

ASTR 4080 - Week 7

- What is dark matter?
- Why do we think it exists?
- How would you go about looking for evidence of its existence?
- Is dark matter a good name?
- Why do we call normal matter "baryonic"?

Rotation Curves of Galaxies



$$a = \frac{v^2}{R}$$

angular acceleration
of circular motion

$$a_s = \frac{GM(<R)}{R^2}$$

assumes spherical
symmetry, so

all interior mass can be assumed
to be @ the center

Therefore

$$\frac{v^2}{R} = \frac{GM(<R)}{R^2}$$

or

$$v = \sqrt{\frac{GM(<R)}{R}}$$

SB of a disk galaxy typically falls off

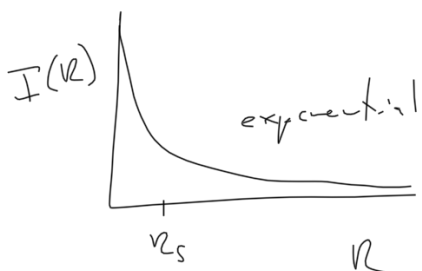
as

$$I(R) = I(0) e^{-R/R_s}$$

$$R_s \sim 4 \text{ kpc (MW)} \quad + \quad R_s \sim 6 \text{ kpc (M31)}$$

-if $M \propto L$, beyond

few $\times R_s$, mass const.



so $v \propto R^{-1/2}$

"Keplerian"

★ Slide on rotation curve

- rearrange eq. $\rightarrow M(R) = \frac{v^2 R}{G}$

MW: $L_{\text{gal},v} \approx 2.0 \times 10^{10} L_{\odot,v}$

so $\langle M/L_{\text{gal},v} \rangle \approx \underline{64} M_{\odot}/L_{\odot,v} \left(\frac{R_{\text{halo}}}{100 \text{ kpc}} \right)$

Avg. M/L of gal. (given their stars)

is $\sim 4 M_{\odot}/L_{\odot,v}$, so clearly lots
of non-stellar mass

$$\left[\begin{array}{l} R_{\text{halo}} \approx 75 \text{ kpc} - 300 \text{ kpc} \\ M_{\text{tot}} \approx (1-4) \times 10^{12} M_{\odot} \\ M/L \approx (50-200) M_{\odot}/L_{\odot,v} \end{array} \right]$$

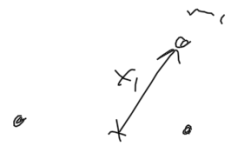
Dark Matter in Galaxy Clusters

→ Galaxy Velocities

Need the virial theorem

- start w/ moment of inertia

$$I = \sum_i m_i |\vec{x}_i|^2$$



- can link to W + K by taking 2nd derivat

$$\ddot{I} = 2 \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i + \dot{\vec{x}}_i \cdot \dot{\vec{x}}_i)$$

position x accel. term,
equiv. to potential E
→ due to grav.

K term

$$K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2$$

$$\ddot{\vec{x}}_i = G \sum_{j \neq i} m_j \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^3}$$

$$\begin{aligned} \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) &= G \sum_{\substack{i,j \\ (i \neq j)}} m_i m_j \frac{\vec{x}_i \cdot (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3} = \sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j) \end{aligned}$$

$$\begin{aligned}
\sum_i m_i (\ddot{\vec{x}}_i \cdot \dot{\vec{x}}_i) &= \frac{1}{2} \left[\sum_i m_i (\dot{\vec{x}}_i \cdot \dot{\vec{x}}_i) + \sum_j m_j (\dot{\vec{x}}_j \cdot \dot{\vec{x}}_j) \right] \\
&= \frac{G}{2} \left[\sum_{i,j} m_i m_j \frac{\dot{\vec{x}}_i \cdot (\dot{\vec{x}}_j - \dot{\vec{x}}_i)}{|\dot{\vec{x}}_j - \dot{\vec{x}}_i|^3} + \sum_{i,j} \dots \right] \\
&= \frac{G}{2} \left[\sum_{i,j} \frac{m_i m_j}{|\dot{\vec{x}}_i - \dot{\vec{x}}_j|^3} \underbrace{(\dot{\vec{x}}_i \cdot \dot{\vec{x}}_j - \dot{\vec{x}}_i^2 + \dot{\vec{x}}_j \cdot \dot{\vec{x}}_i - \dot{\vec{x}}_j^2)}_{-(\dot{\vec{x}}_i - \dot{\vec{x}}_j)^2} \right] \\
&= -\frac{G}{2} \sum_{i,j} \frac{m_i m_j}{|\dot{\vec{x}}_i - \dot{\vec{x}}_j|} = W \quad \text{pot. energy}
\end{aligned}$$

So $\boxed{\ddot{I} = 2W + 4K}$

Steady state, $\ddot{I} = 0$ + $K = -\frac{W}{2}$

Can write $W = -\alpha \frac{GM^2}{v_h}$ (simple v of 1 part. w/ const factor α)

$\alpha K = \frac{1}{2} M \langle v^2 \rangle$

where $\langle v^2 \rangle$ is mass-weighted velocity of galaxies

$K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2$ if $\langle v^2 \rangle = \frac{1}{M} \sum_i m_i |\dot{\vec{x}}_i|^2$ + $M = \sum_i m_i$

So this gives $\frac{1}{2} M \langle v^2 \rangle = \frac{\alpha}{2} \frac{GM^2}{R}$

$$M = \frac{\langle v^2 \rangle r_h}{\alpha G} \leftarrow \begin{array}{l} \text{half-mass} \\ \text{radius} \end{array}$$

Velocity dispersion σ_v is what can be measured, which is l.o.s. spread in velocities

→ since true vel. are 3D

$$\langle v^2 \rangle = 3 \sigma_v^2$$

★ Slides for Coma

$$\langle v^2 \rangle \approx 3 (880 \text{ km/s})^2 \quad \left. \vphantom{\langle v^2 \rangle} \right\} M_{\text{Coma}} \approx 2 \times 10^{14}$$

$$r_h \sim 1.5 \text{ Mpc}$$

$$[L_{\text{Coma},V} \approx 5 \times 10^{12} L_{\odot,V}]$$

$$\left[\begin{array}{l} M_{*,\text{Coma}} \sim 2 \times 10^{13} \\ M_{\text{gas,Coma}} \sim 2 \times 10^{14} \end{array} \right]$$

$$\langle M/L \rangle \sim 400 M_{\odot} / L_{\odot,V}$$