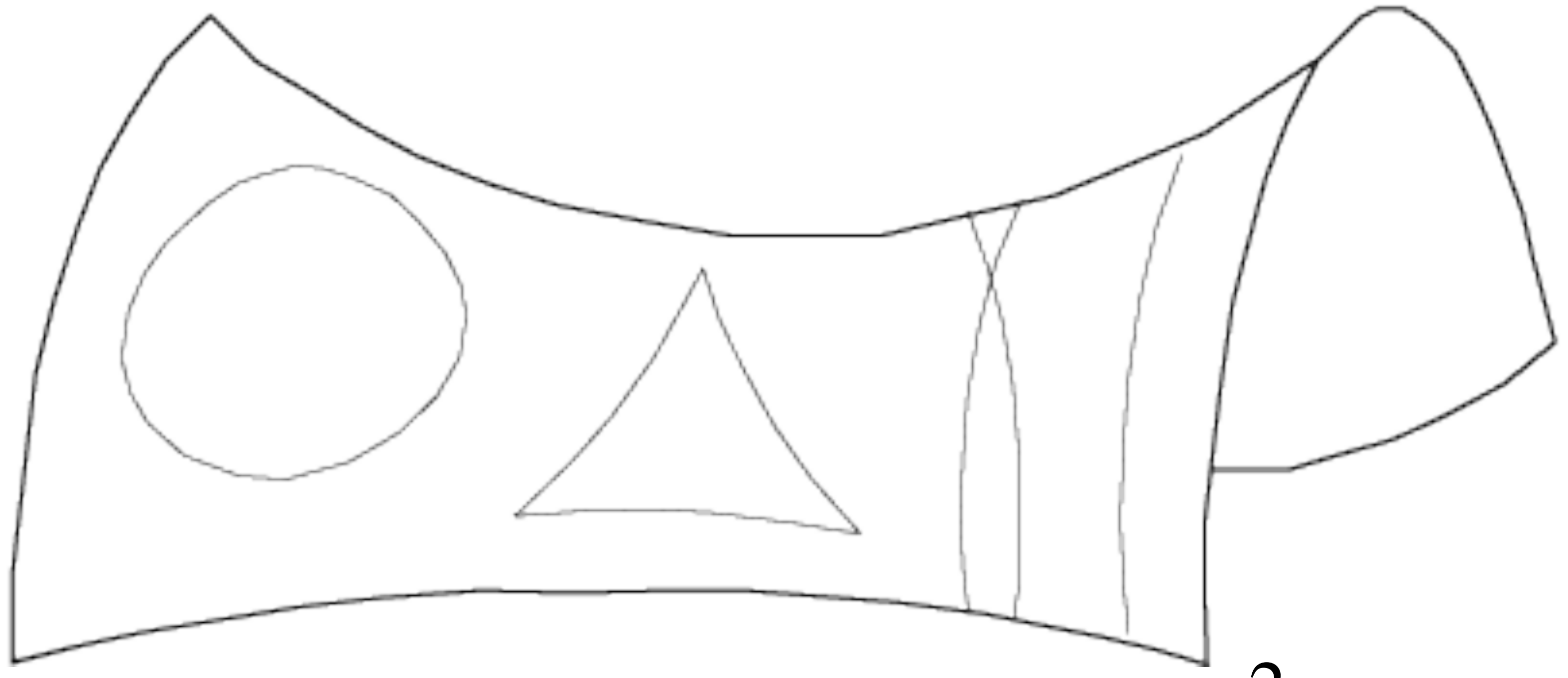
Curvature



$$\alpha + \beta + \gamma = \pi + A/R^2$$



$$\alpha + \beta + \gamma = \pi$$



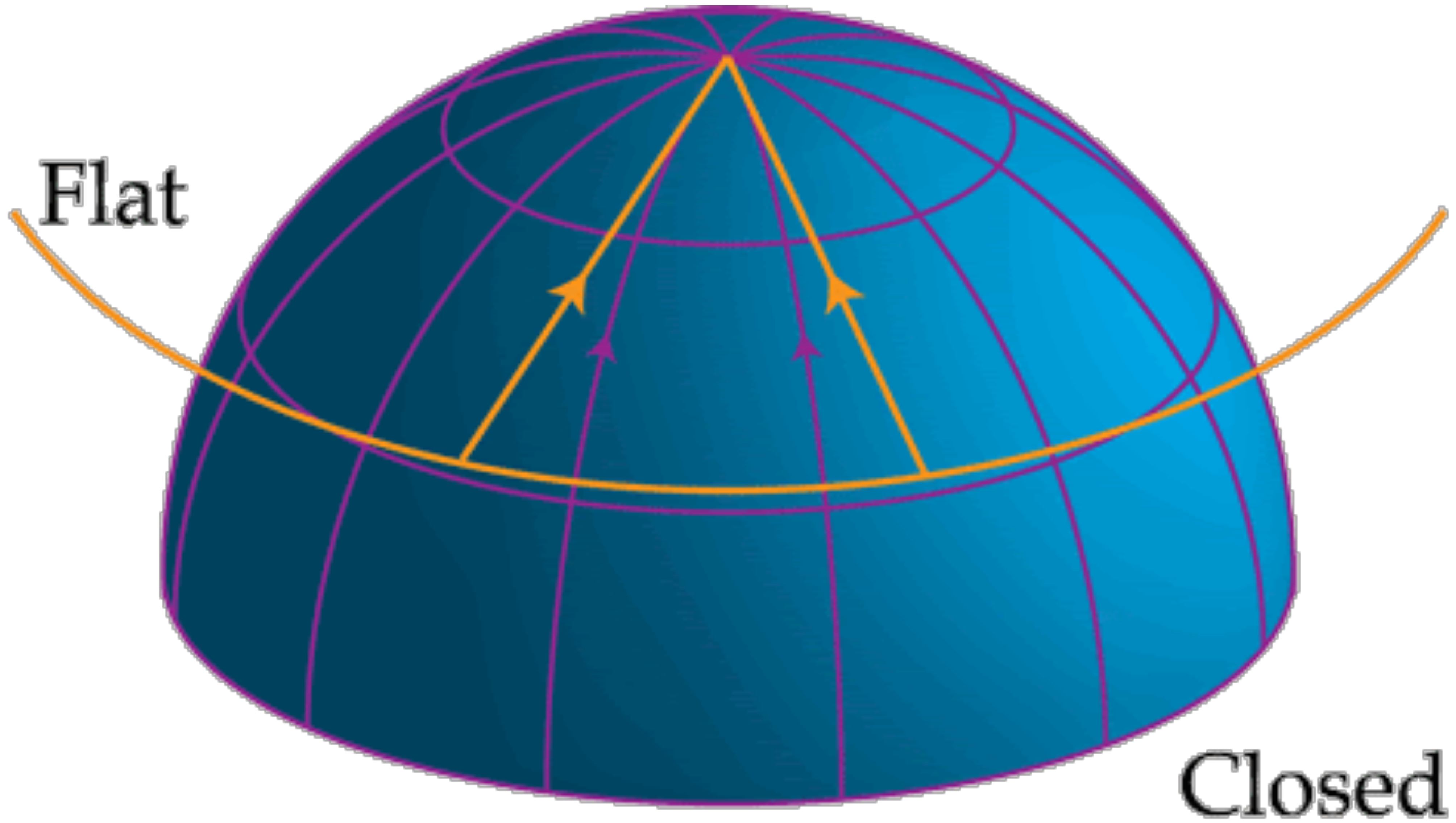
$$\alpha + \beta + \gamma = \pi - A/R^2$$

Robertson-Walker Metric

$$ds^2 = -c^2 dt^2 + a(t)[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

$$S_\kappa(r) = \begin{cases} R \sin \frac{r}{R} & (\kappa = +1) \\ r & (\kappa = 0) \\ R \sinh \frac{r}{R} & (\kappa = -1) \end{cases}$$

Curvature



Einstein's Field Equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

<u>Einstein Tensor</u>

4D curvature
of space-time

<u>Stress-Energy Tensor</u>

Distribution of energy
within space-time

"Mass-energy tells space-time how to curve, and space-time curvature tells mass-energy how to move" - John Wheeler

$(E = mc^2)$

$$ds^2 = \sum g_{\alpha\beta} dx_\alpha dx_\beta$$

metric tensor

Robertson-Walker metric
derived from metric tensor

$$ds^2 = -c^2 dt^2 + a(t)[dr^2 + S_\kappa(r)^2 d\Omega^2]$$

Friedmann Equation

Newtonian form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2}\frac{1}{a^2}$$

General relativistic form:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c^2}{R_0^2}\frac{1}{a^2}$$