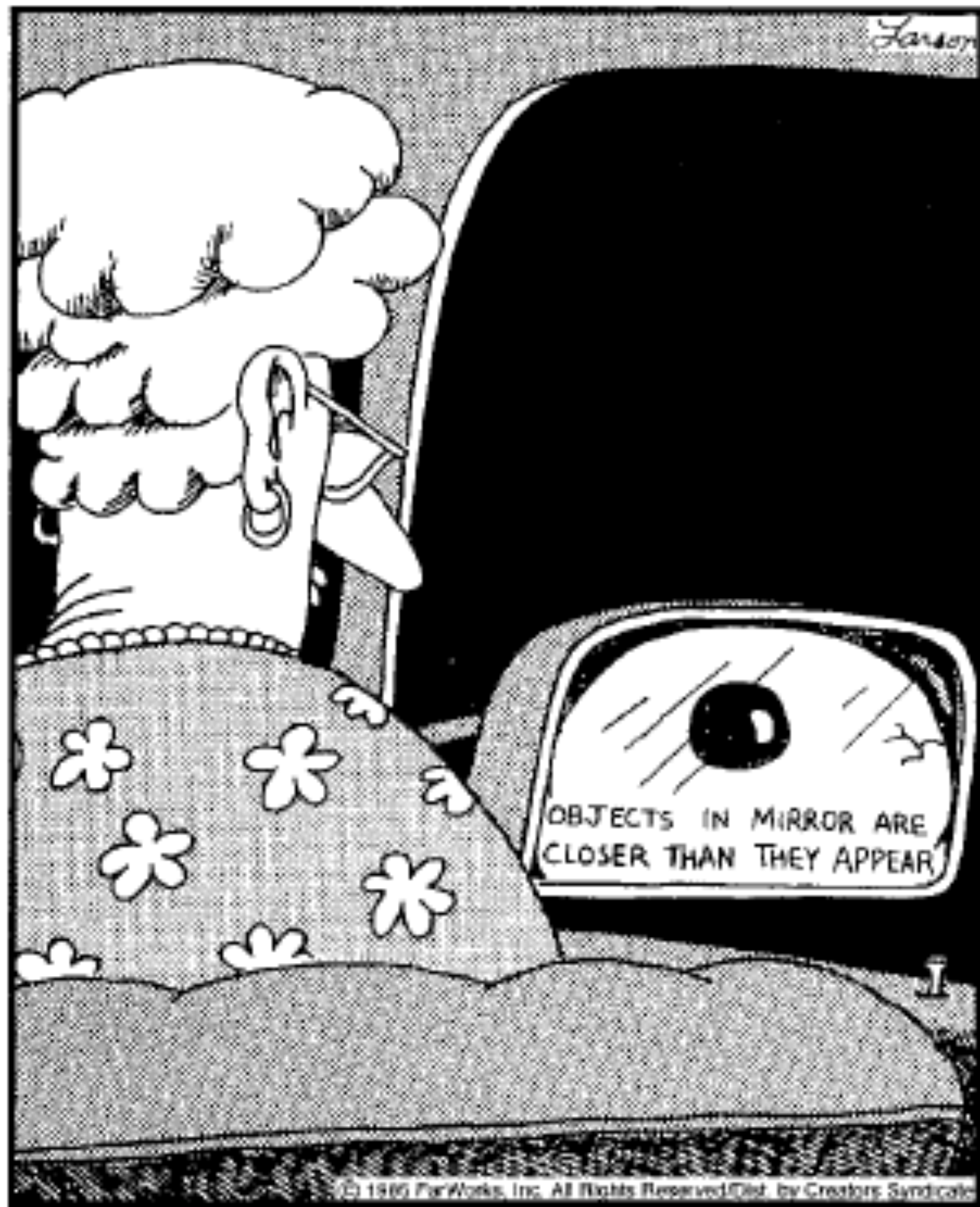


# Measuring Distances

ASTR/PHYS 4080: Intro to Cosmology  
Week 6



The Far Side® by Gary Larson © 1985 FarWorks, Inc. All Rights Reserved. Used with permission.

HWs 1-3 graded and returned  
Midterm 1 grading in progress

HW 4 due on Thursday

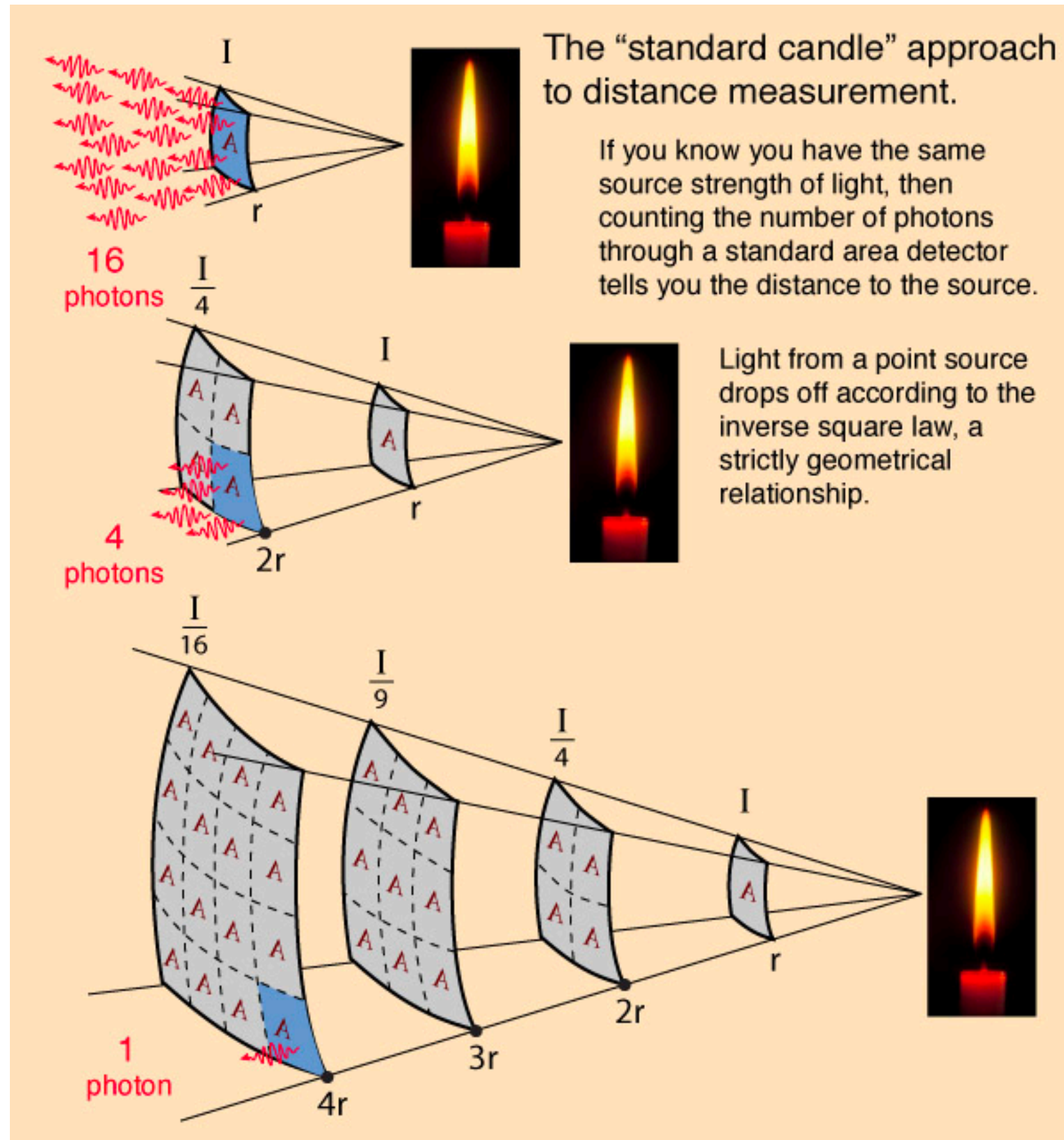
Today:

Different Model Universes (finish Ch. 5)  
Measuring Cosmological Parameters (start Ch. 6)

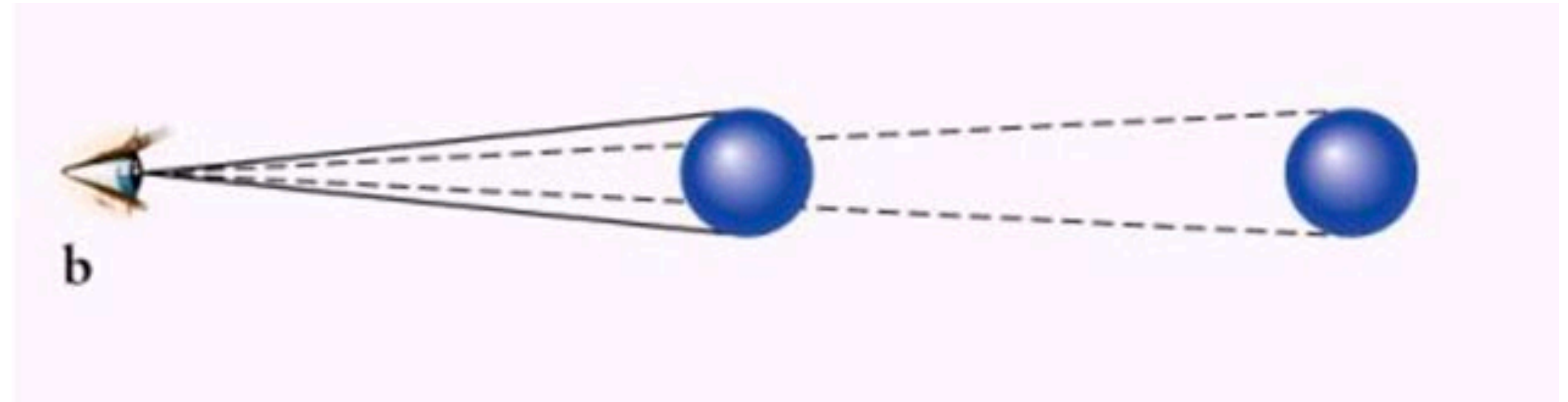
# How do we measure the distances to astronomical objects?

# Practical Distance Measures

## Luminosity Distance



## Angular Diameter Distance



# $d_L$ and $d_A$ in a universe with curvature

## Robertson-Walker Metric

$$ds^2 = -c^2 dt^2 + a(t) [dr^2 + S_\kappa(r)^2 d\Omega^2]$$

$$S_\kappa(r) = \begin{cases} R \sin \frac{r}{R} & (\kappa = +1) \\ r & (\kappa = 0) \\ R \sinh \frac{r}{R} & (\kappa = -1) \end{cases}$$

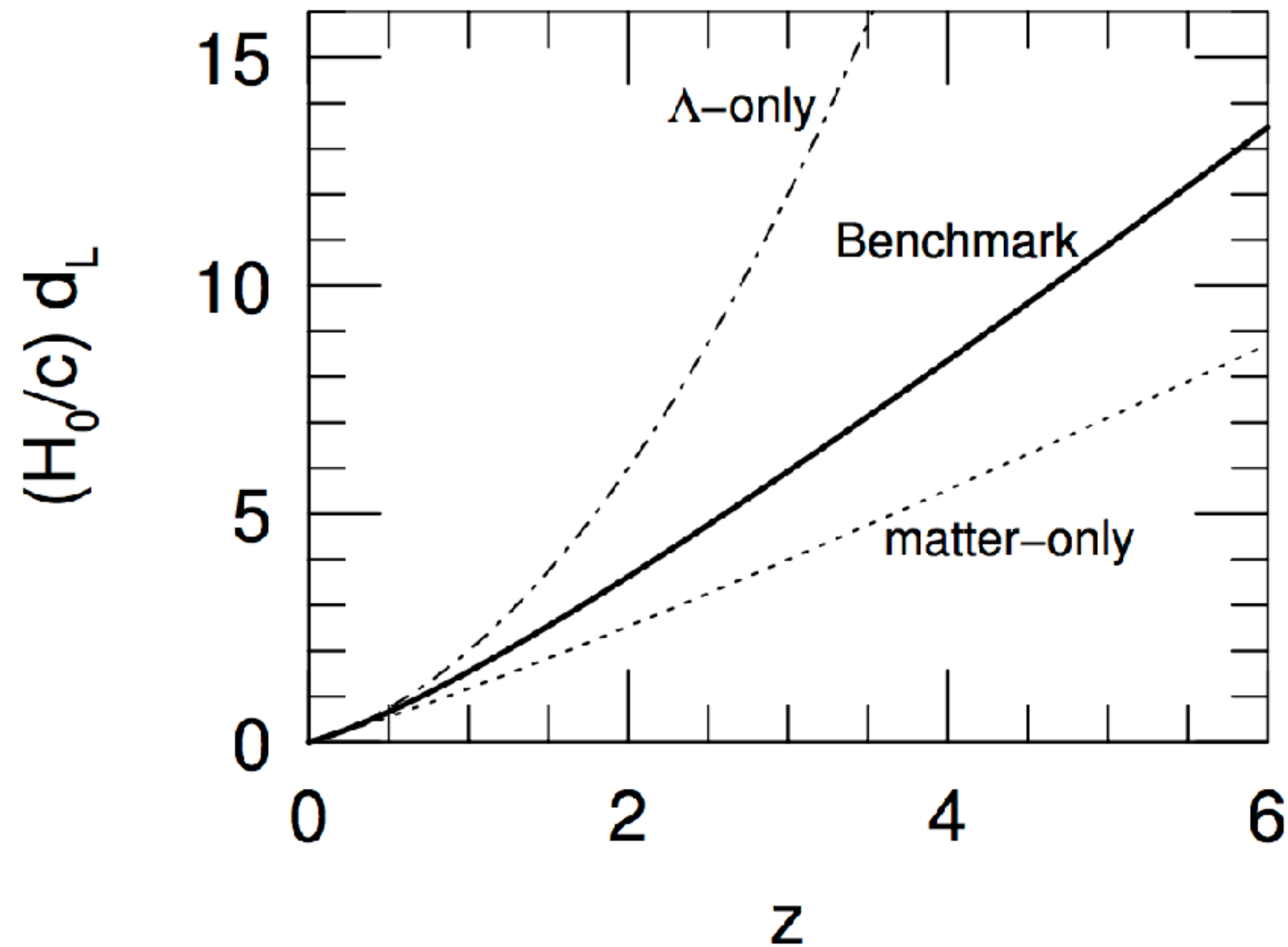
flux affected by area of  
expanding shell of light

angular extent affected by  
curvature of geodesics

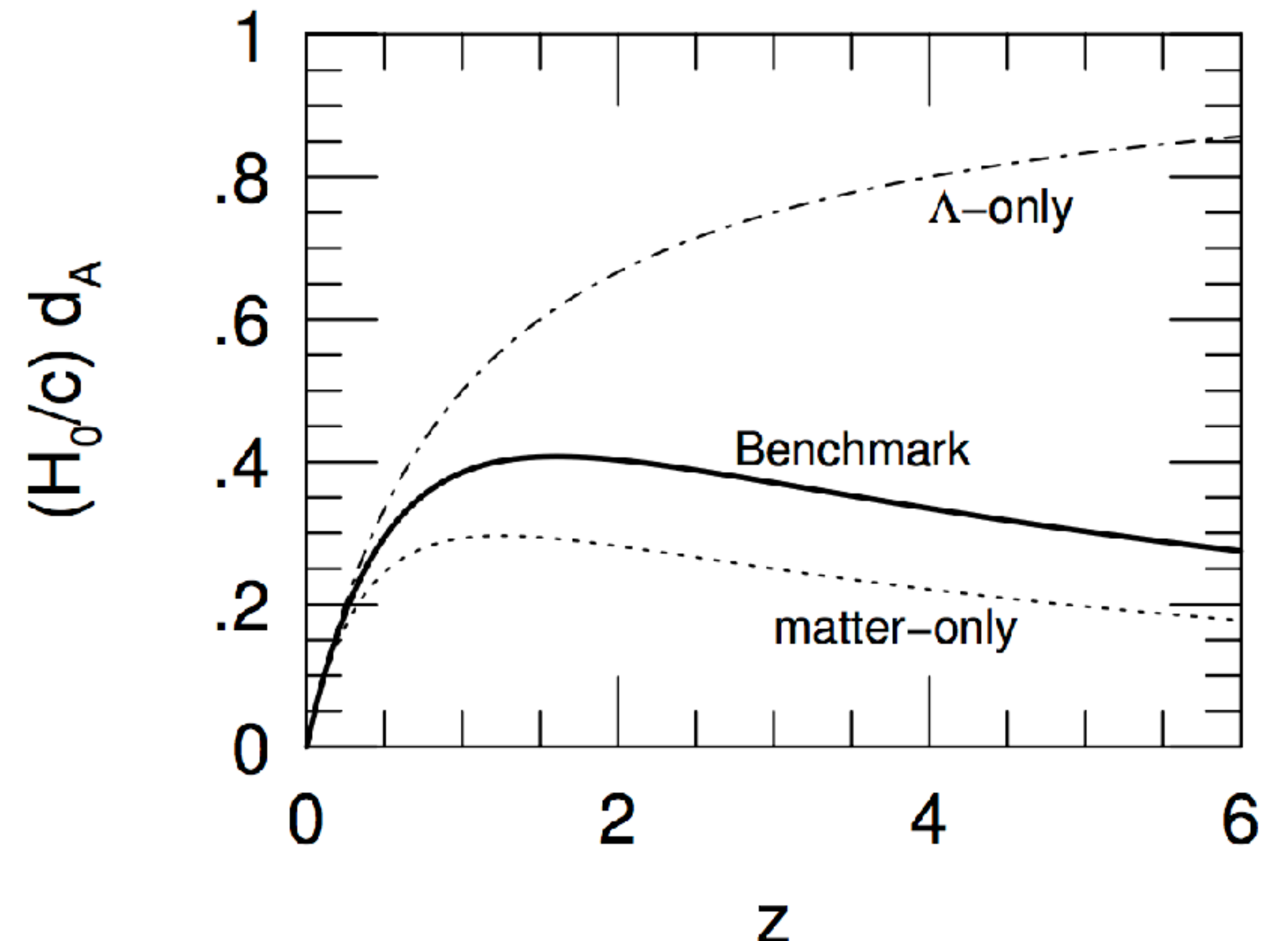


# How distances are affected by underlying cosmology

## Luminosity Distance



## Angular Diameter Distance

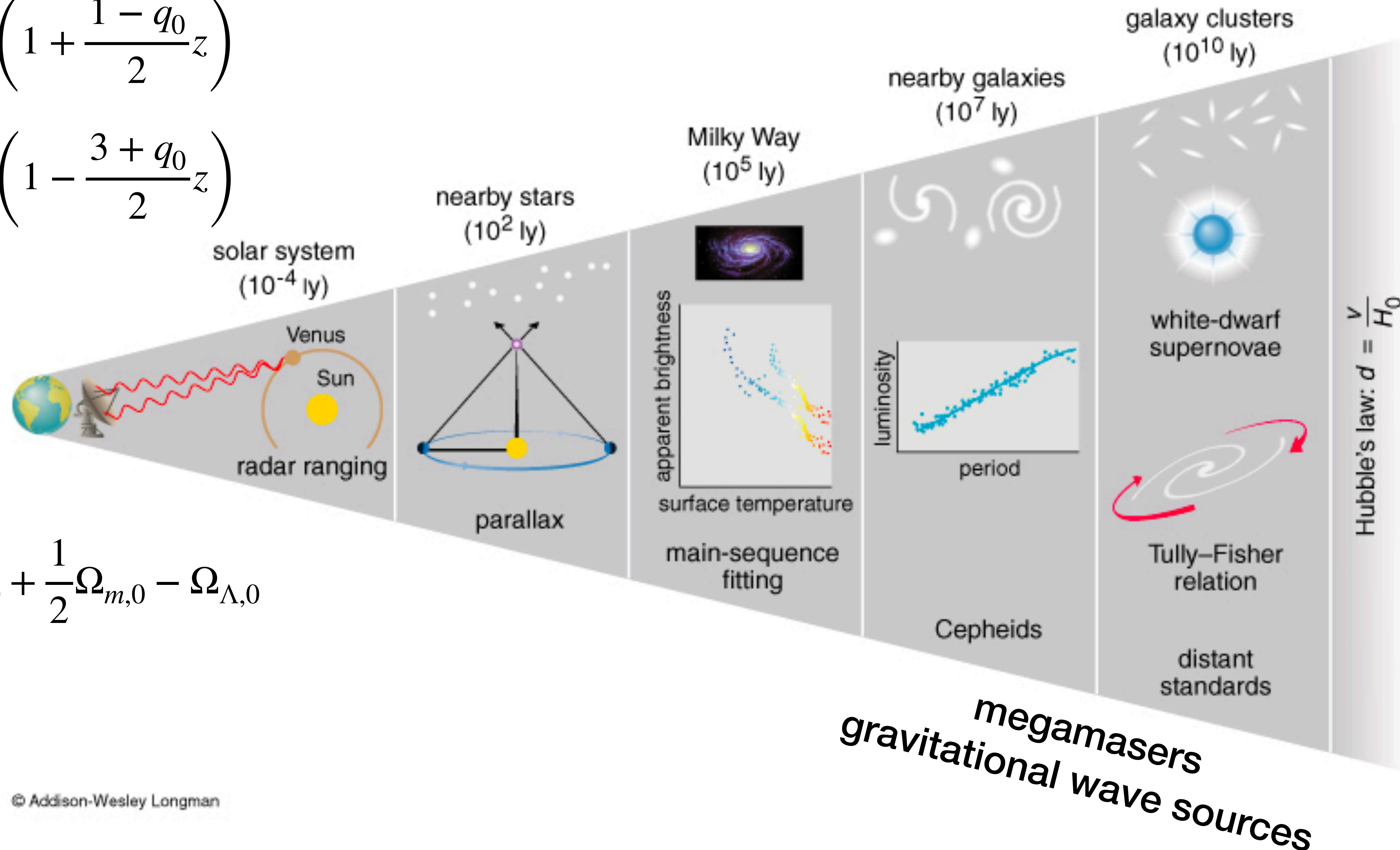


# Distance Ladder

$$d_L \approx \frac{cz}{H_0} \left( 1 + \frac{1 - q_0}{2} z \right)$$

$$d_A \approx \frac{cz}{H_0} \left( 1 - \frac{3 + q_0}{2} z \right)$$

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}$$



© Addison-Wesley Longman

# Magnitudes (alternative units of flux)

$$m \equiv -2.5 \log_{10}(f/f_{\text{ref}}) \quad f_{\text{ref}} = 2.53 \times 10^{-8} \text{ W m}^{-2}$$

$$M \equiv -2.5 \log_{10}(L/L_{\text{ref}}) \quad L_{\text{ref}} = 78.7 L_{\odot} \quad m = M \text{ at } 10 \text{ pc}$$

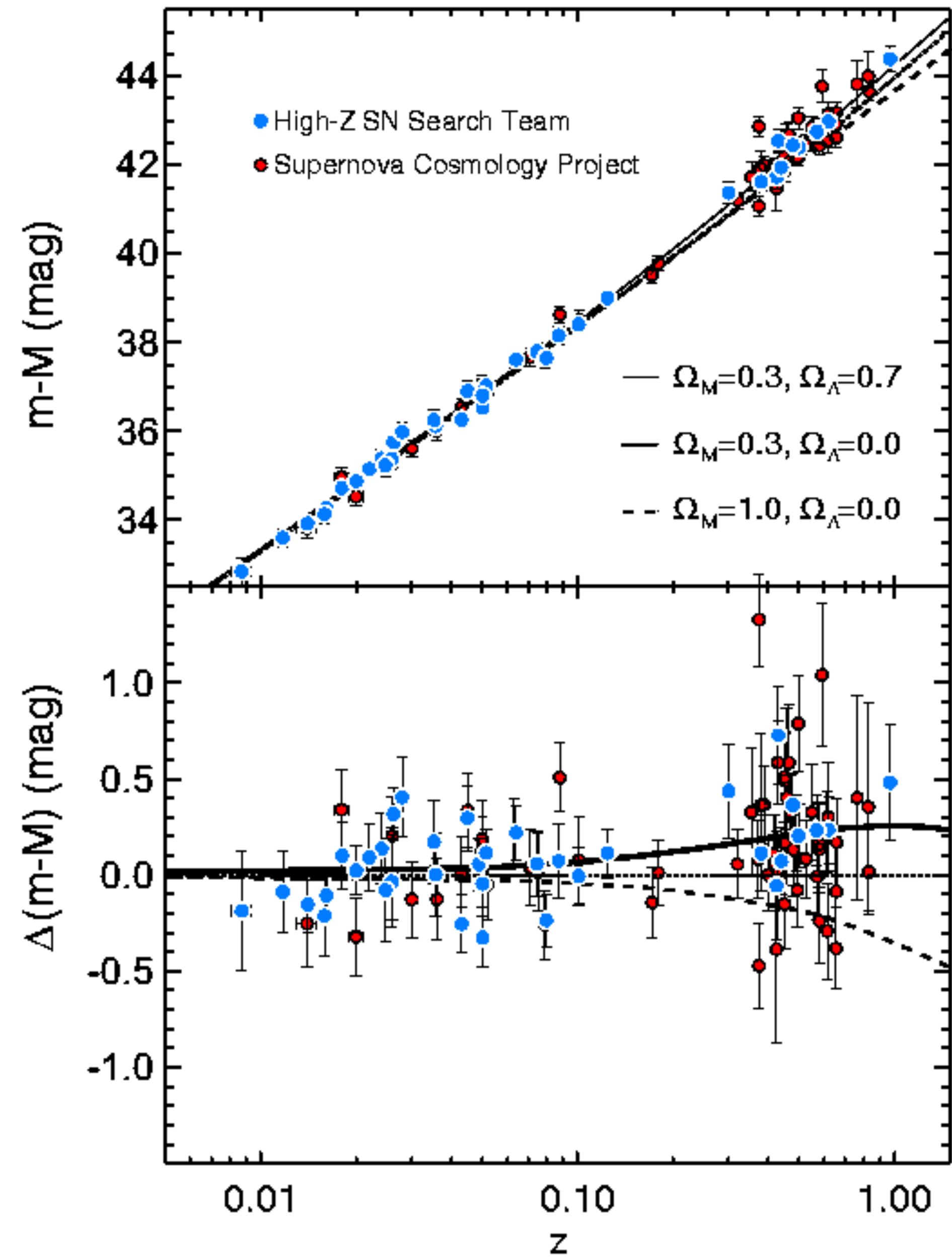
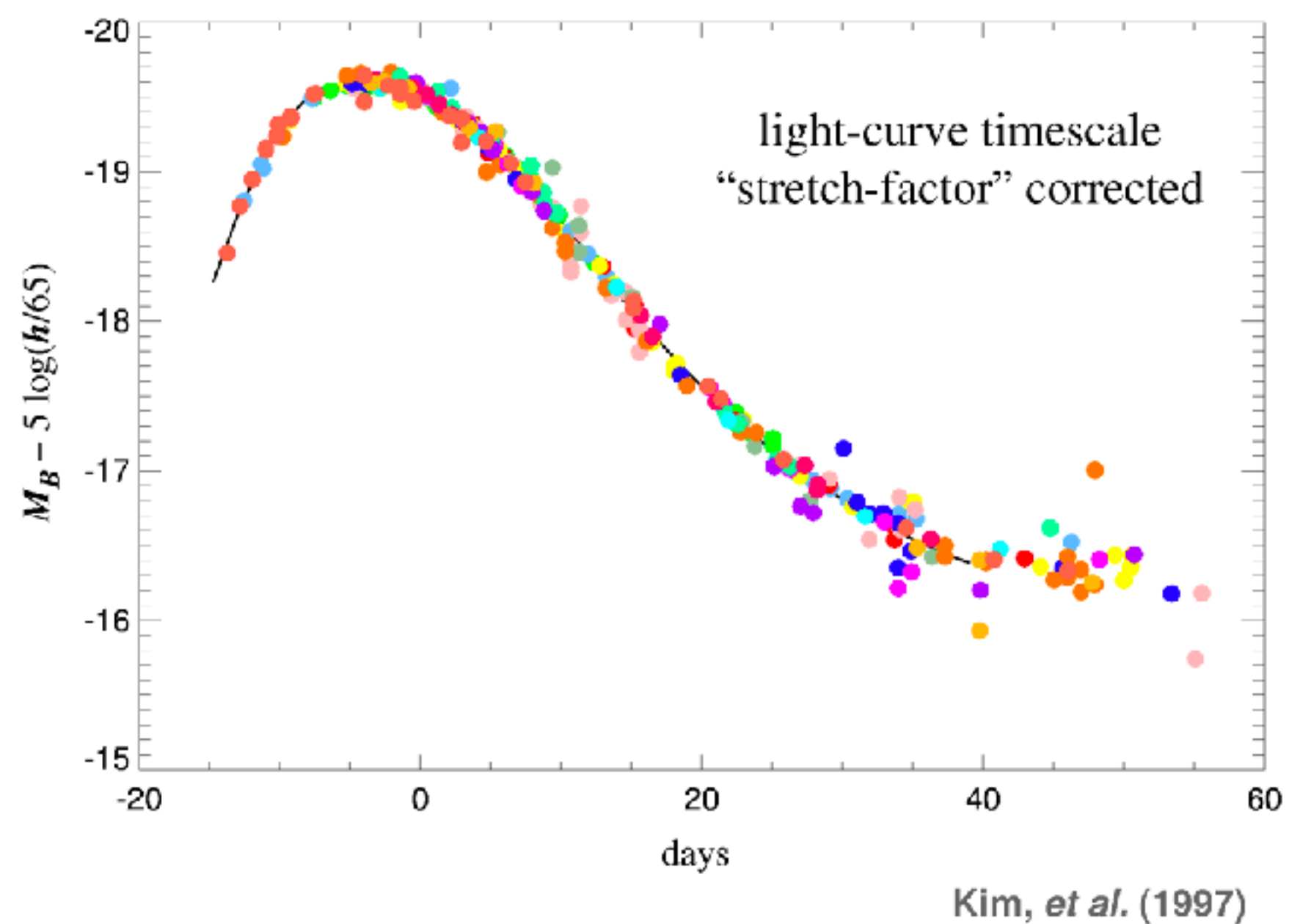
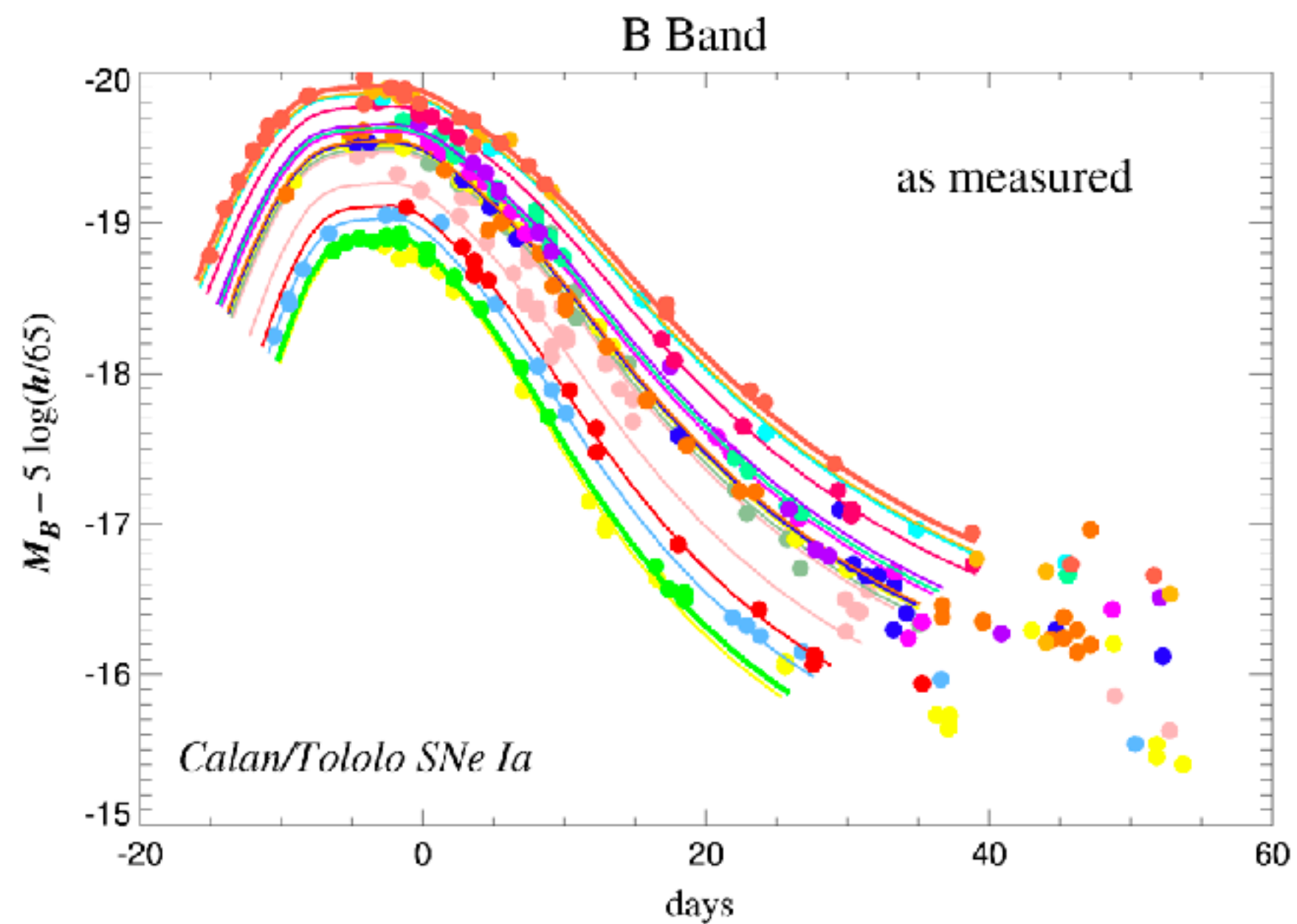
Combine:

$$M = m - 5 \log_{10} \left( \frac{d_L}{10 \text{ pc}} \right) = m - 5 \log_{10} \left( \frac{d_L}{1 \text{ Mpc}} \right) - 25$$

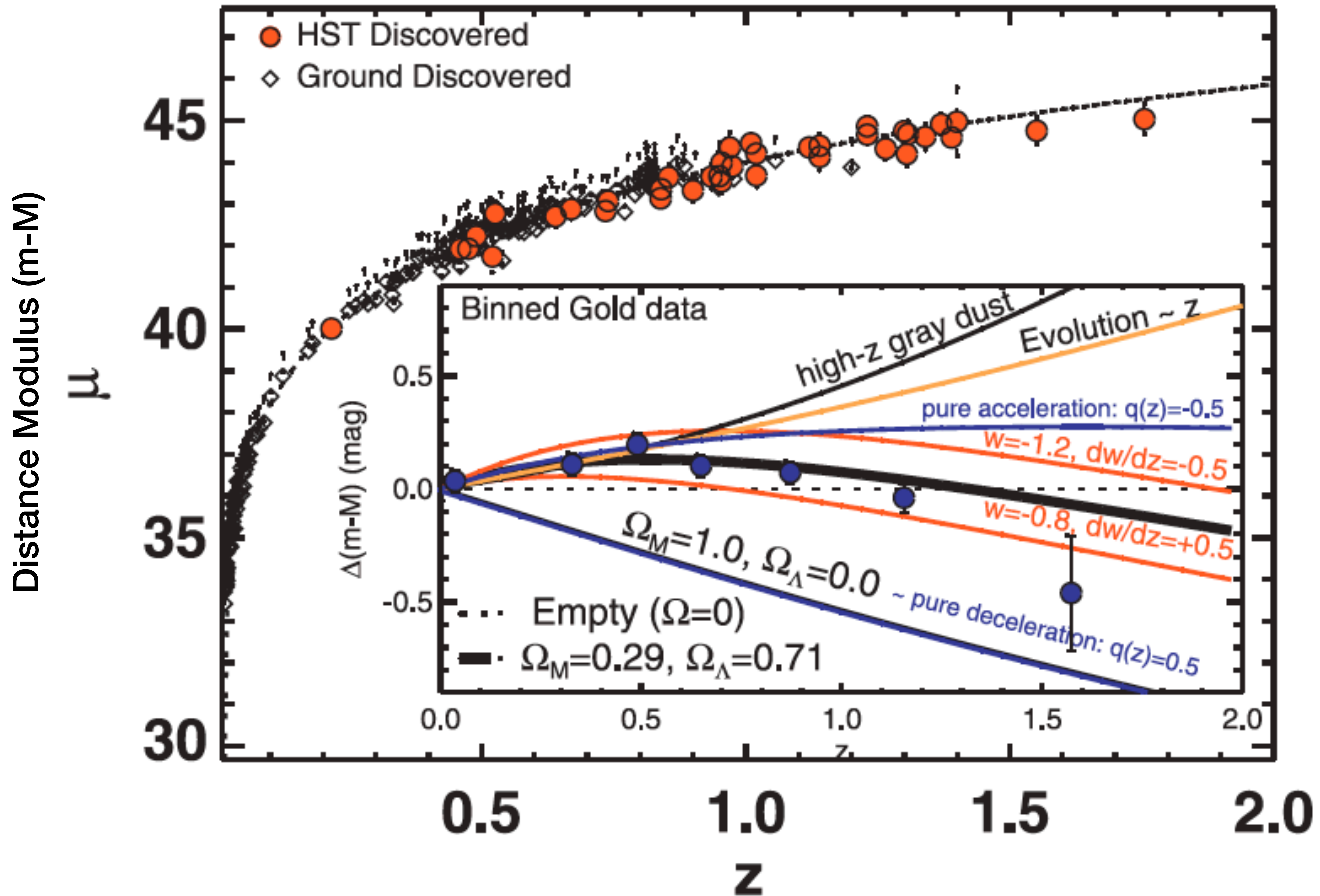
Substitute expression for  $d_L$ :

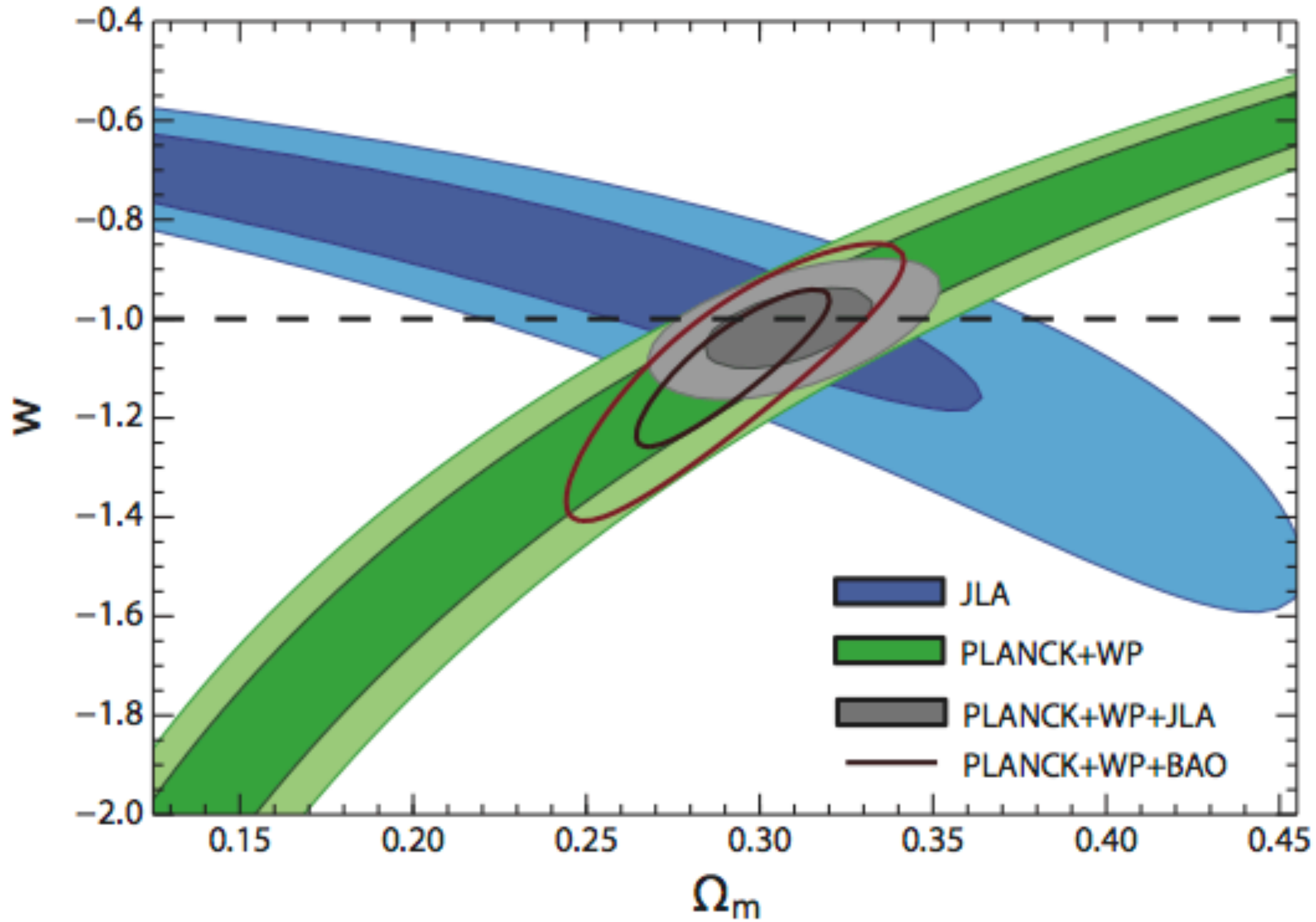
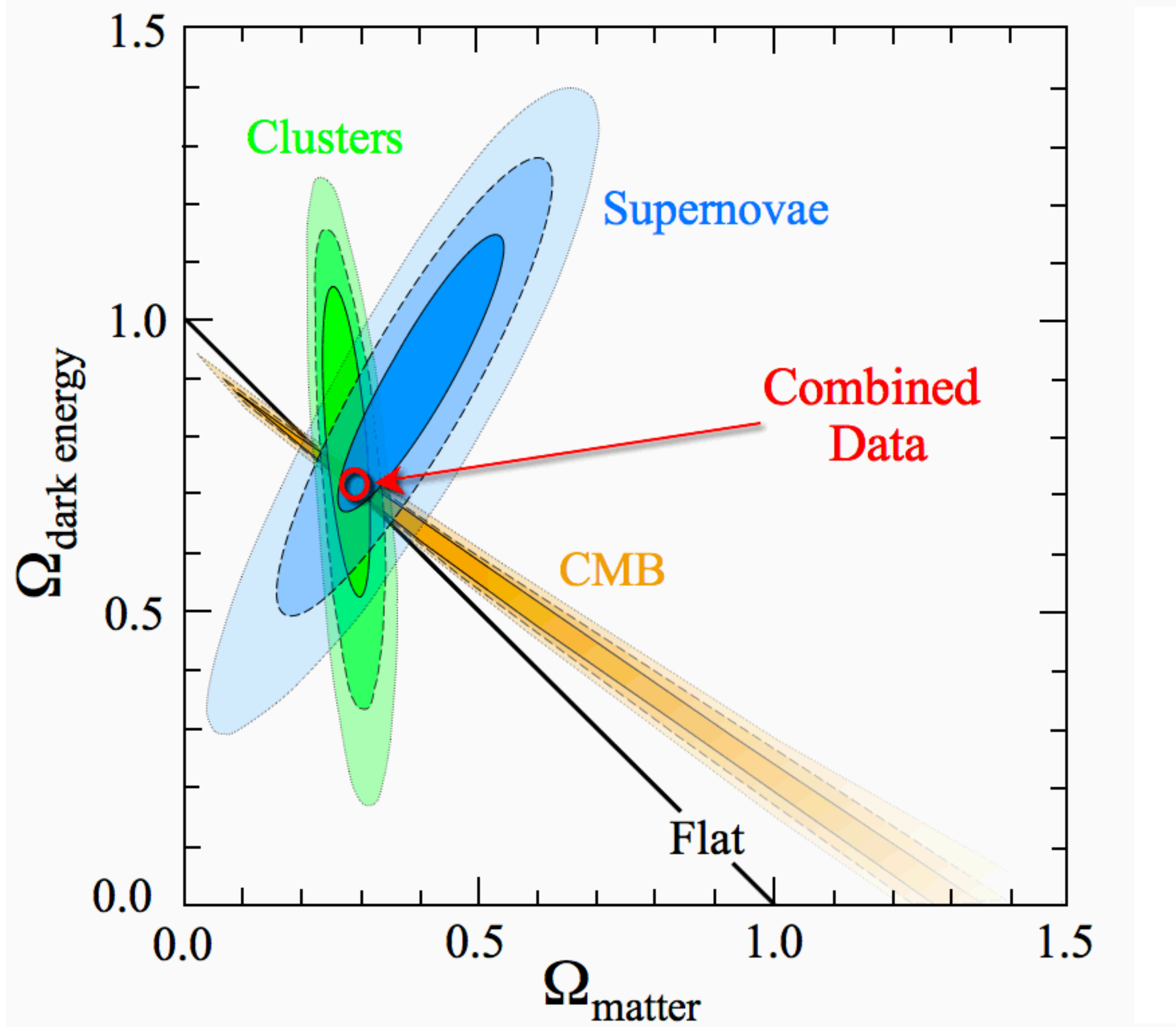
$$h = \frac{H_0}{68 \text{ km s}^{-1} \text{ Mpc}^{-1}}$$

$$m - M \approx 43.23 - 5 \log_{10} h + 5 \log_{10} z + 1.086(1 - q_0)z$$









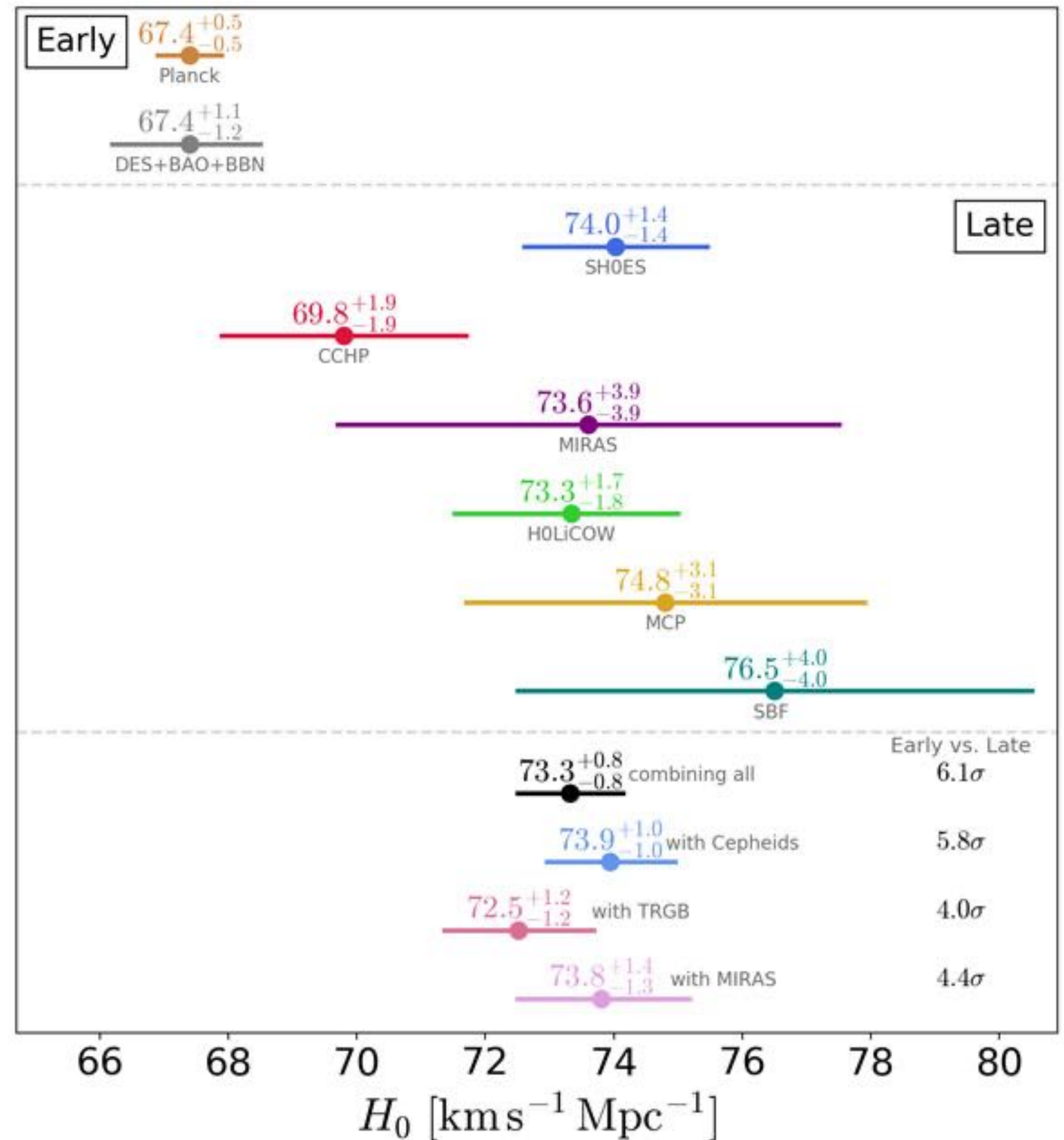


# Measurements of $H_0$

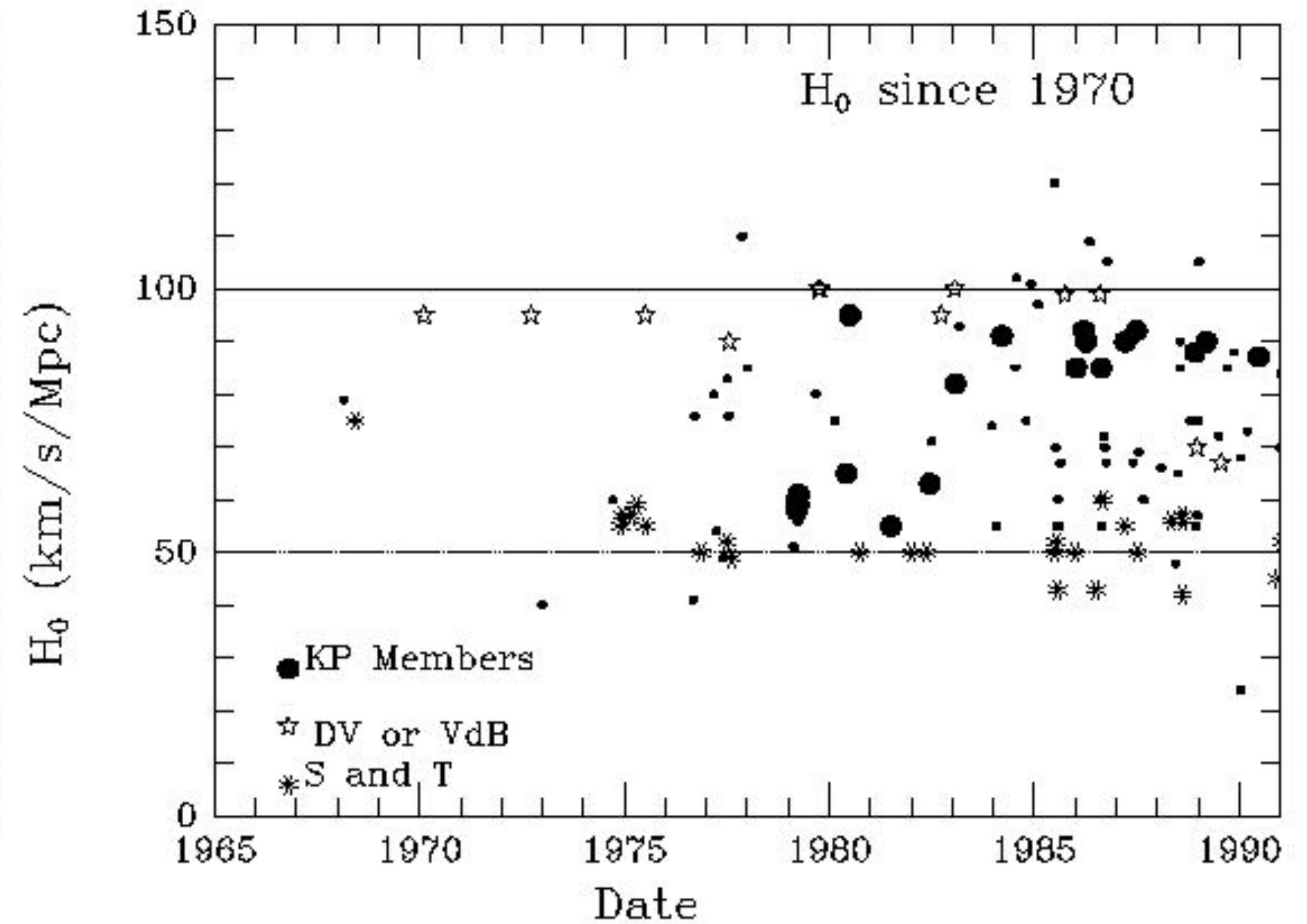
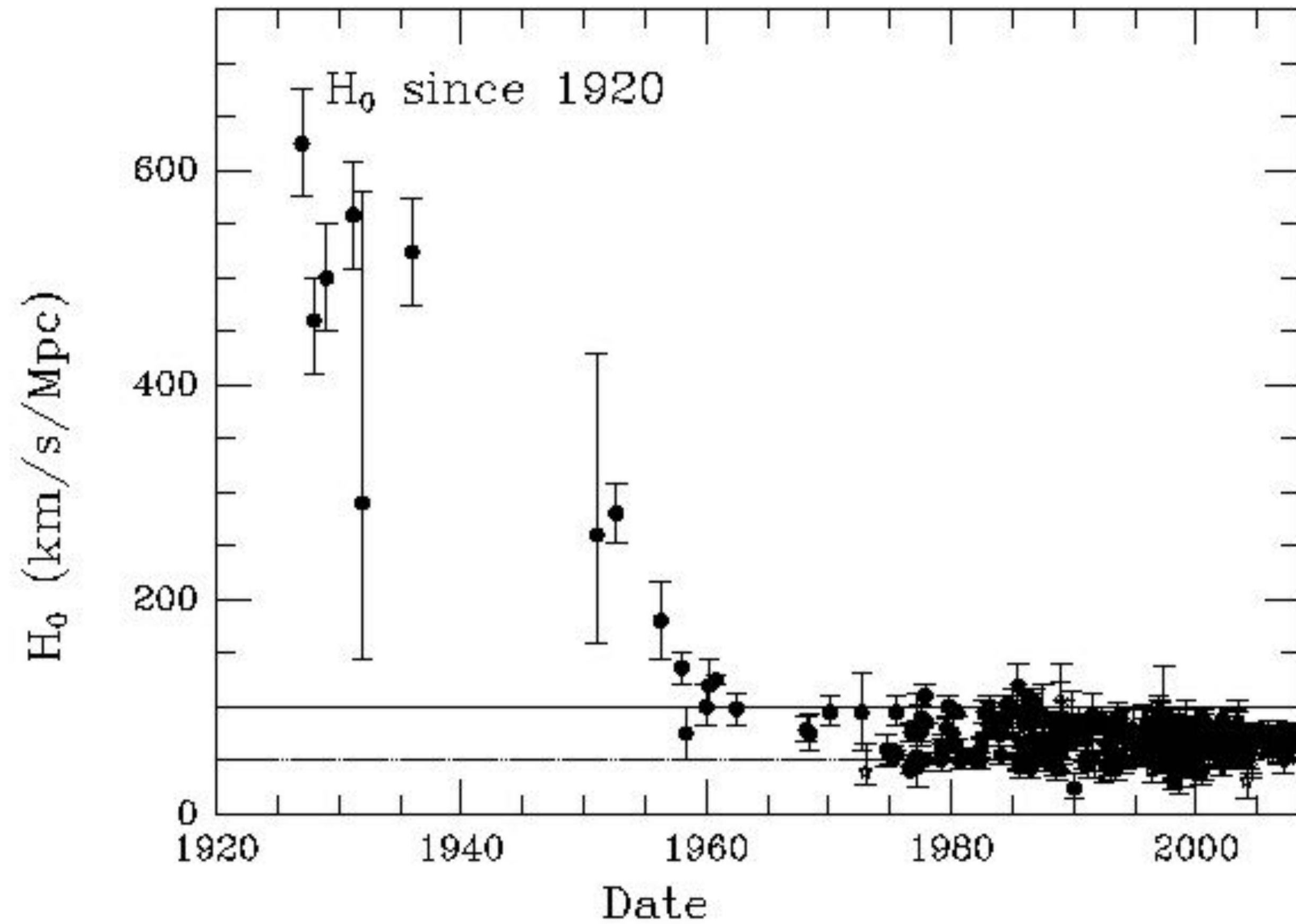
Supernovae Luminosity Distances

Megamaser Potential

Promise of GW sources as  
“standard sirens”



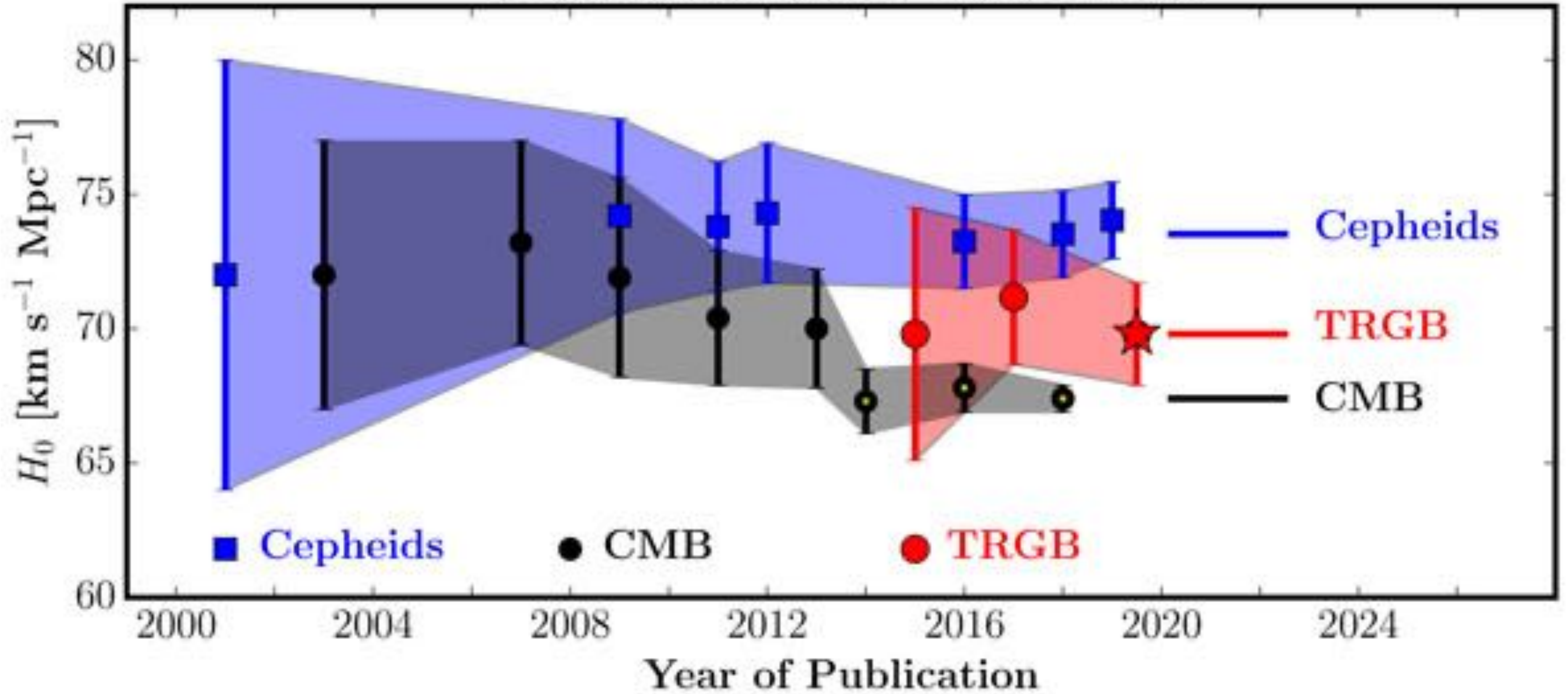
# Distances are hard, estimating $H_0$ is hard



Copyright J. Huchra 2008

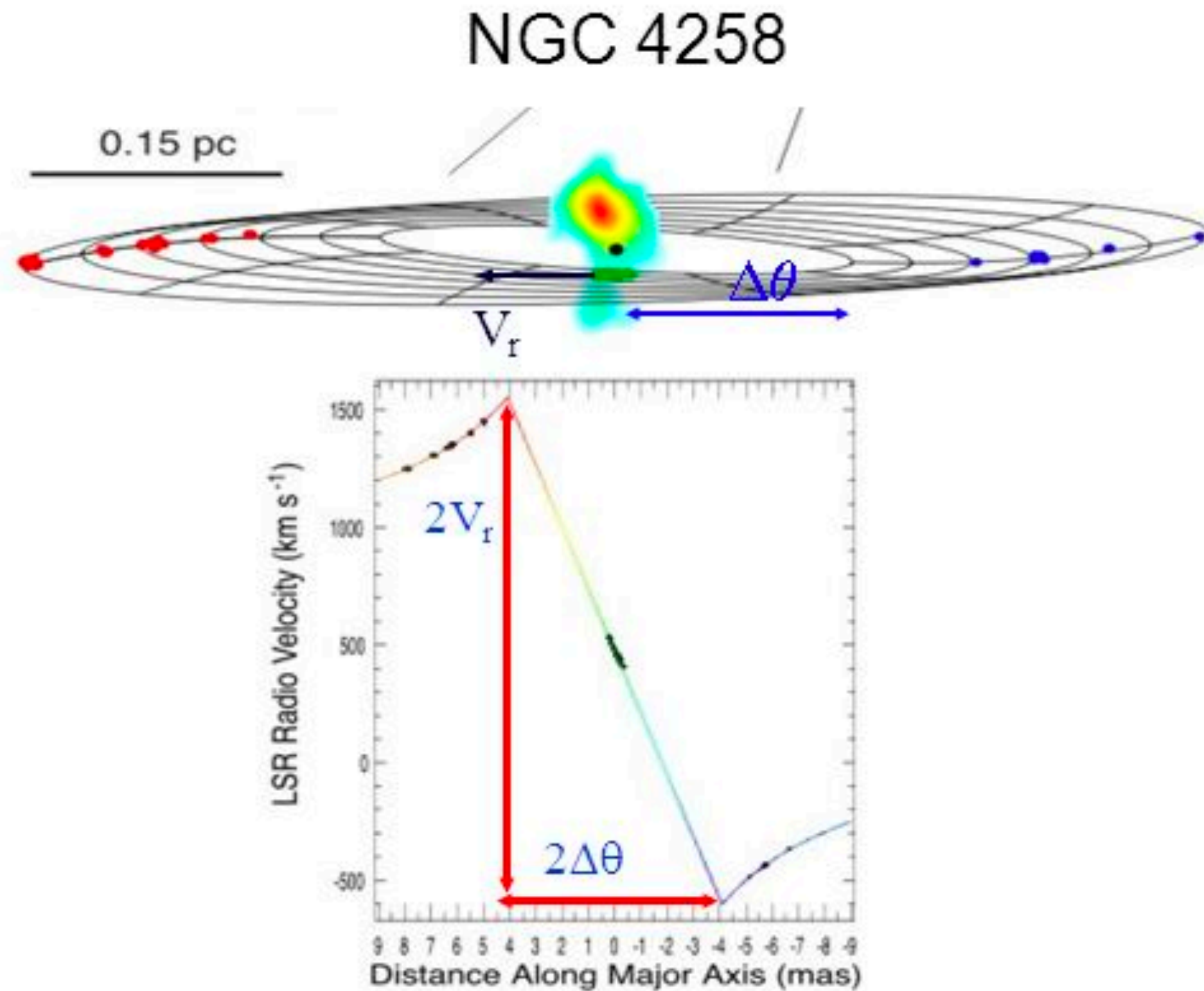


# Hubble Constant Over Time





# Measuring Distances to H<sub>2</sub>O Megamasers



Thin-ring model:

$$D = a^{-1} k^{2/3} \Omega^{4/3}$$

$a$  = acceleration

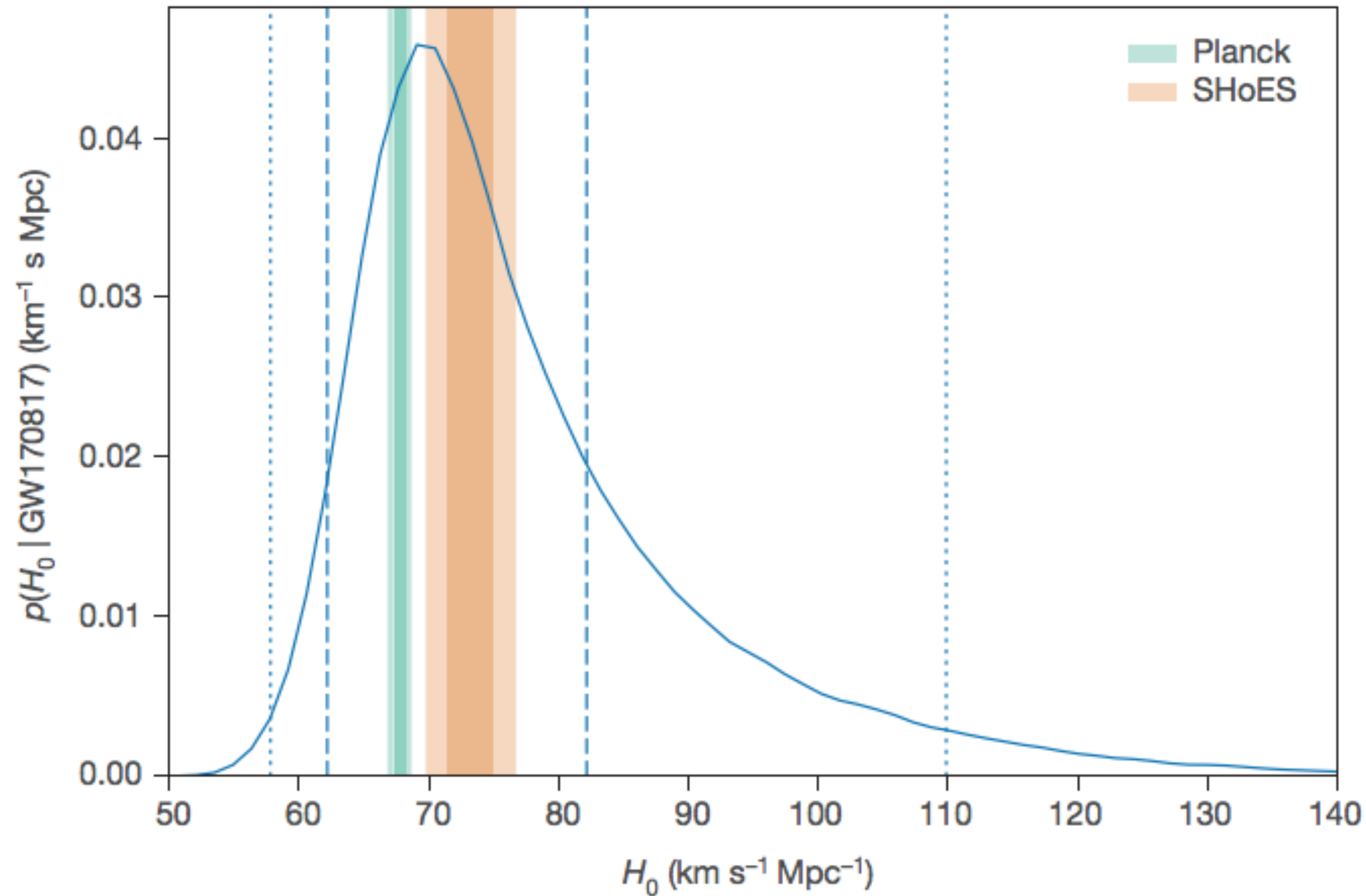
$$v = k r^{-1/2}$$

$\Omega$  = slope of sys features

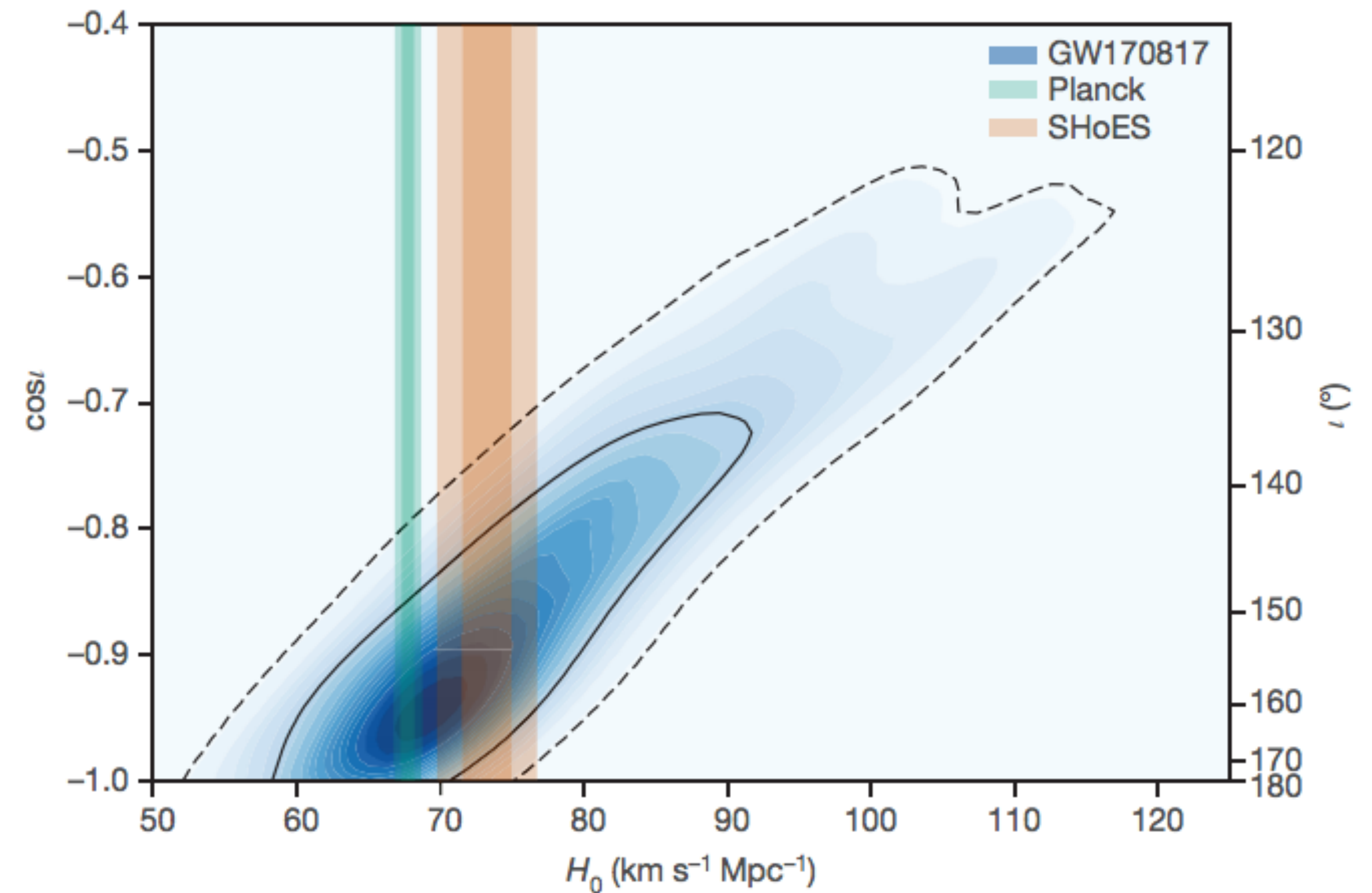
$$\frac{\sigma_{Da}}{D_a} \simeq \sqrt{\left(\frac{\sigma_a}{a}\right)^2 + \frac{4}{9} \left(\frac{\sigma_k}{k}\right)^2 + \frac{16}{9} \left(\frac{\sigma_\Omega}{\Omega}\right)^2}$$







**Figure 1 | GW170817 measurement of  $H_0$ .** The marginalized posterior density for  $H_0$ ,  $p(H_0 | \text{GW170817})$ , is shown by the blue curve. Constraints at  $1\sigma$  (darker shading) and  $2\sigma$  (lighter shading) from Planck<sup>20</sup> and SHoES<sup>21</sup> are shown in green and orange, respectively. The maximum a posteriori value and minimal 68.3% credible interval from this posterior density function is  $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The 68.3% ( $1\sigma$ ) and 95.4% ( $2\sigma$ ) minimal credible intervals are indicated by dashed and dotted lines, respectively.



**Figure 2 | Inference on  $H_0$  and inclination.** The posterior density of  $H_0$  and  $\cos i$  from the joint gravitational-wave–electromagnetic analysis are shown as blue contours. Shading levels are drawn at every 5% credible level, with the 68.3% ( $1\sigma$ ; solid) and 95.4% ( $2\sigma$ ; dashed) contours in black. Values of  $H_0$  and  $1\sigma$  and  $2\sigma$  error bands are also displayed from Planck<sup>20</sup> and SHoES<sup>21</sup>. Inclination angles near  $180^\circ$  ( $\cos i = -1$ ) indicate that the orbital angular momentum is antiparallel to the direction from the source to the detector.



