Week 4

Read thru Chapter 5
Also read the Key Concepts for those chapters

Today: Eqn of States & Model Universes

HW 3 due Thursday
HW 4 available

Only available first 20min on Tuesday

<u>Office Hours:</u>
Tuesdays 2-3pm
Wednesdays 11am-12pm

Equations of the Universe

Friedmann Eqn: <u>gravity</u>

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3}$$

$$\varepsilon_\Lambda \equiv \frac{c^2}{8\pi G}\Lambda$$

Fluid Eqn: <u>conserve energy</u>

$$\dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

Eqn of State:
<u>relations b/t thermodynamic properties</u>

$$P = w\varepsilon$$

Ideal Gas Law

$$PV = NRT \qquad P = nkT \qquad P = \frac{\rho}{\mu}kT$$

$$\varepsilon = \rho c^2$$

$$P \approx \frac{kT}{\mu c^2}\varepsilon$$

Acceleration Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c^2}{R_0^2 a^2} + \frac{\Lambda}{3} \quad \xrightarrow{+} \quad = \quad \xleftarrow{+} \quad \dot{\varepsilon} + 3\frac{\dot{a}}{a}(\varepsilon + P) = 0$$

$$\downarrow$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$$

$$P < -\frac{1}{3}\varepsilon \qquad \Lambda = 0 \quad \longrightarrow \quad \ddot{a} > 0$$

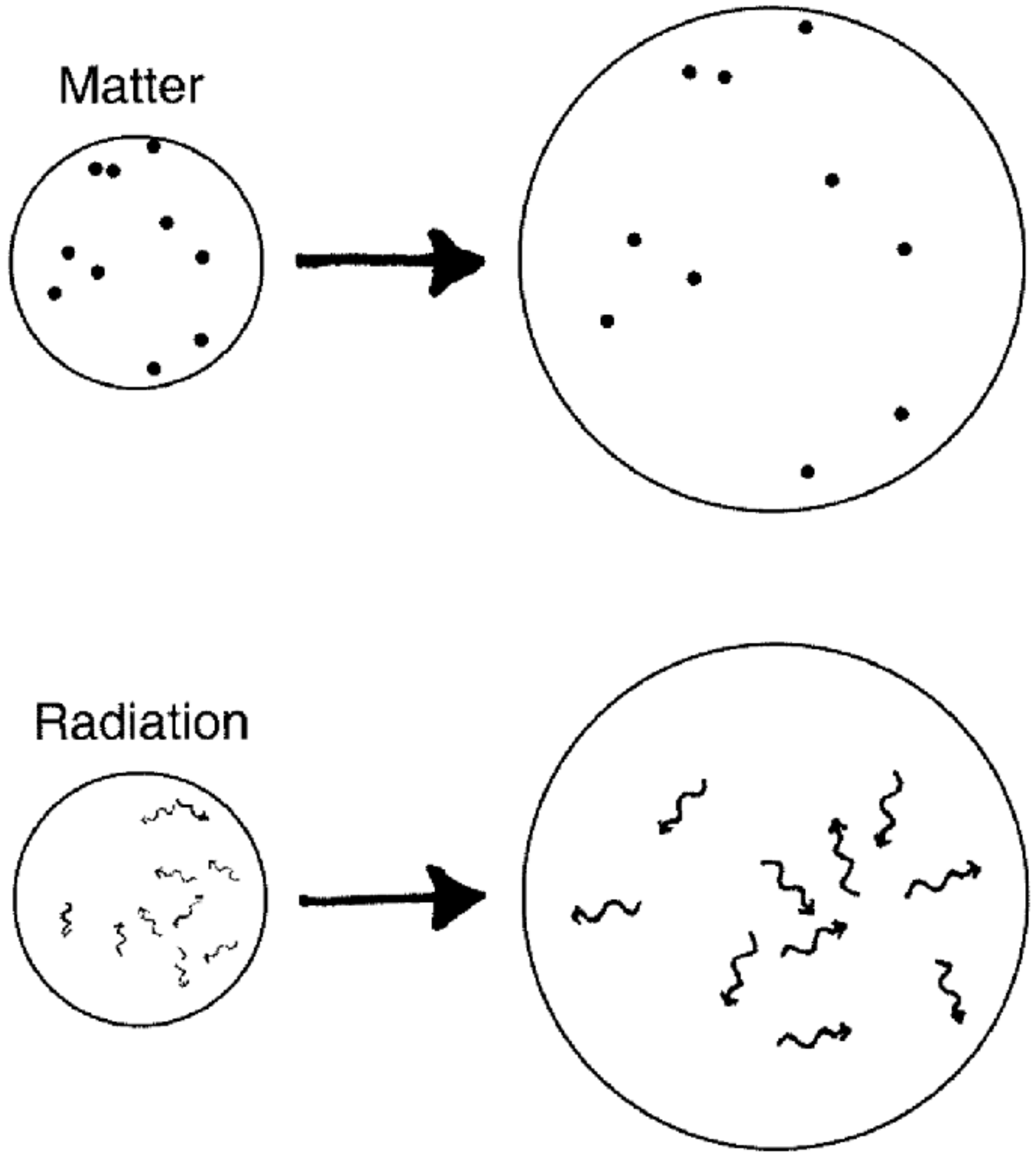
If Λ const, ε also const b/c $\varepsilon_\Lambda \equiv \frac{c^2}{8\pi G}\Lambda$

so $P = -\varepsilon_\Lambda = -\frac{c^2}{8\pi G}\Lambda$

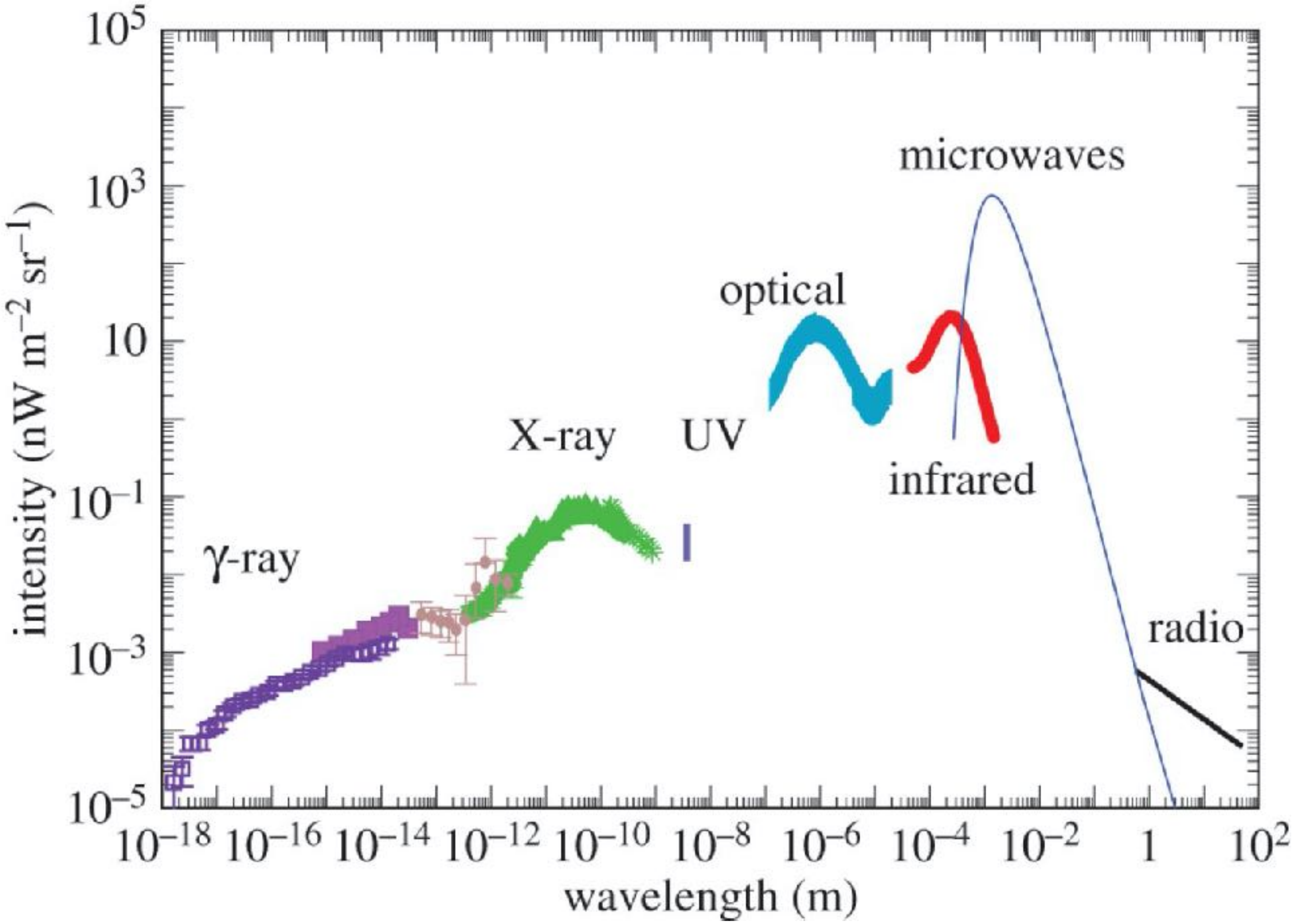
Evolution of Components

$$\varepsilon = \sum_i \varepsilon_i$$

$$P = \sum_i w_i \varepsilon_i$$



Relativistic Component: “Radiation” = Photons + Neutrinos



CMB dominates energy density of photons in universe today

Don’t know the mass of neutrinos, but energy density dominated by momentum and decoupling in early universe. Calculated to be:

$$\varepsilon_\nu = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\varepsilon_{\rm CMB} = 0.227\varepsilon_{\rm CMB}$$

True for each neutrino flavor, so total is 3x above.

$$\varepsilon_{\rm CMB,0} = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

Relativistic, Non-rel., & Λ Components

Radiation

$$\varepsilon_{\text{CMB},0} = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

$$\varepsilon_{\text{crit},0} = 4870 \text{ MeV m}^{-3}$$

$$\Omega_{r,0} = \frac{1.681\varepsilon_{\text{CMB},0}}{\varepsilon_{\text{crit},0}} = 9 \times 10^{-5}$$

(neutrinos have ~68% of the energy density of CMB photons)

Matter: $\Omega_{m,0} \approx 0.31$

Dark Energy: $\Omega_{\Lambda,0} \approx 0.69$

Using evolution of each component with a, can compute when matter-dark energy and matter-radiation components were comparable

Model Universes

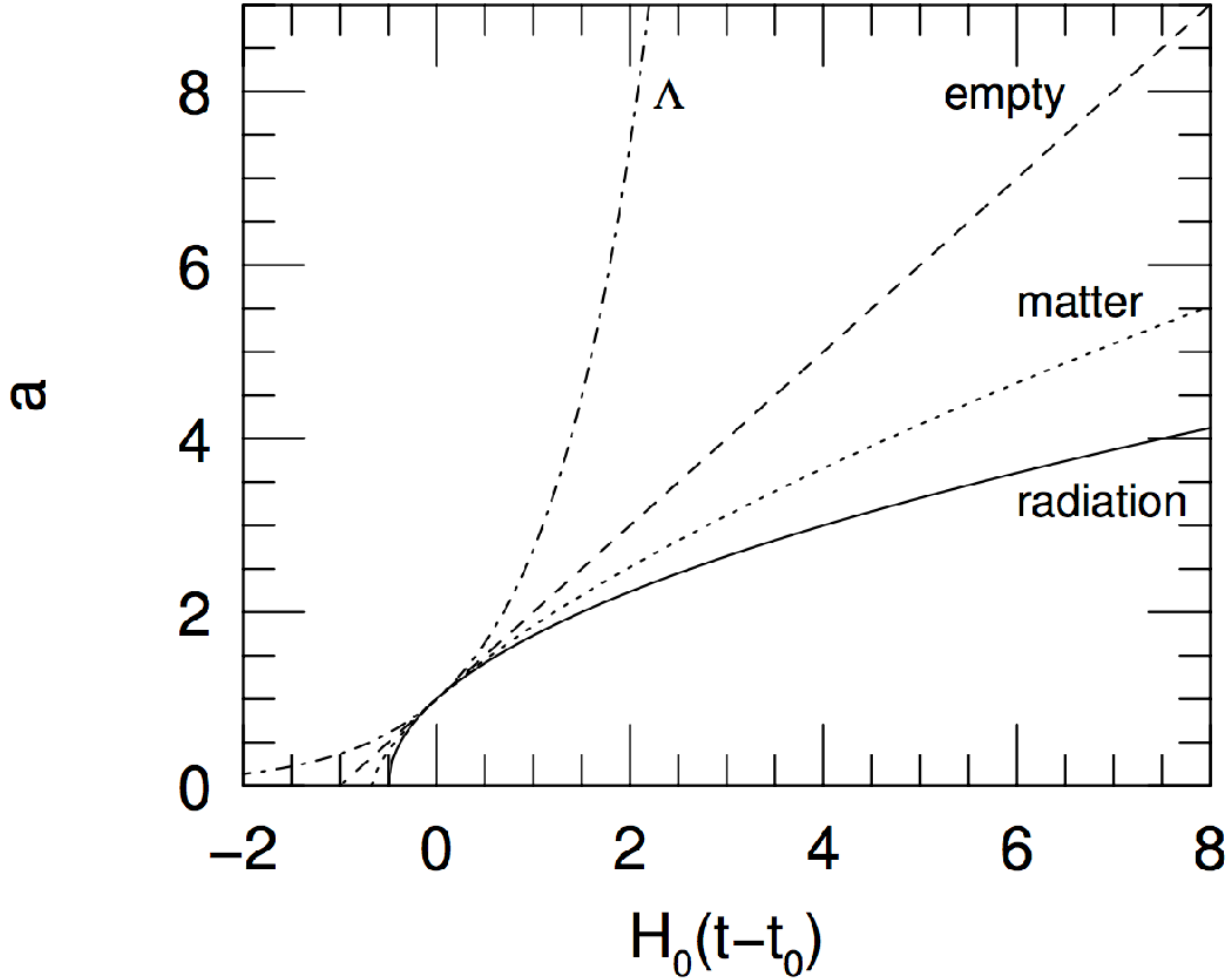
Can now solve for a(t) generically, if not necessarily analytically:

$$\dot{a}^2 = \frac{8\pi G}{3c^2} \sum_i \varepsilon_{i,0} a^{-1-3w_i} - \frac{\kappa c^2}{R_0^2}$$

- Empty
- Matter only
  - classic case: open, closed, flat
- Radiation only (+curvature)
- Lambda only (+curvature)
- Various Combinations!

Empty

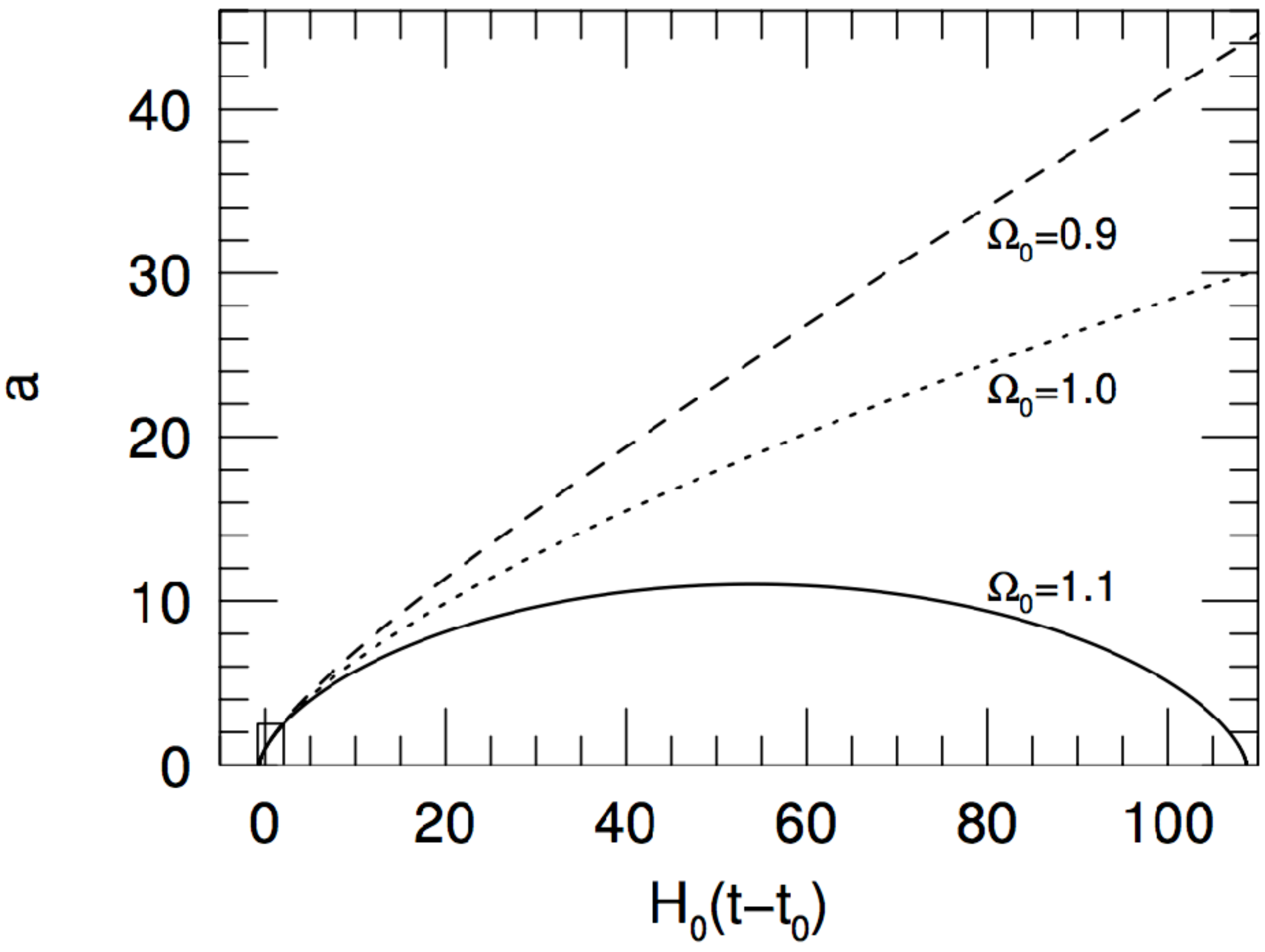
Only 1 Constituent in a Flat Spacetime



Only 1 Constituent in a Flat Spacetime

Classical Cosmology: Matter + Curvature

Classical Cosmology: Matter + Curvature



$$\kappa = 0,\ \Omega_0 = 1;\ a(t) = (t/t_0)^{2/3}$$

$$\kappa = +1,\ \Omega_0 > 1;\ a(\theta) = \frac{1}{2}\frac{\Omega_0}{\Omega_0 - 1}(1 - \cos\theta)$$

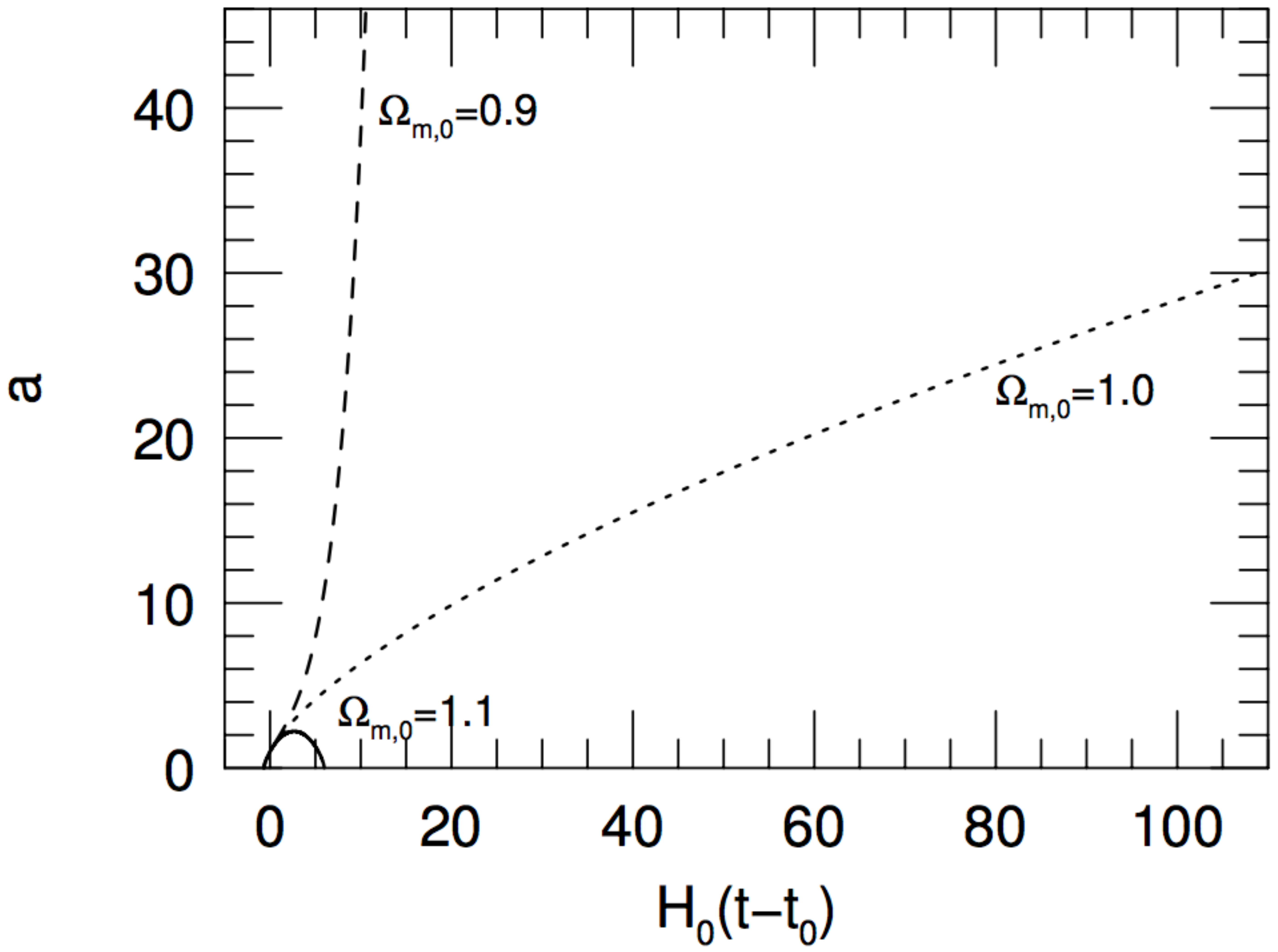
$$t(\theta) = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(\theta - \sin\theta)$$

$$\kappa = -1,\ \Omega_0 < 1;\ a(\eta) = \frac{1}{2}\frac{\Omega_0}{1 - \Omega_0}(\cosh\eta - 1)$$

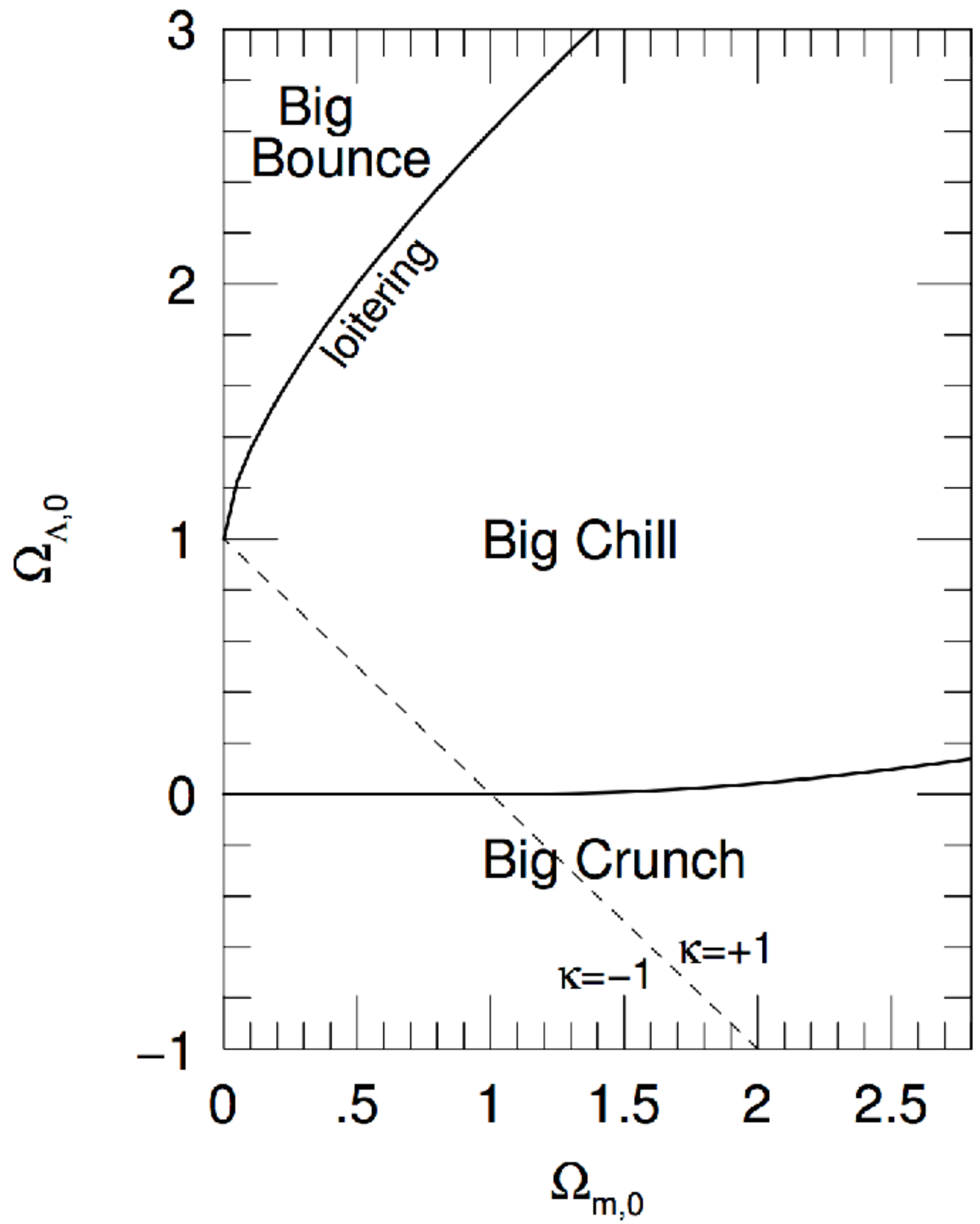
$$t(\eta) = \frac{1}{2H_0}\frac{\Omega_0}{(1 - \Omega_0)^{3/2}}(\sinh\eta - \eta)$$

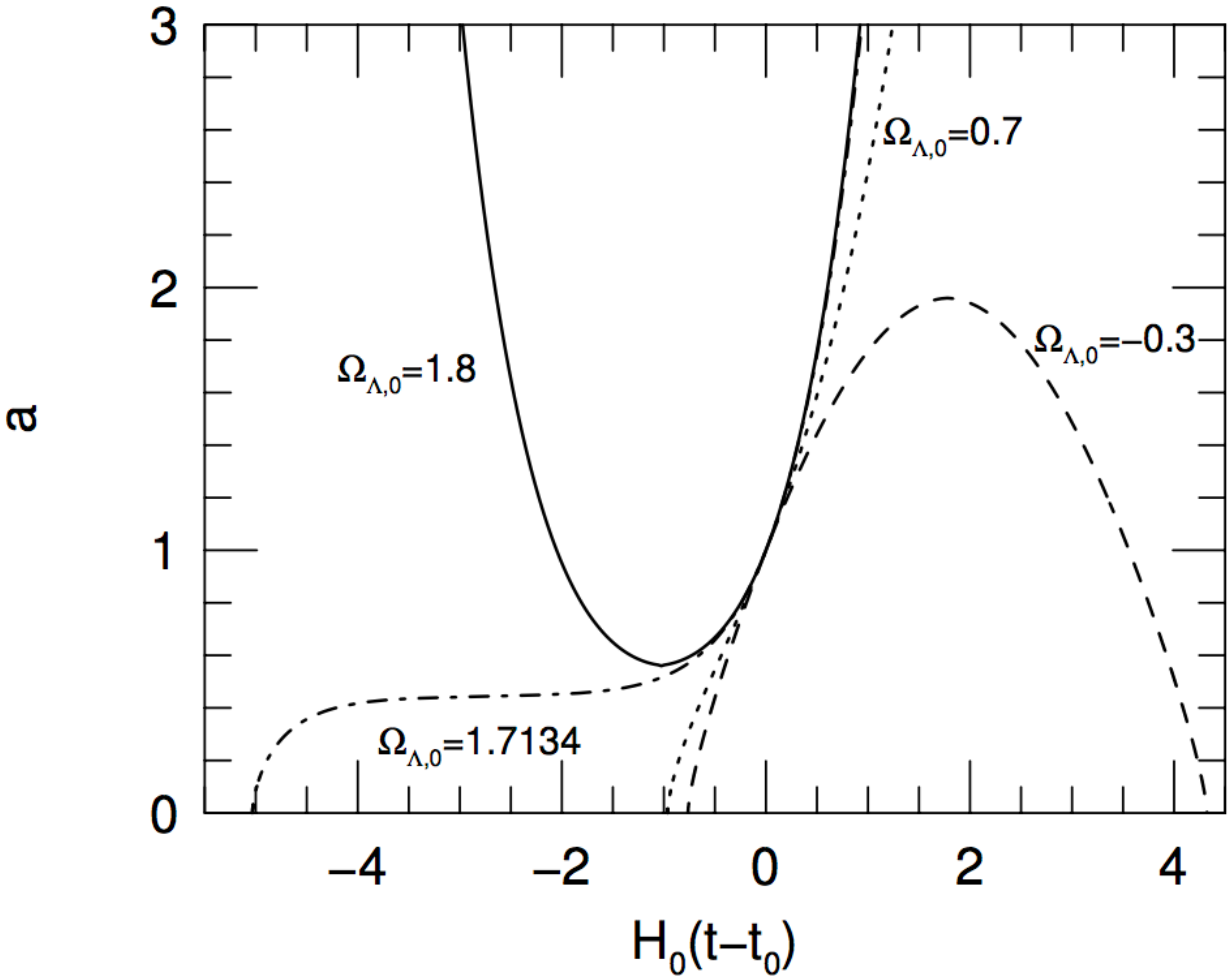
Density —> Destiny

Flat: Matter + Cosmological Constant

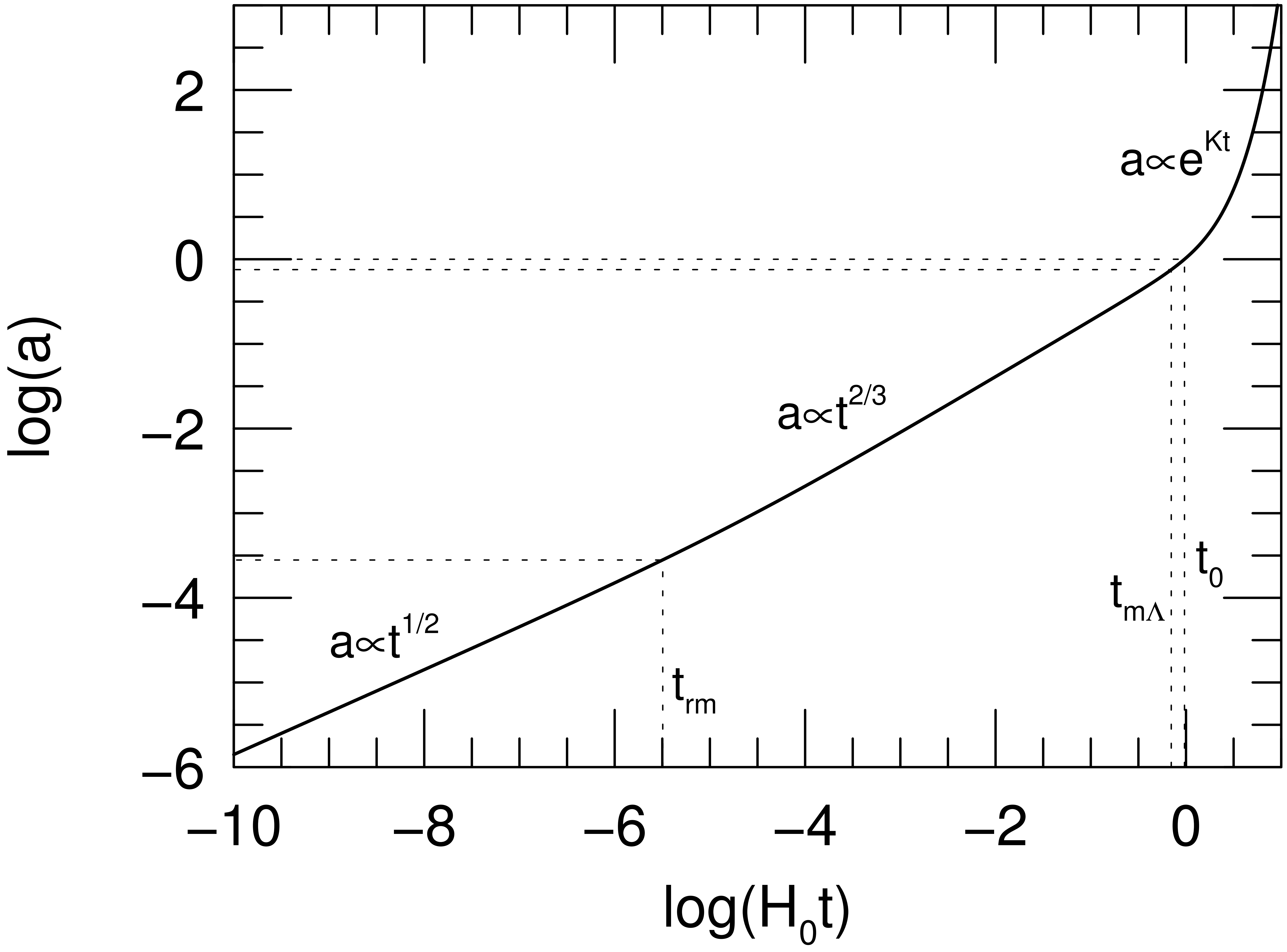


Matter + Lambda + Curvature





Benchmark Model



$$\kappa = 0$$

$$\Omega_r + \Omega_m + \Omega_\Lambda = 1$$

$$\Omega_r = \Omega_\gamma + \Omega_\nu$$

$$\Omega_m = \Omega_{bary} + \Omega_{dm}$$

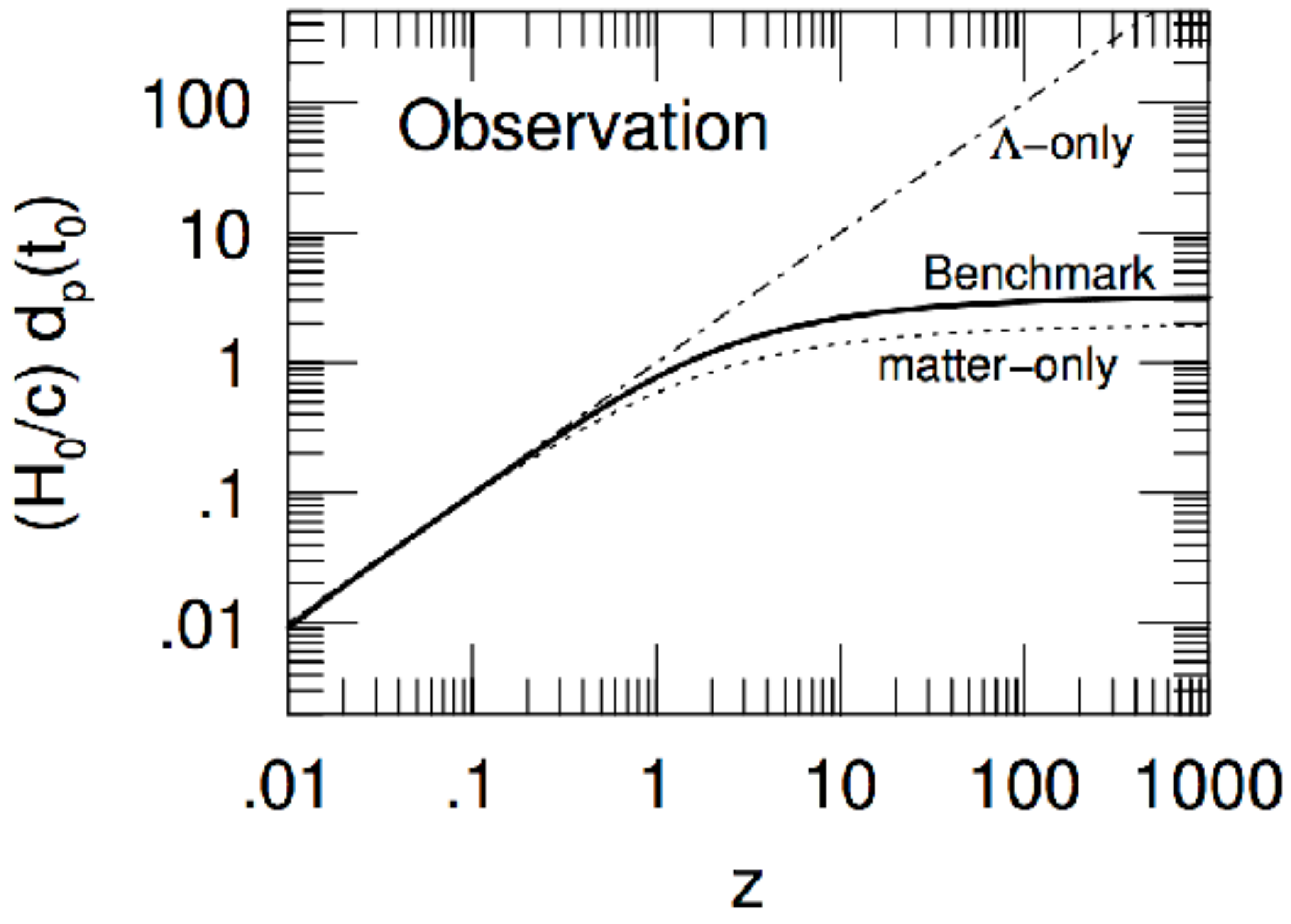
$\Omega_{r,0} = 0.31$
$\Omega_{bary,0} = 0.048$
$\Omega_{dm,0} = 0.262$

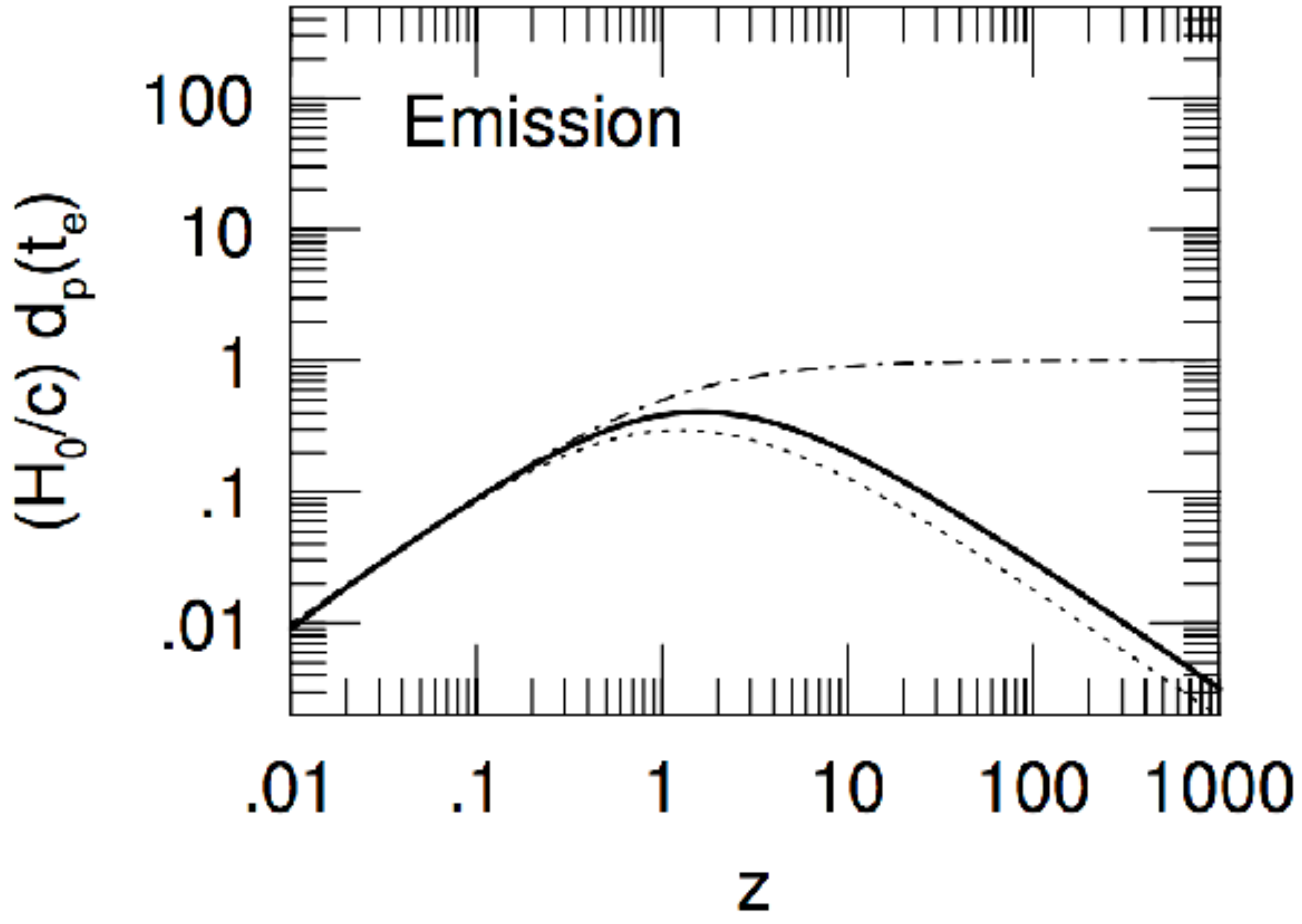
$\Omega_{r,0} = 9.0 \times 10^{-5}$
$\Omega_{\gamma,0} = 5.35 \times 10^{-5}$
$\Omega_{\nu,0} = 3.65 \times 10^{-5}$

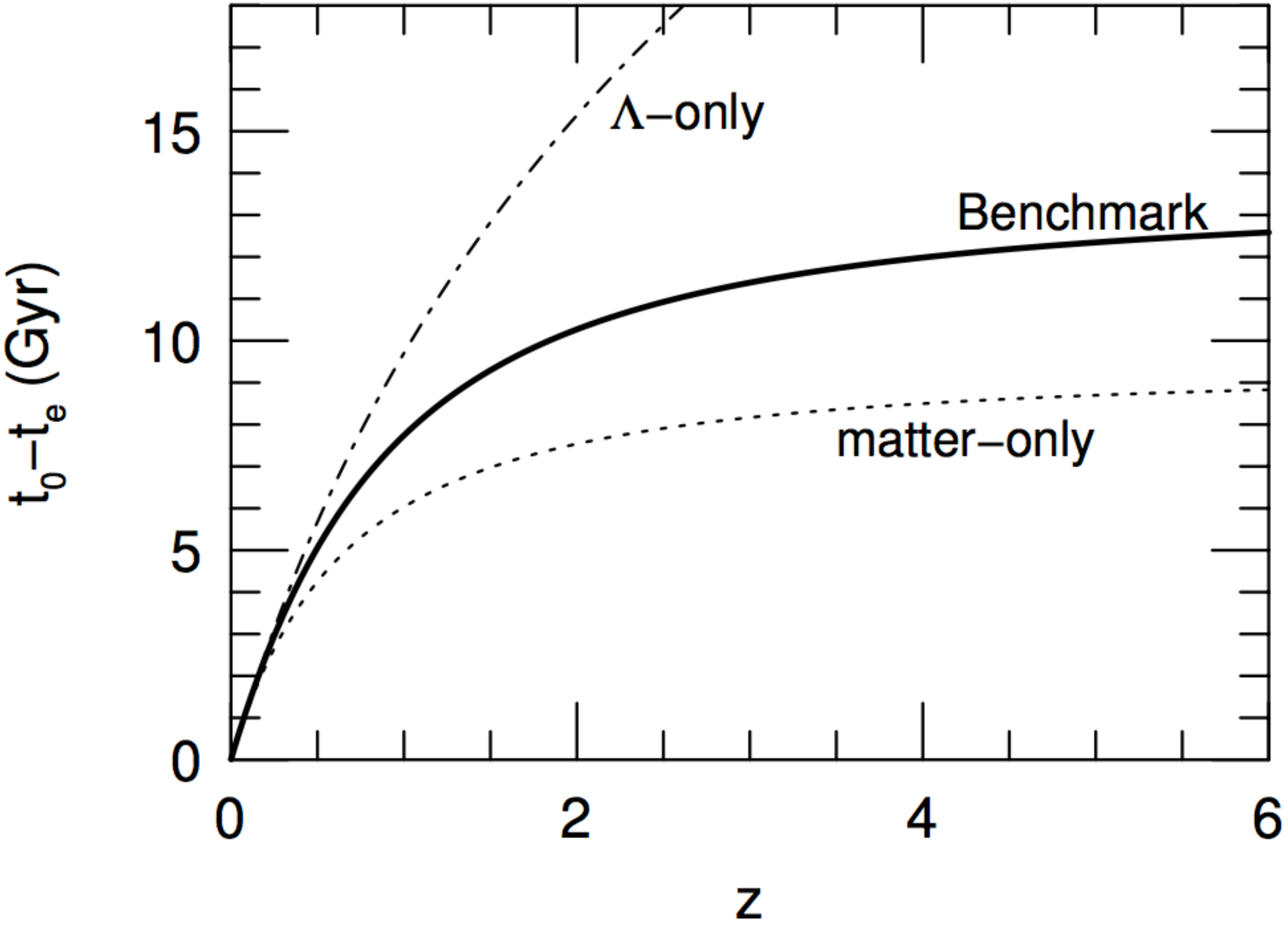
$$\Omega_{\Lambda,0} \approx 0.69$$

Rad. − Matter :	$a_{rm} = 2.9 \times 10^{-4}$	$t_{rm} = 50$ kyr
Matter − Λ :	$a_{m\Lambda} = 0.77$	$t_{m\Lambda} = 10.2$ Gyr
Now :	$a_0 = 1$	$t_0 = 13.7$ Gyr

Benchmark Model







https://tritonstation.wordpress.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/