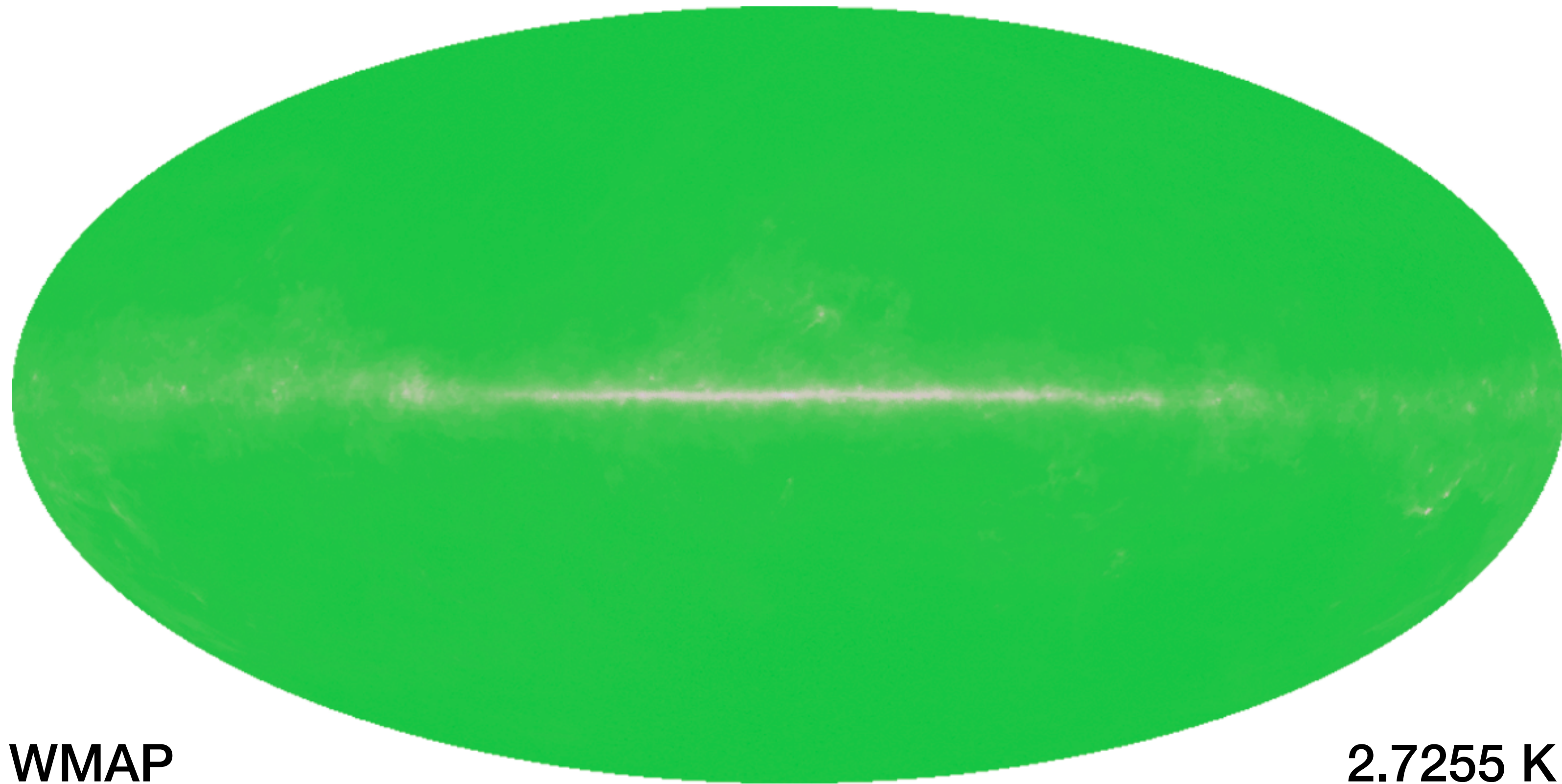


# Cosmic Microwave Background

ASTR/PHYS 4080: Intro to Cosmology  
Week 8



HW 6 due on Monday  
HW 7 due on Friday next week

Today:  
Dark Matter (finish Ch. 7)  
CMB (start Ch. 8)

# Brief History

- 1934 (Richard Tolman) blackbody radiation in an expanding universe cools but retains its thermal distribution and remains a blackbody
- 1941 (Andrew McKellar) excitation of interstellar CN doublet absorption lines gives an effective temperature of space of  $\sim 2.3\text{K}$
- 1946 (Gamow) to match observed abundance, nuclei should be built up out of equilibrium in hot early universe (high expansion rate, assume matter dominates)
- 1948 (Gamow)  $T \sim 10^9\text{ K}$  when deuterium formed, argues for radiation domination in early universe; the existence of CMB
- 1948 (Alpher, Bethe, & Gamow [ $\alpha\beta\gamma$  paper]), element synthesis in an expanding universe; calculations based on previous ideas
- 1948 (Alpher & Herman) make corrections to previous results; state that present radiation temperature should be  $\sim 5\text{K}$  (close! but largely a coincidence; incorrect assumptions - neutron dominated initial state); no mention of the observability.

# Brief History

- 1957 (Shmaonov) horn antenna at 3.2cm, find the absolute effective temperature of radio emission background  $4\pm 3\text{K}$ , independent of time and direction
- Early 1960s, (Zel'dovich, Doroshkevich, Novikov) estimate expected background temperature from helium abundance; realize Bell Labs telescope can constrain
- 1964 (Hoyle & Tayler) essentially correct version of primordial helium abundance calculation (no longer pure neutron initial state; weak interaction for neutron vs proton)
- 1965 (Dicke, Peebles, Roll, & Wilkinson) realize oscillating or singular universe might have thermal background; build detector to search; then they hear about its discovery
- 1965 (Penzias & Wilson) antenna has isotropic noise of  $3.5\pm 1.0\text{K}$  at wavelength of 7.35cm; careful experiment (e.g., shooed away pigeons roosted in the antenna; cleaned up “the usual white dielectric” generated by pigeons); explanation could be that of Dicke et al.

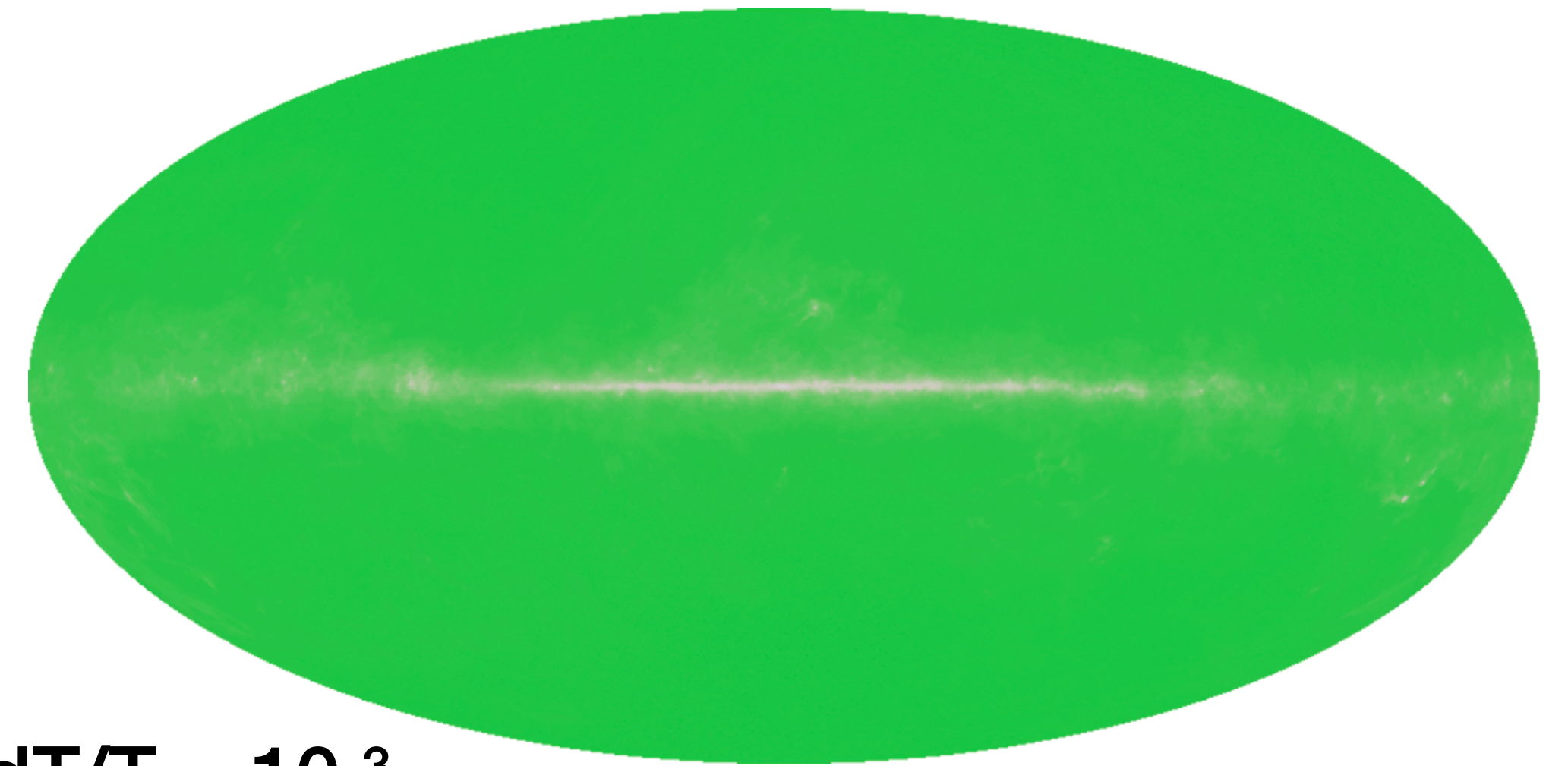
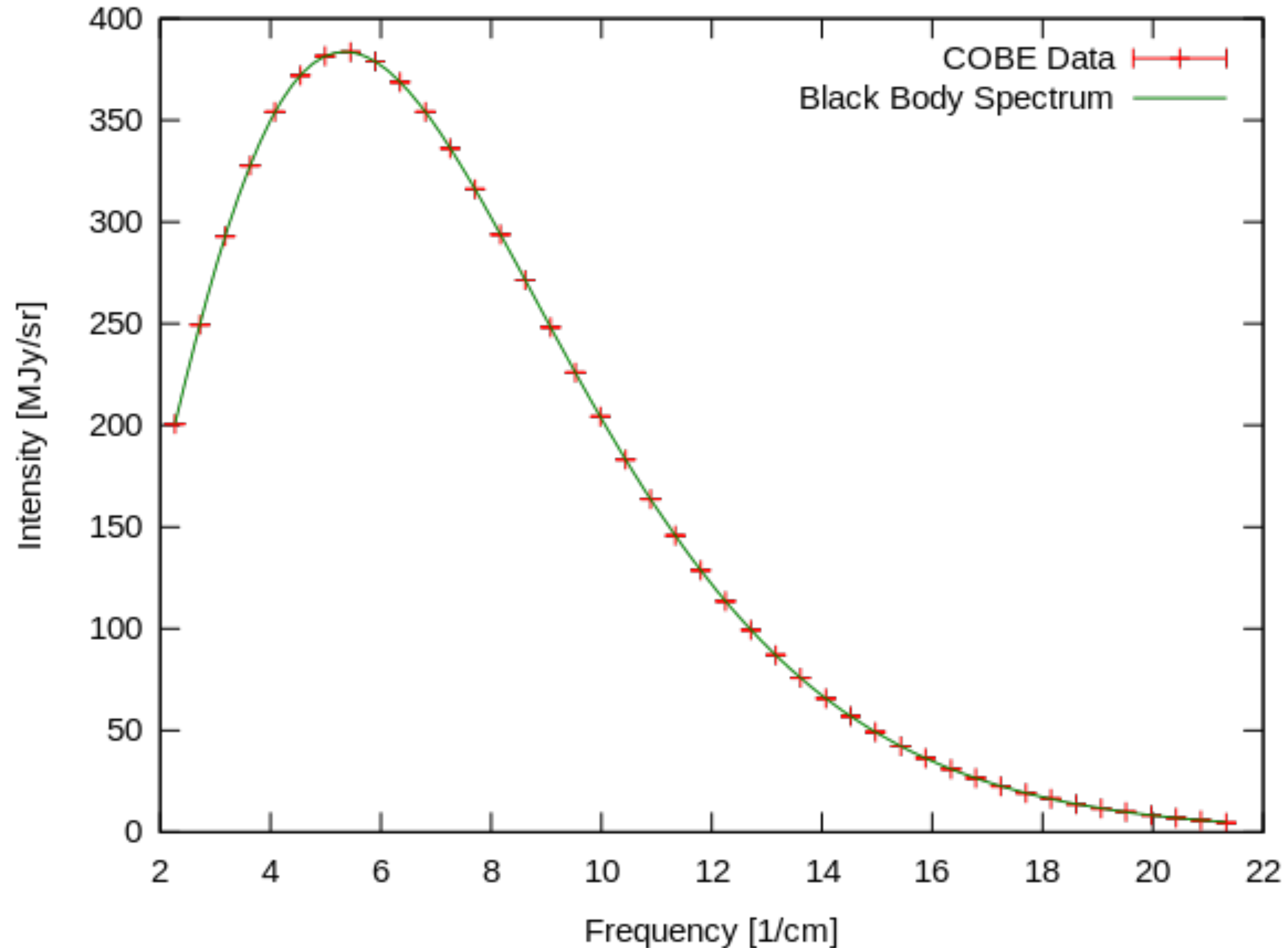
# Brief History

- 1965 (Roll & Wilkinson) detect the radiation background at 3.2cm, with amplitude consistent with Penzias & Wilson for blackbody spectrum; isotropic to 10%
- 1966-1967 (Field & Hitchcock, Shklovsky, Thaddeus & Clauser, Thaddeus [following a suggestion by Woolf]) independently show that the excitation of interstellar CN is caused by CMB (McKellar's 1941 observation explained!)
- 1970s-1980s, ground, balloon, satellite observations
- 1992, NASA's COsmic Background Explorer (COBE) satellite confirms CMB as nearly perfect isotropic blackbody and discovers the anisotropies.
- Era of "precision cosmology" begins, especially with SNe measurements a few years later and then the launches of WMAP (2001) and Planck (2009)

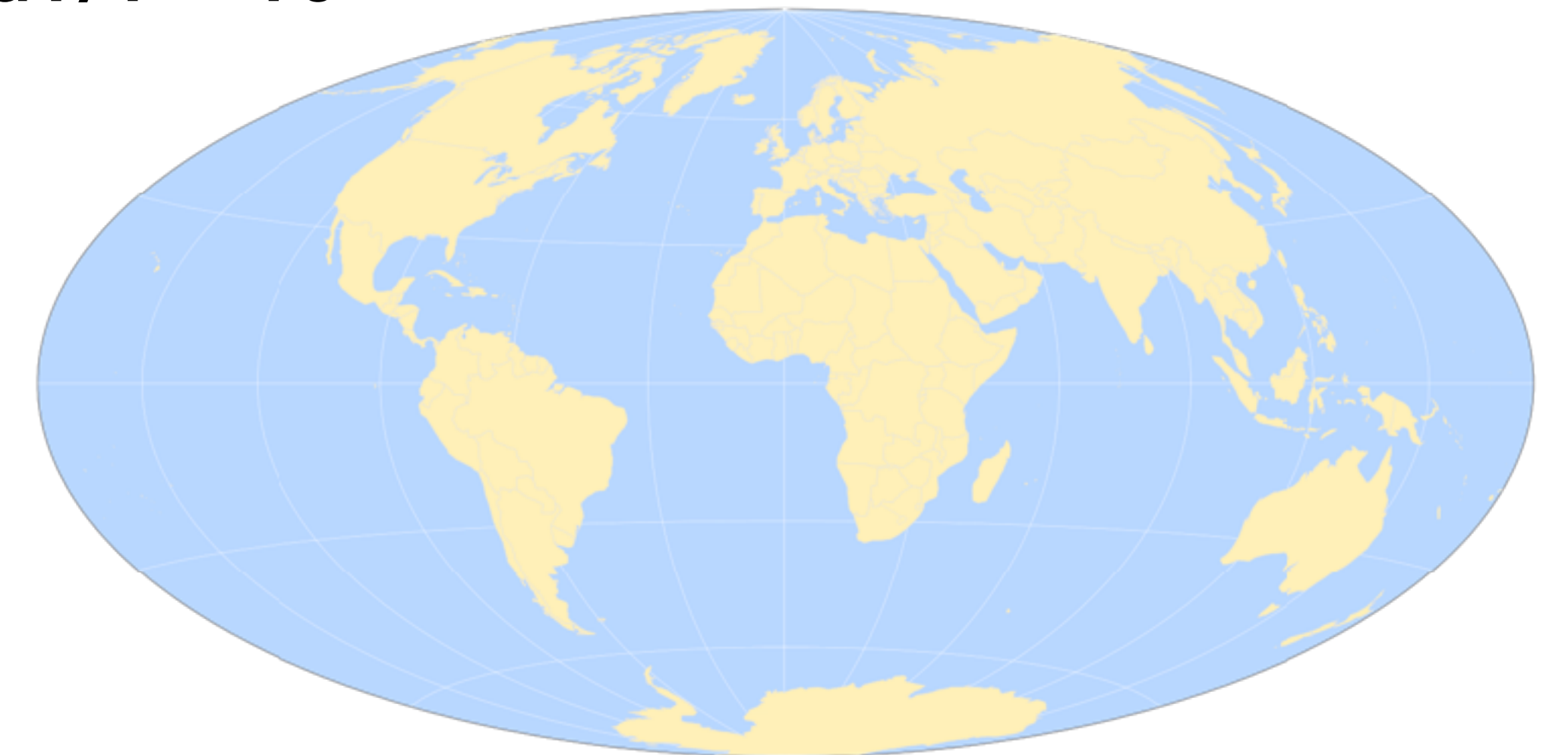


# Near perfect BB everywhere on the sky

Cosmic Microwave Background Spectrum from COBE



$dT/T \sim 10^{-3}$

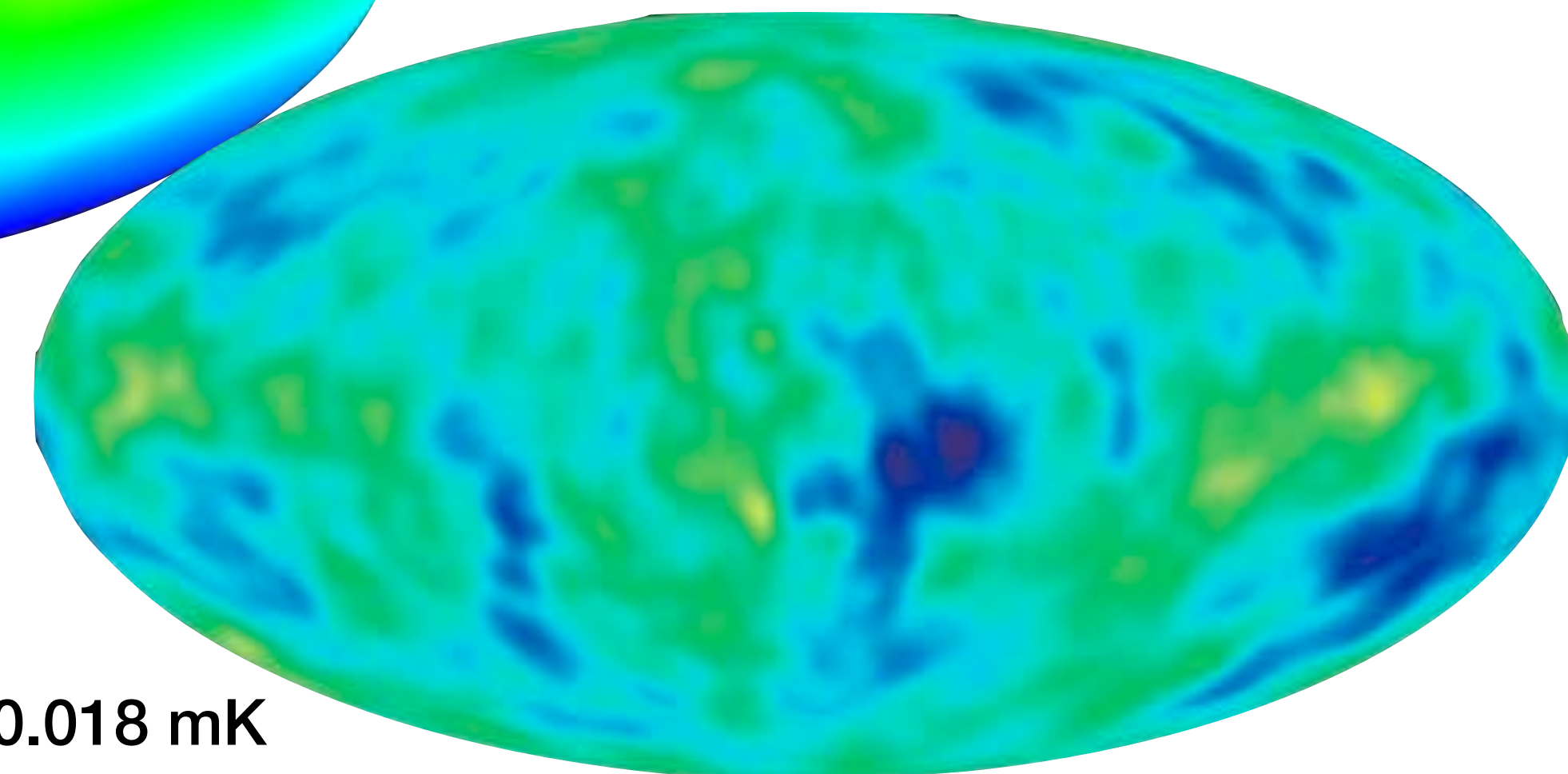
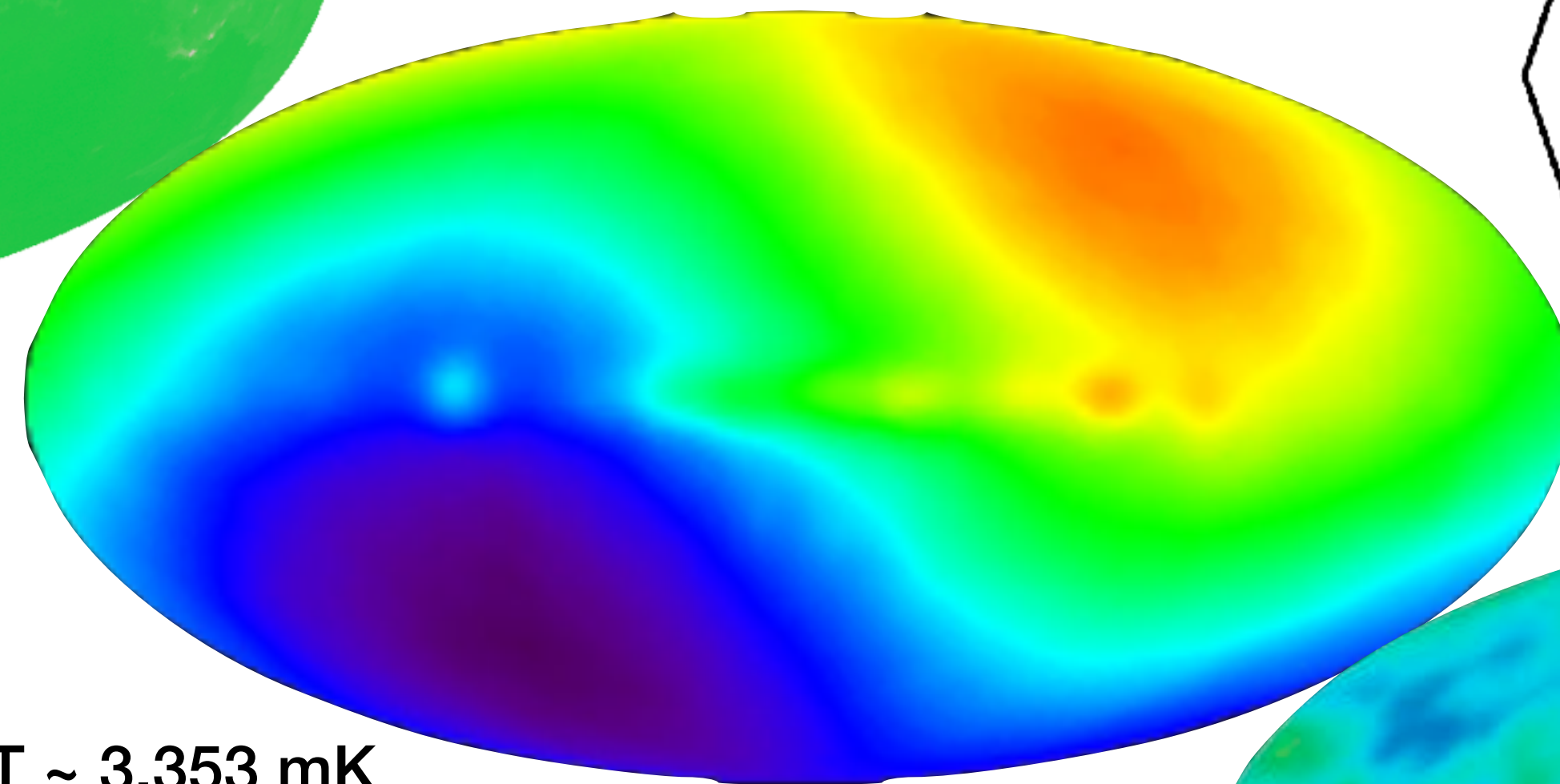
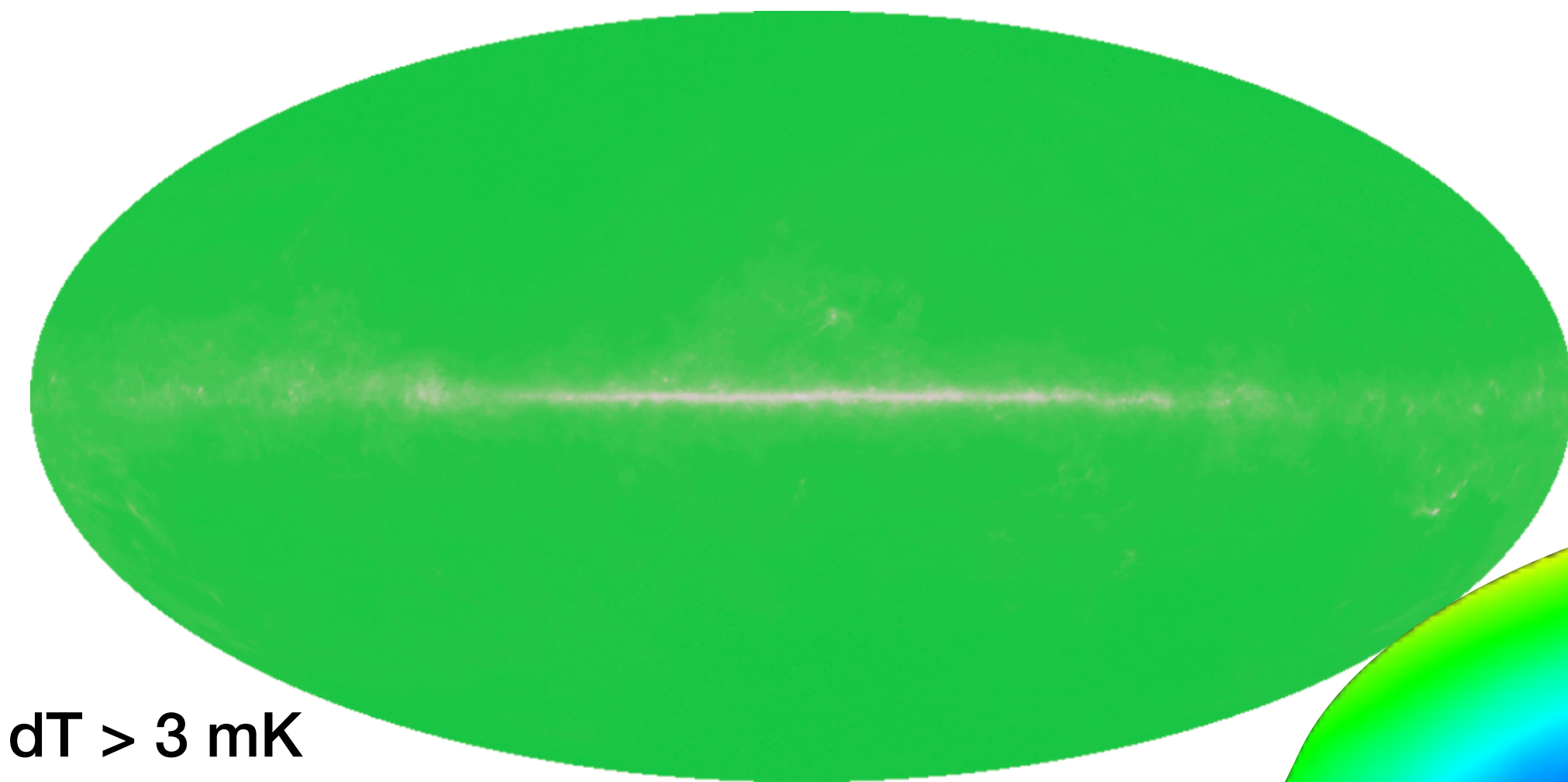




# Spatial variations on different scales

$$\frac{\delta T}{T}(\theta, \phi) \equiv \frac{T(\theta, \phi) - \langle T \rangle}{T}$$

$$\left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5}$$



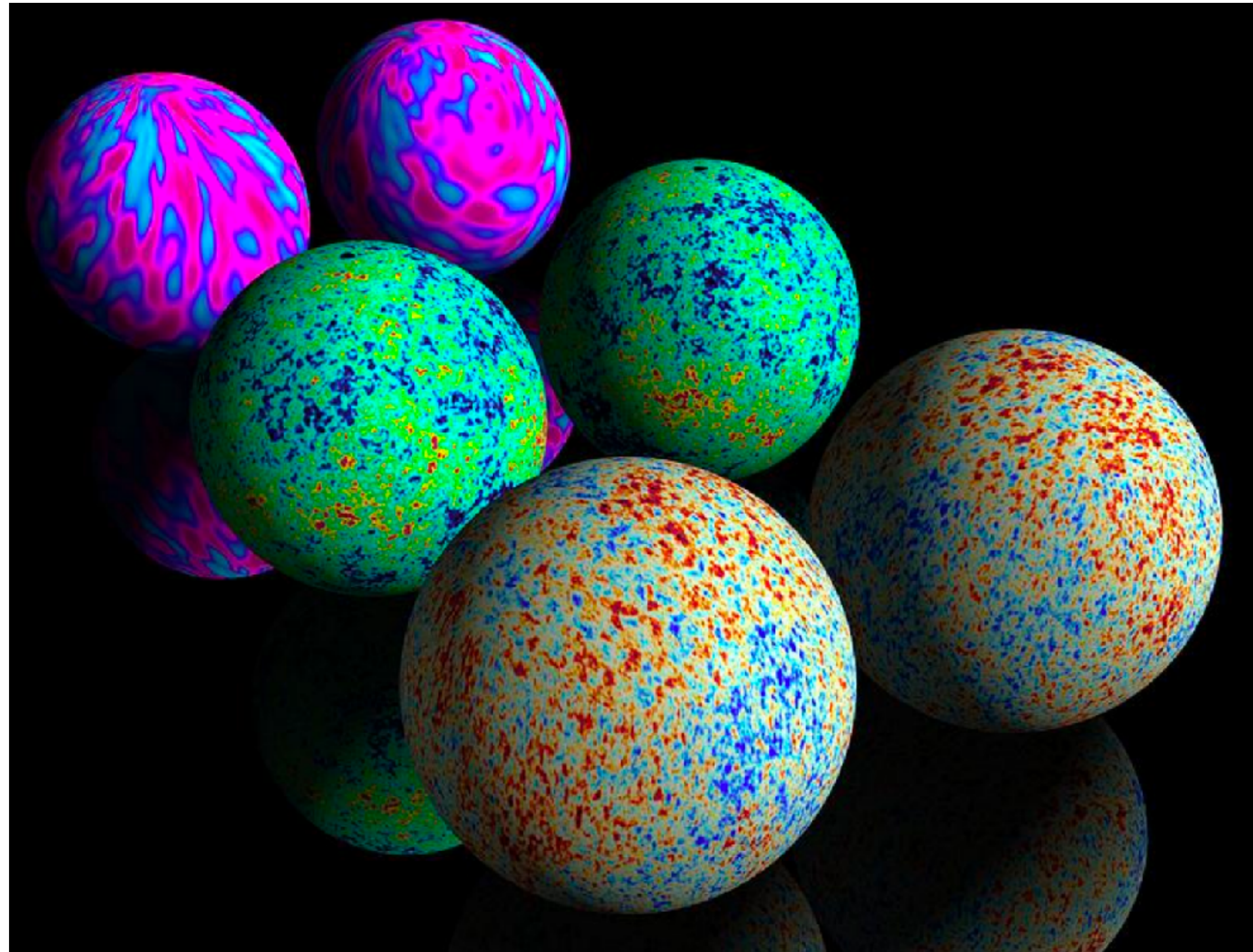


# History of CMB space measurements

COBE  
1990

WMAP  
2003

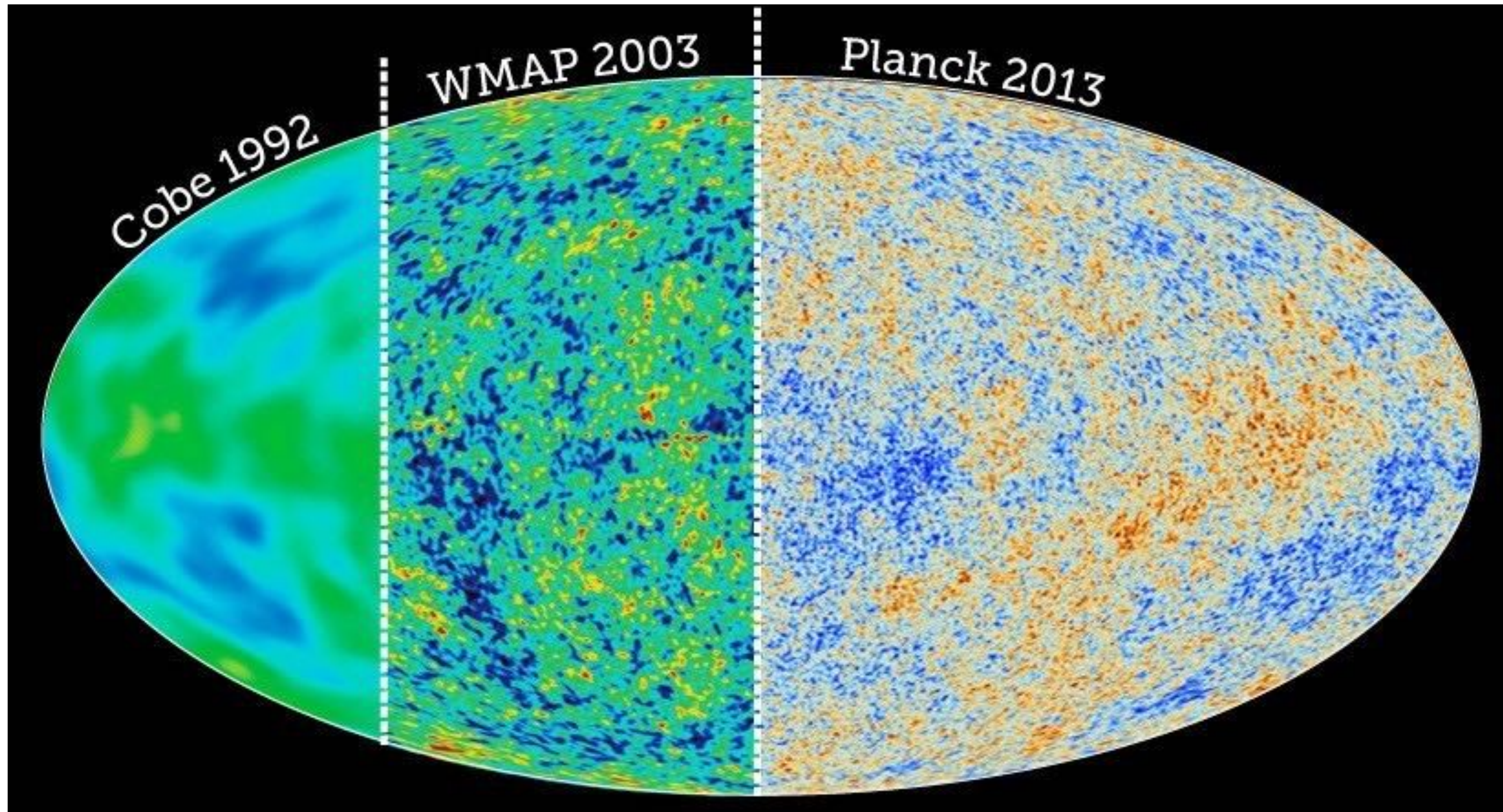
Planck  
2013



<https://fineartamerica.com/featured/cosmic-microwave-background-radiation-carlos-clarivan.html>

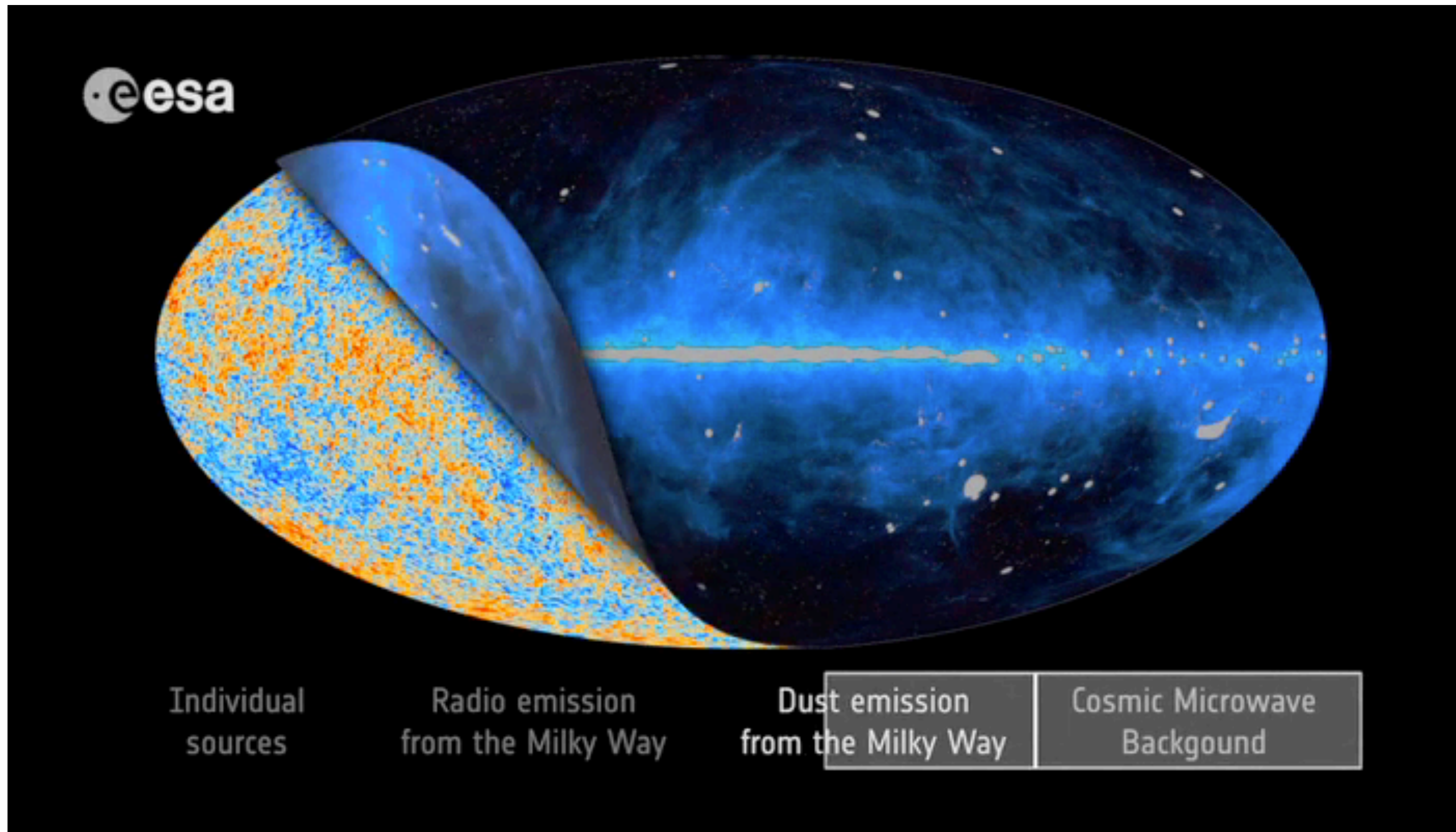


# Primary aim to measure small-scale fluctuations





# Observing the CMB





# What produces the CMB and features we see?

In the early universe, many interactions between particles (just like at the LHC) quarks, electrons, photons, neutrinos all transform into each other

As universe expands, densities decrease and protons/electrons/photons dominate baryon soup

Eventually, electrons can be captured by protons to form atoms that are not immediately broken up by energetic photons  
—> recombination

Soon thereafter, the density of free electrons is too low to scatter photons, and the universe becomes transparent  
—> photon decoupling

As the universe expands further, a time comes when a CMB photon scatters off an electron for one last time  
—> last scattering



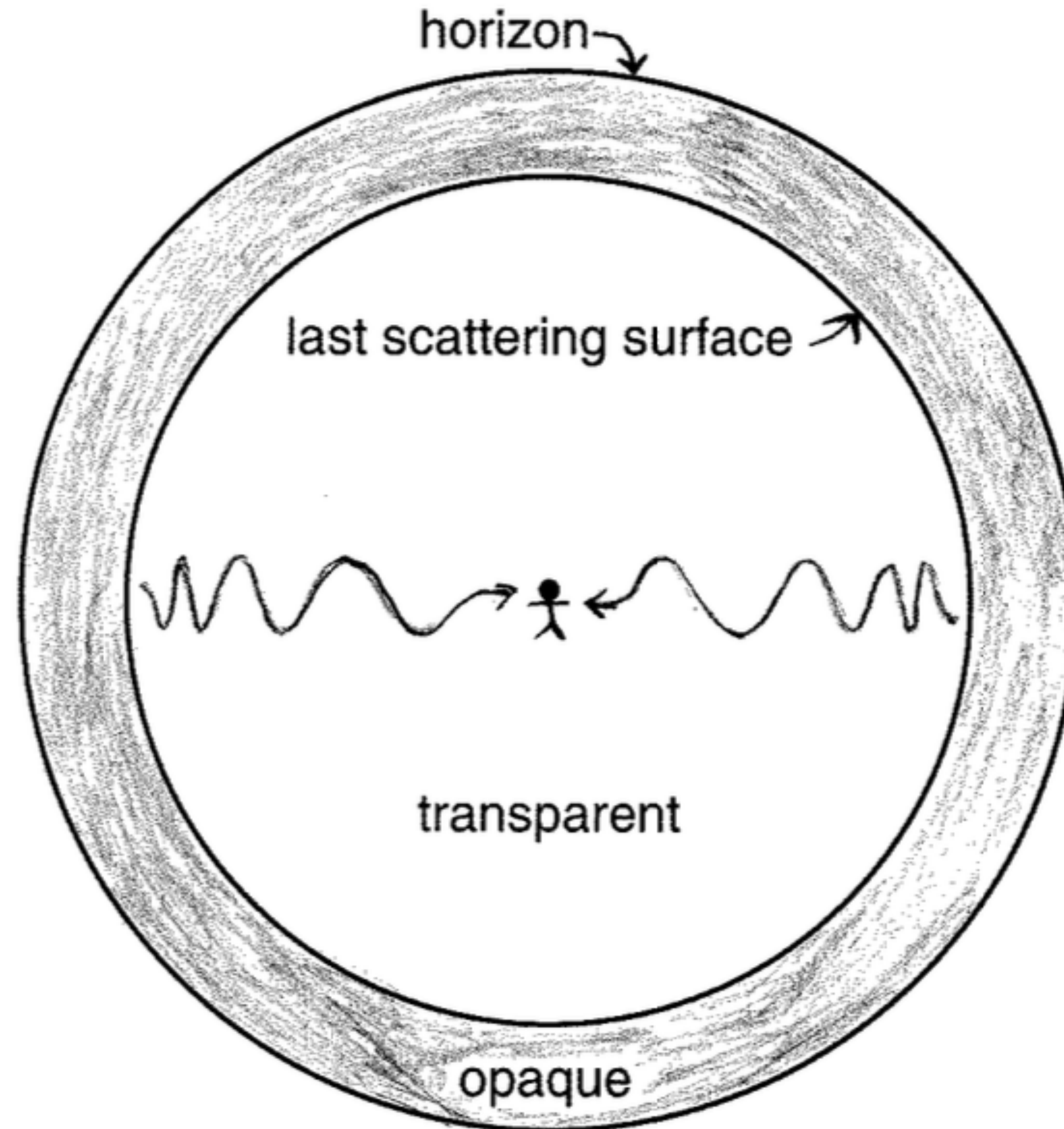
# Surface of Last Scattering

Thomson cross-section  
("size" of an electron)

$$\sigma_e = 6.65 \times 10^{-29} \text{ m}^2$$

mean free path of a photon

$$\lambda_{\text{mfp}} = \frac{1}{n_e \sigma_e}$$



Scattering interaction rate

$$\Gamma = \frac{c}{\lambda} = n_e \sigma_e c$$

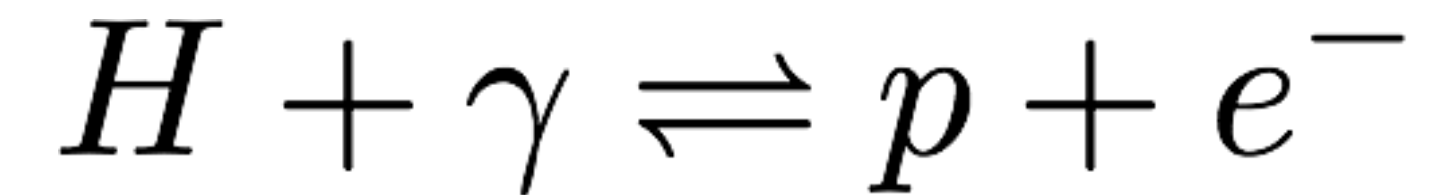
When photons scatter off of electrons at the same rate as the expansion rate:

$$H(z) = \Gamma(z)$$

Photons and baryons will "decouple" from each other



# Recombination



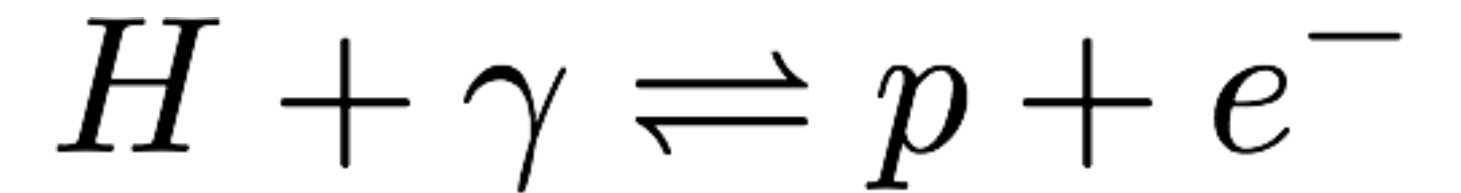
$$\langle h\nu \rangle < H_{\text{ionize}} = Q = 13.6 \text{ eV}$$

$$\langle h\nu \rangle = 2.7kT \text{ (BB spectrum)}$$

Implies  $T \sim 60,000 \text{ K} \rightarrow$  much too high: BB spectrum has a tail



# Recombination



# density of particles of type  $x$  with momenta b/t

$$p \rightarrow p + dp$$

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{\exp([E - \mu_x]/kT) \pm 1}$$

(minus for bosons, plus for fermions)

$g \rightarrow 2$  (for non-nucleons,  $g_H=4$ )  
chemical potential of photons = 0

$$\mu_H = \mu_p + \mu_e$$

$$n_\gamma = \frac{2.4041}{\pi^2} \left( \frac{kT}{\hbar c} \right)^3$$

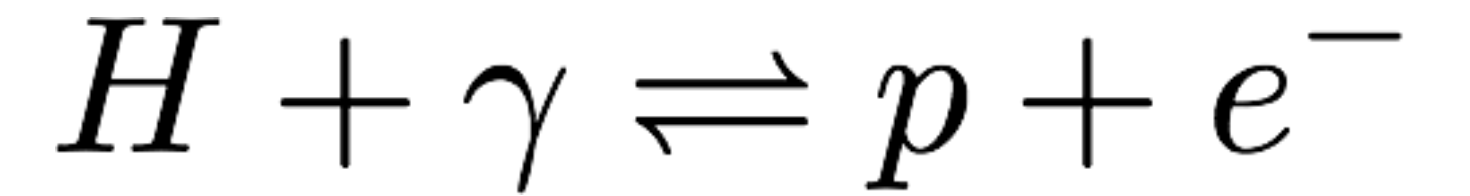
$$n_x = g_x \left( \frac{m_x kT}{2\pi \hbar^2} \right)^{3/2} \exp \left( \frac{-m_x c^2 + \mu_x}{kT} \right)$$

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left( \frac{m_H}{m_p m_e} \right)^{3/2} \left( \frac{kT}{2\pi \hbar^2} \right)^{-3/2} \exp \left( \frac{[m_p + m_e - m_H]c^2}{kT} \right) = \left( \frac{m_e kT}{2\pi \hbar^2} \right)^{-3/2} \exp \left( \frac{Q}{kT} \right)$$

Saha Equation



# Recombination



$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left( \frac{m_H}{m_p m_e} \right)^{3/2} \left( \frac{kT}{2\pi\hbar^2} \right)^{-3/2} \exp\left( \frac{[m_p + m_e - m_H]c^2}{kT} \right) = \left( \frac{m_e kT}{2\pi\hbar^2} \right)^{-3/2} \exp\left( \frac{Q}{kT} \right)$$

Defined as when protons and H atoms are equal:

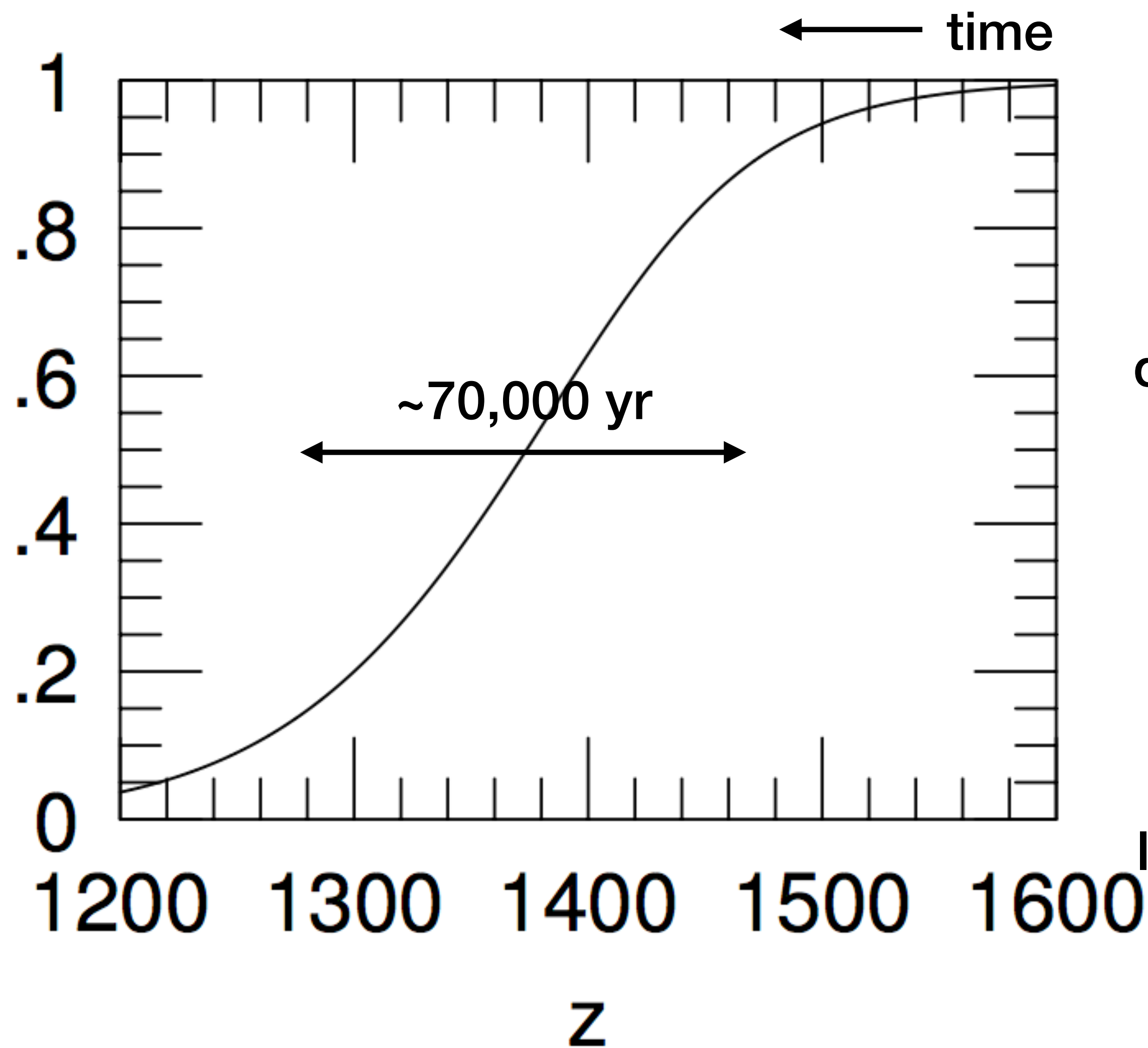
$$X \equiv \frac{n_p}{n_p + n_H} = 1/2$$

$$\frac{1-X}{X} = n_p \left( \frac{m_e kT}{2\pi\hbar^2} \right)^{-3/2} \exp\left( \frac{Q}{kT} \right) \quad \eta = \frac{n_p}{X n_\gamma}, \quad n_\gamma = \frac{2.4041}{\pi^2} \left( \frac{kT}{\hbar c} \right)^3$$

(set by current baryon/photon density)

$$\frac{1-X}{X^2} = 3.84\eta \left( \frac{kT}{m_e c^2} \right)^{3/2} \exp\left( \frac{Q}{kT} \right) = S \quad \longrightarrow \quad X = \frac{-1 + \sqrt{1 + 4S}}{2S} \quad \longrightarrow \quad kT_{\text{rec}} = \frac{Q}{42}$$

# Redshift of recomb., decoupling, & scattering



$$T = T_0 a^{-1} = T_0(1 + z)$$

recombination:  $z = 1380$  when  $T = 3760\text{K}$   
 $t_{\text{age}} = 250,000 \text{ yr}$

decoupling: when expansion rate surpasses  
 scattering rate:  $\Gamma(z) = H(z)$

$$1 + z = \frac{39.3}{X(z)^{2/3}}$$

$z \sim 1090$  (incl. non-eq. effects)

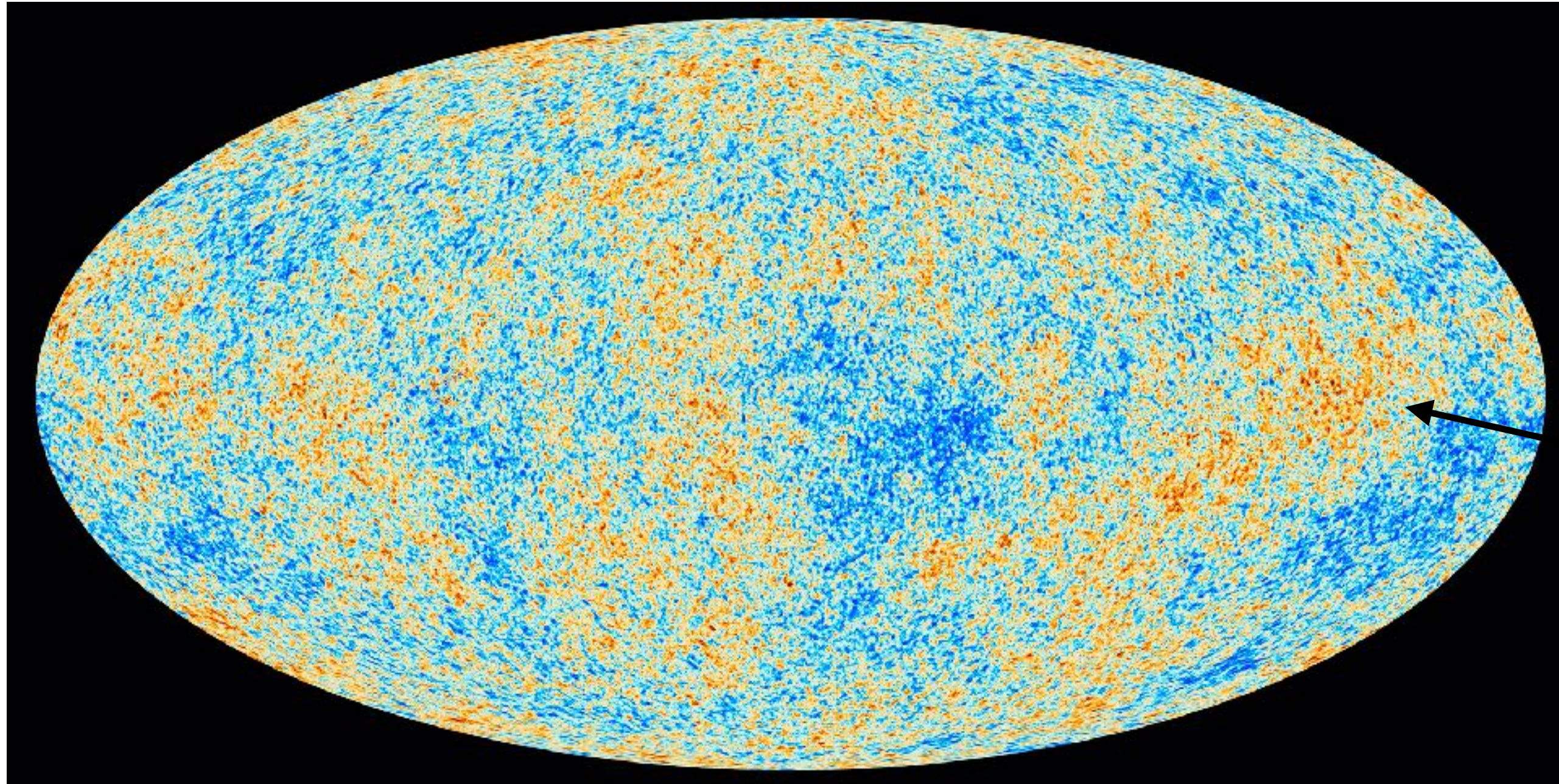
last scattering: when the optical depth is  $\sim 1$

$$\tau(t) = \int_t^{t_0} \Gamma(t) dt$$

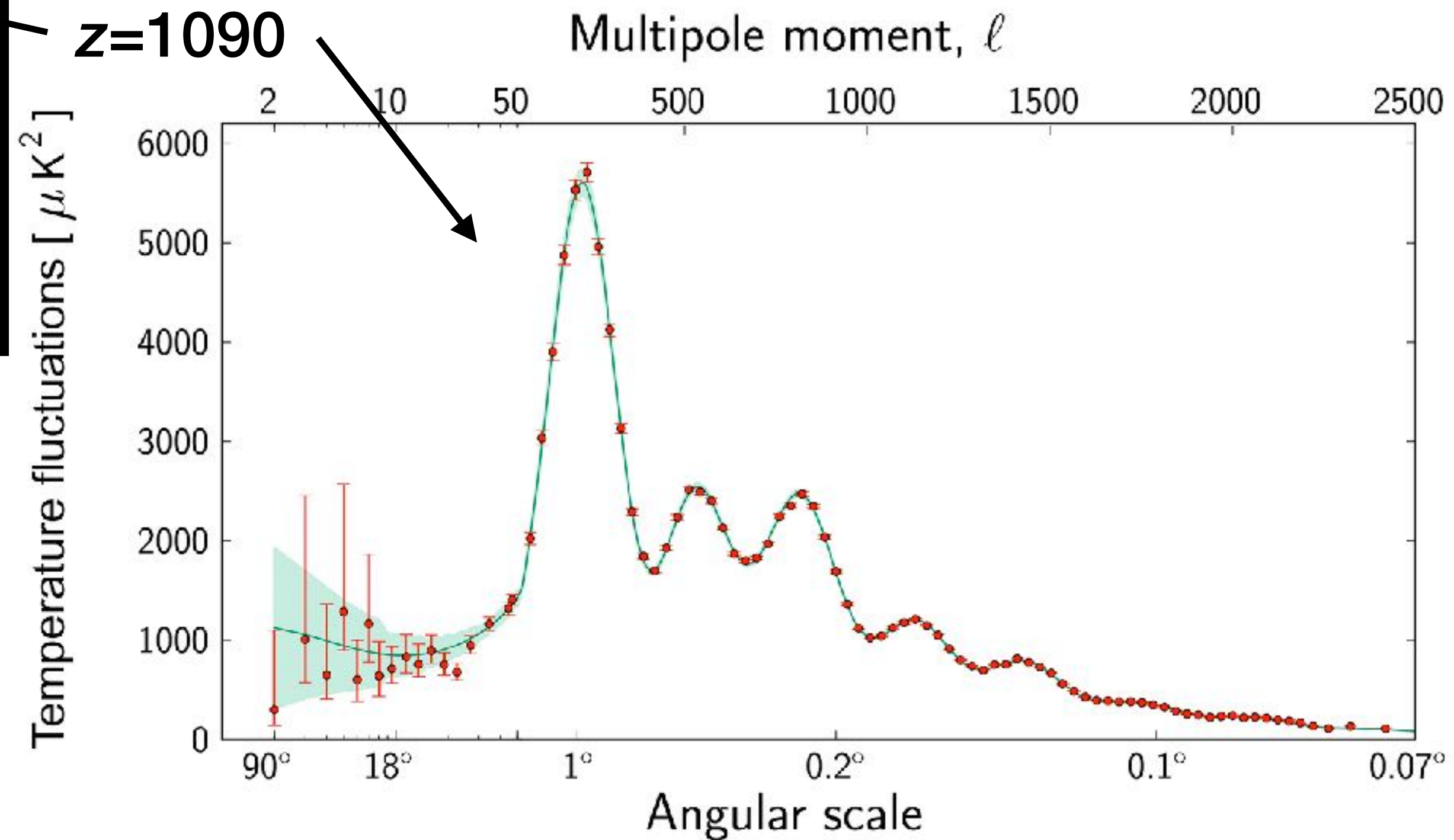
redshift same as decoupling



# Temperature fluctuations



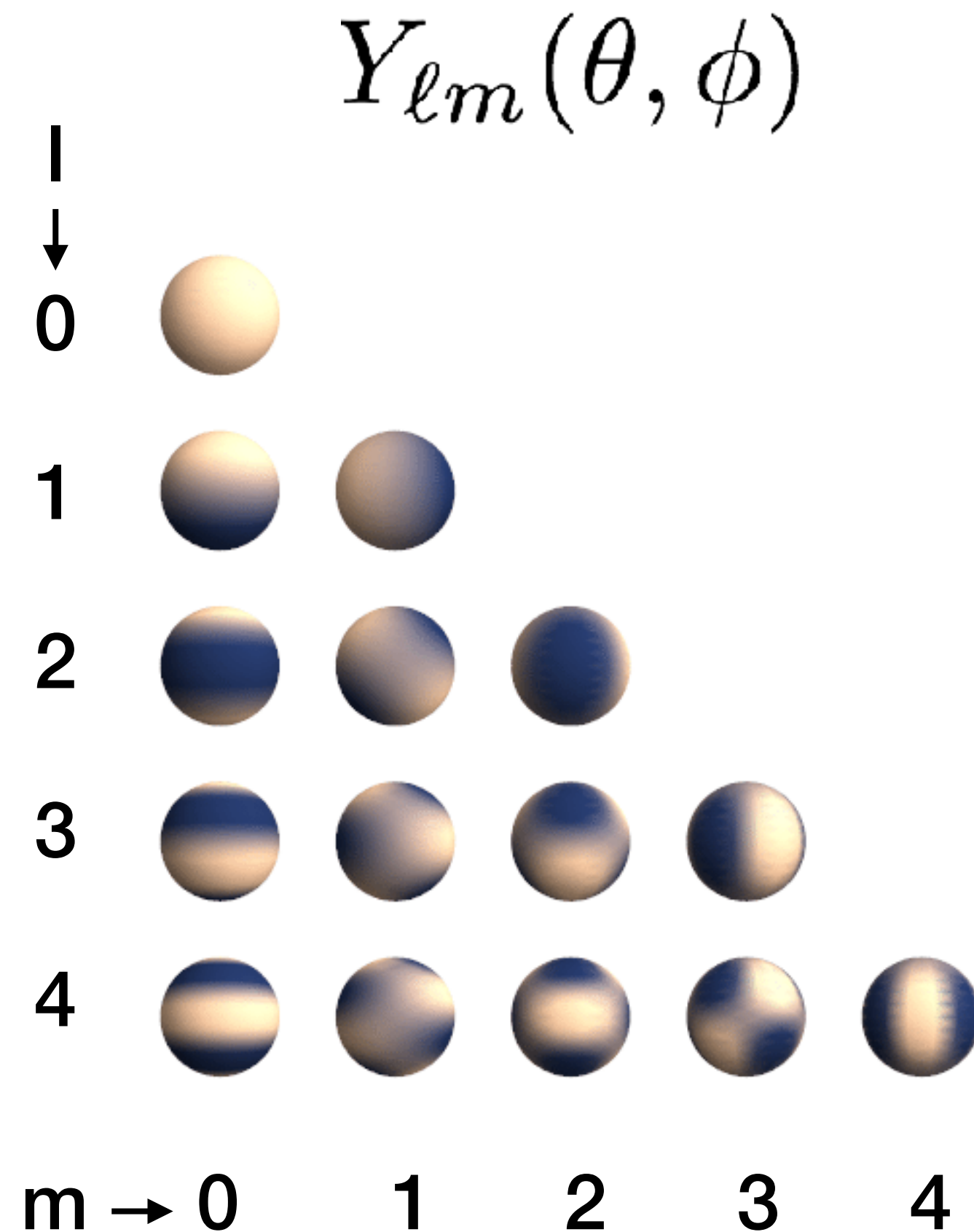
$$d_A(z \rightarrow \infty) \approx \frac{d_{\text{hor}}(t_0)}{z} = \frac{14 \text{ Gpc}}{1090} = 12.8 \text{ Mpc}$$



$$\ell = d_A \cdot \delta\theta = 12.8 \text{ Mpc} \left( \frac{\delta\theta}{1 \text{ rad}} \right) = 3.7 \text{ kpc} \left( \frac{\delta\theta}{1 \text{ arcmin}} \right)$$



# Spherical Harmonics & Power Spectrum



Represent function in terms of spherical harmonics

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi)$$

sum Y over  
m, get  
Legendre  
polynomials

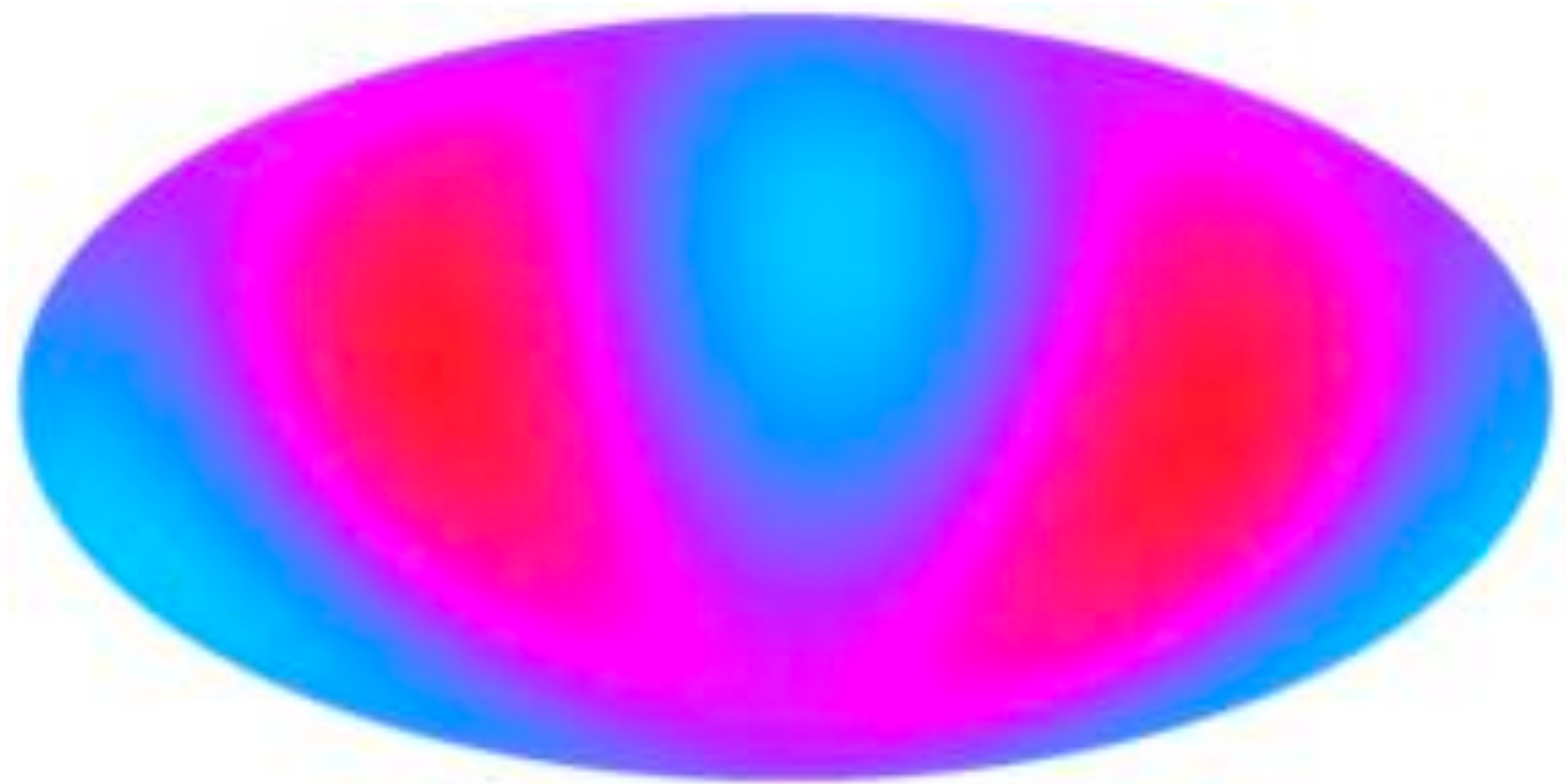
Power Spectrum

$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n} \cdot \hat{n}' = \cos \theta}$$

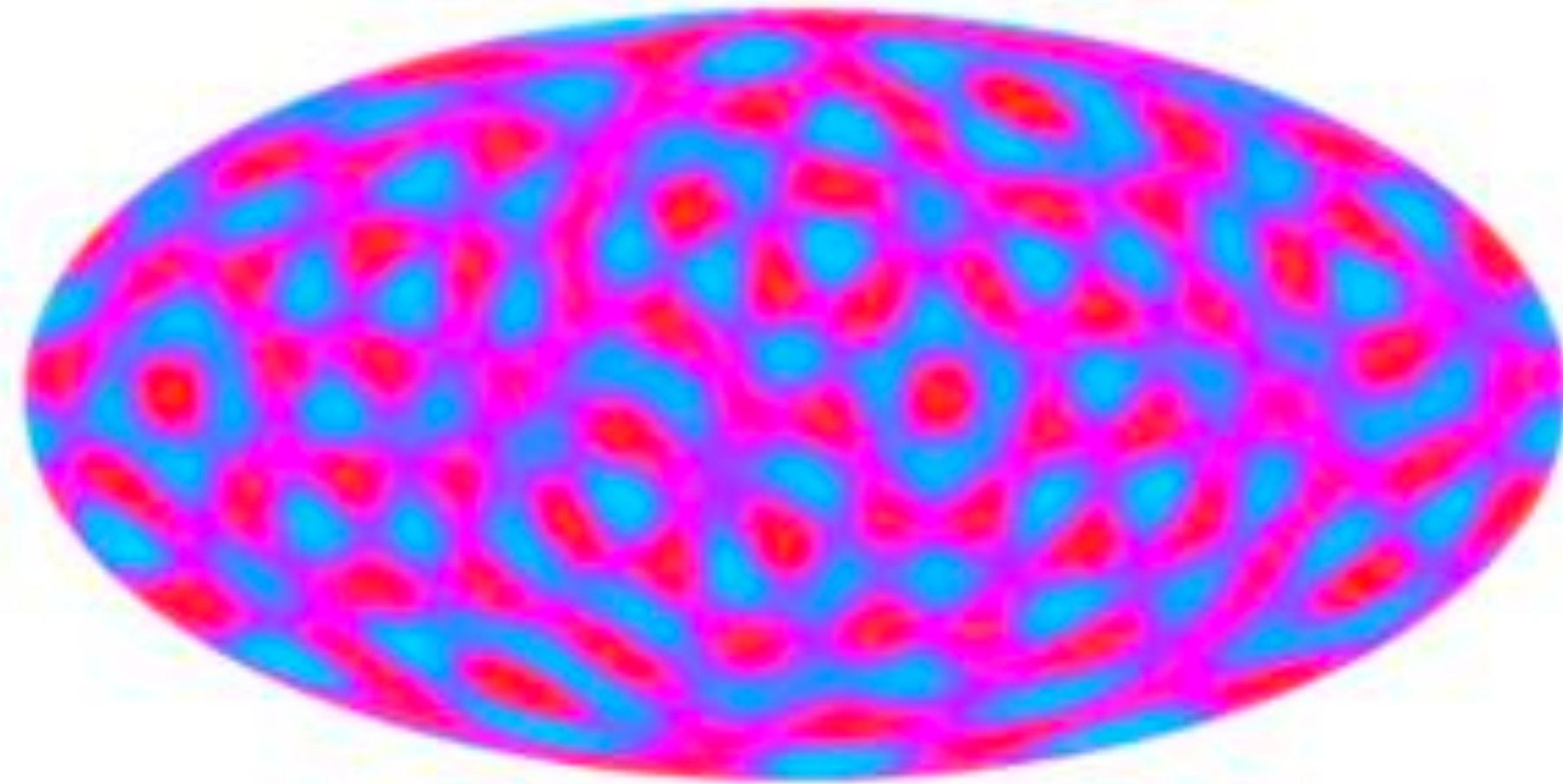
$$= \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell + 1) C_{\ell} P_{\ell}(\cos \theta)$$



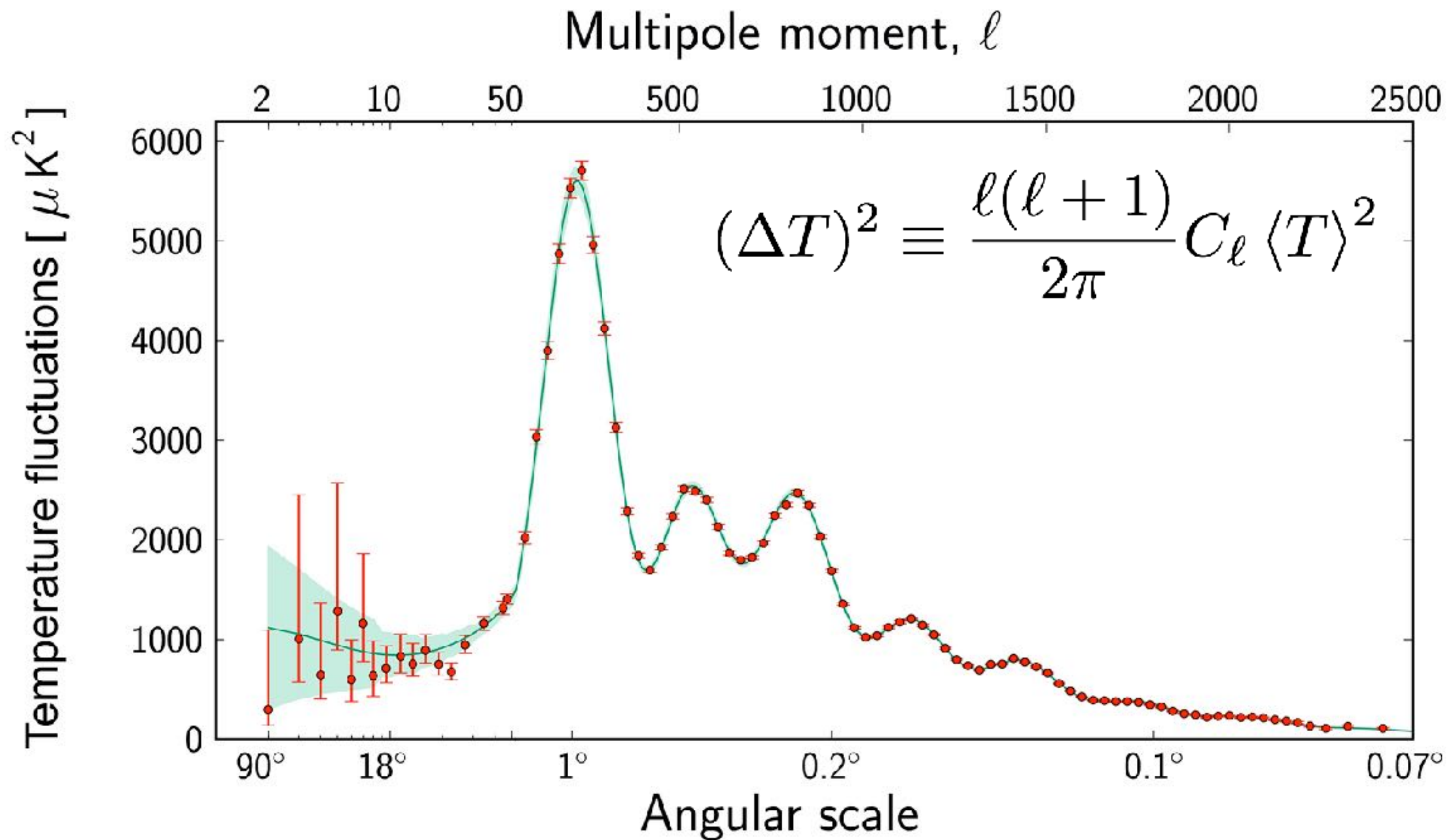
# (2 point) Correlation function



$l = 2$



$l = 16$



# Where do the peaks come from?

First, let's define the scale at which pieces of the universe could be in causal contact

At last scattering, universe evolves as if there's only radiation and matter, so we can easily calculate the horizon distance

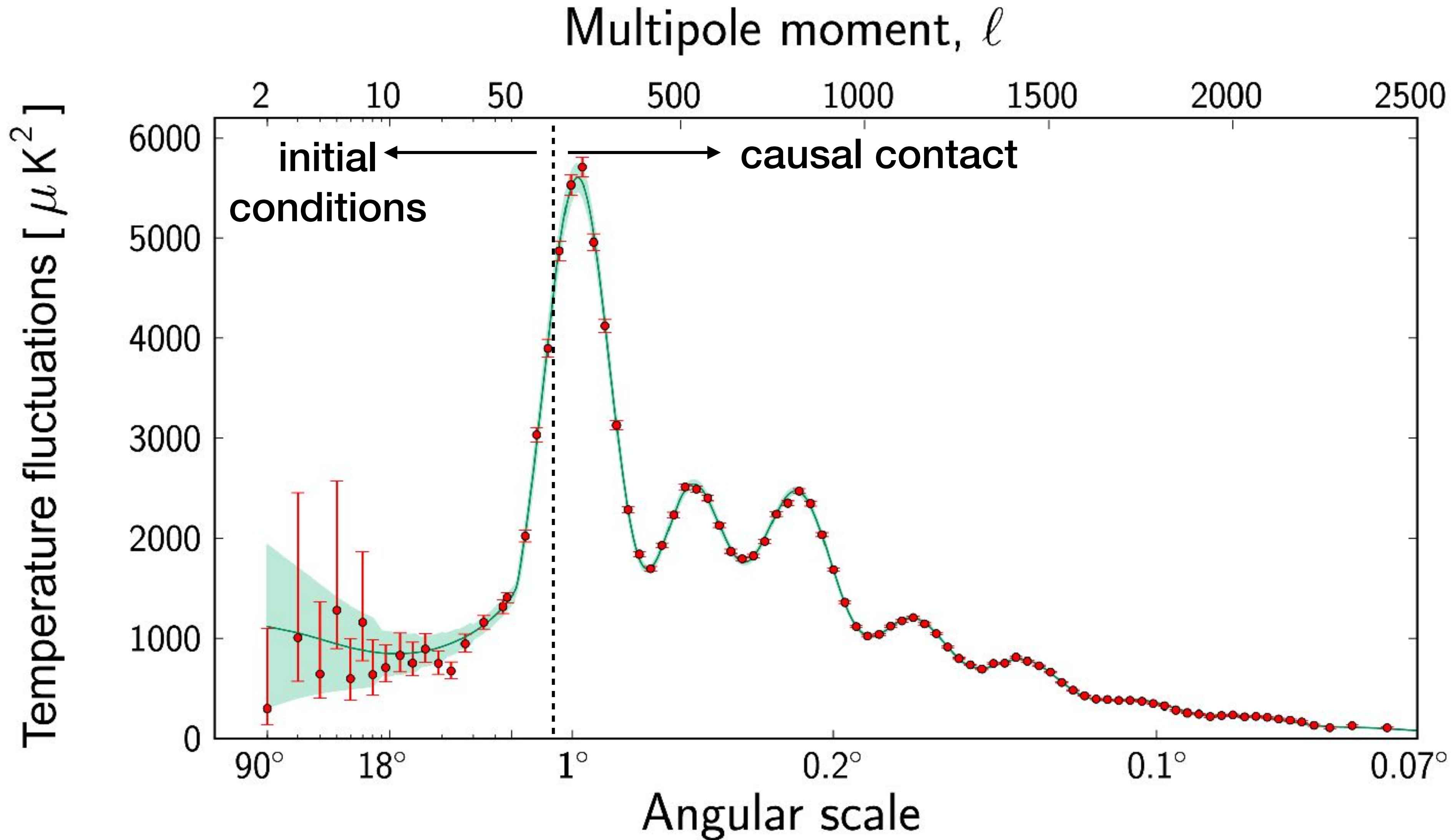
$$d_{\text{hor}}(t_{\text{ls}}) = a(t_{\text{ls}})c \int_0^{t_{\text{ls}}} \frac{dt}{a(t)} = 2.24ct_{\text{ls}} \approx 250 \text{ kpc}$$

By definition, the angular scale this occurs at is given by the angular diameter distance

$$\theta_{\text{hor}} = \frac{d_{\text{hor}}(t_{\text{ls}})}{d_A} = \frac{250 \text{ kpc}}{12.8 \text{ Mpc}} \approx 1.1^\circ$$



# Sachs-Wolfe Effect

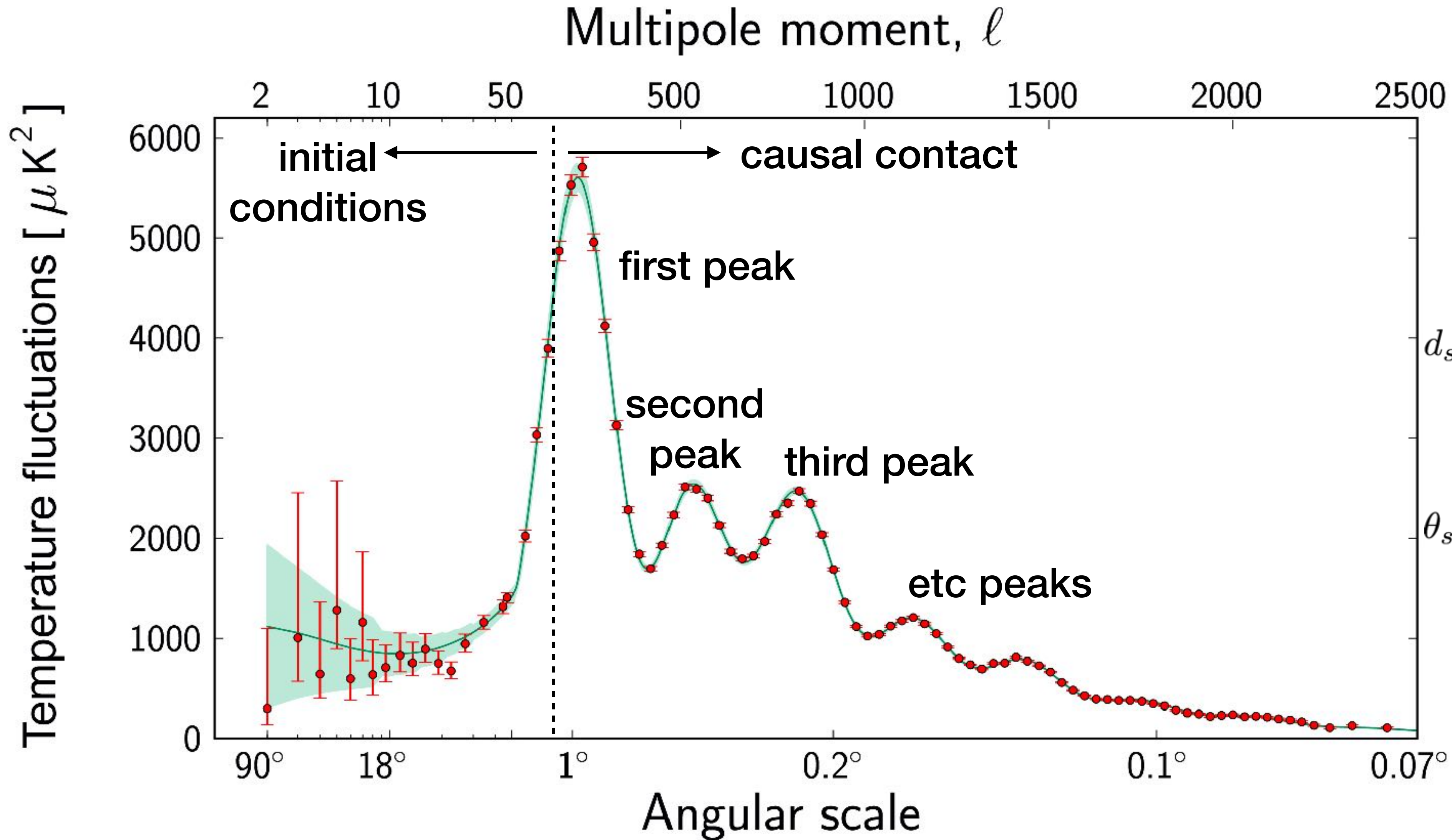


$$\varepsilon(\vec{r}) = \bar{\varepsilon} + \delta\varepsilon(\vec{r})$$

$$\nabla^2(\delta\Phi) = \frac{4\pi G}{c^2} \delta\varepsilon$$

$$\frac{\delta T}{T} = \frac{1}{3} \frac{\delta\Phi}{c^2}$$

# Acoustic peaks



$$d_s(t_{\text{ls}}) = a(t_{\text{ls}}) \int_0^{t_{\text{ls}}} \frac{c_s(t) dt}{a(t)}$$

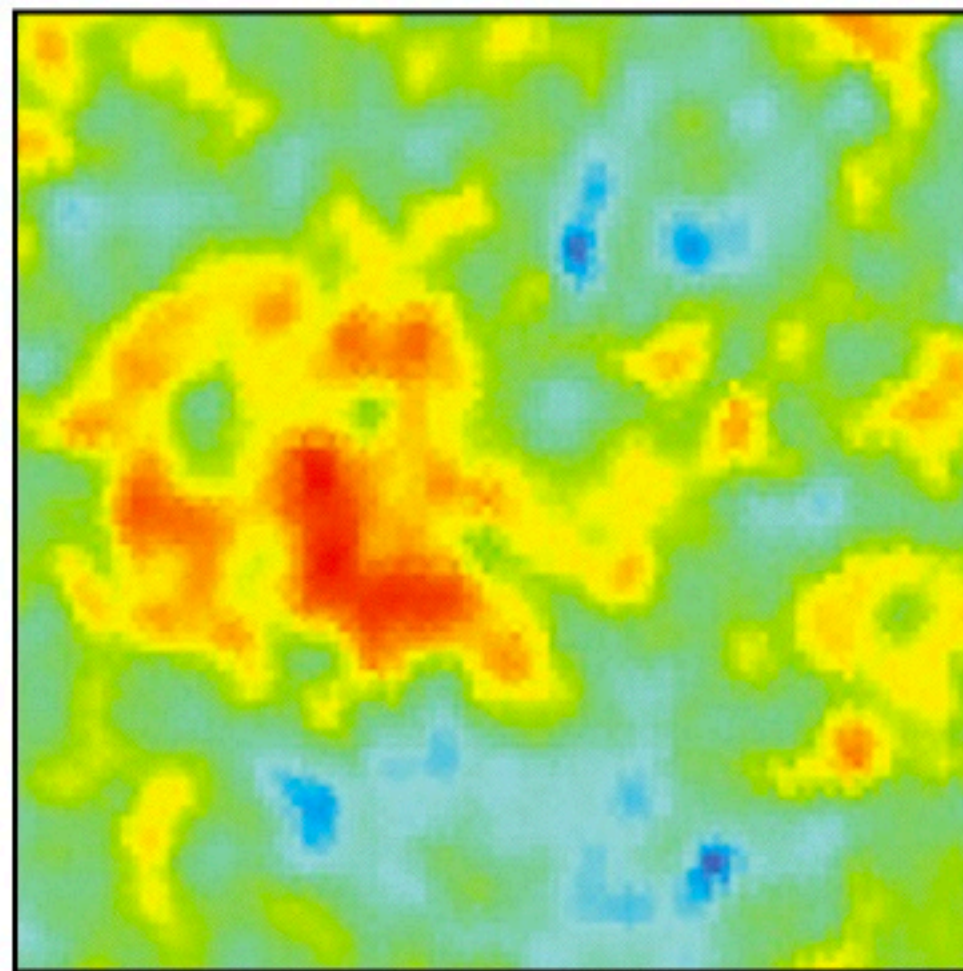
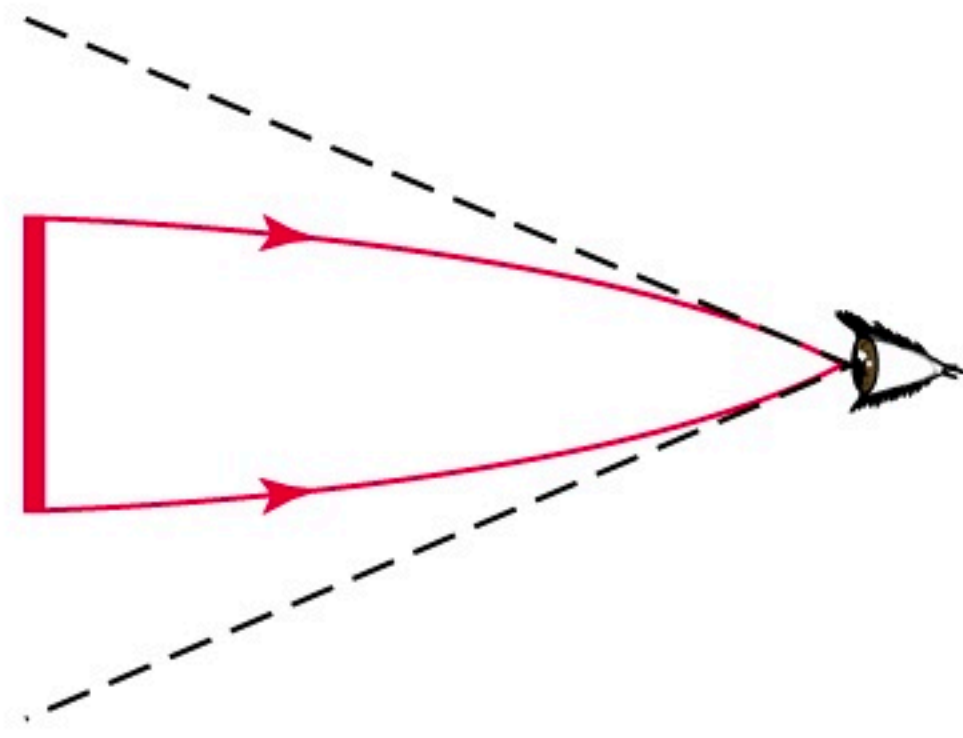
$$d_s(t_{\text{ls}}) \approx \frac{1}{\sqrt{3}} d_{\text{hor}}(t_{\text{ls}}) \approx 145 \text{ kpc}$$

$$\theta_s \approx \frac{d_s(t_{\text{ls}})}{d_A} \approx \frac{145 \text{ kpc}}{12.8 \text{ Mpc}} \approx 0.7^\circ$$

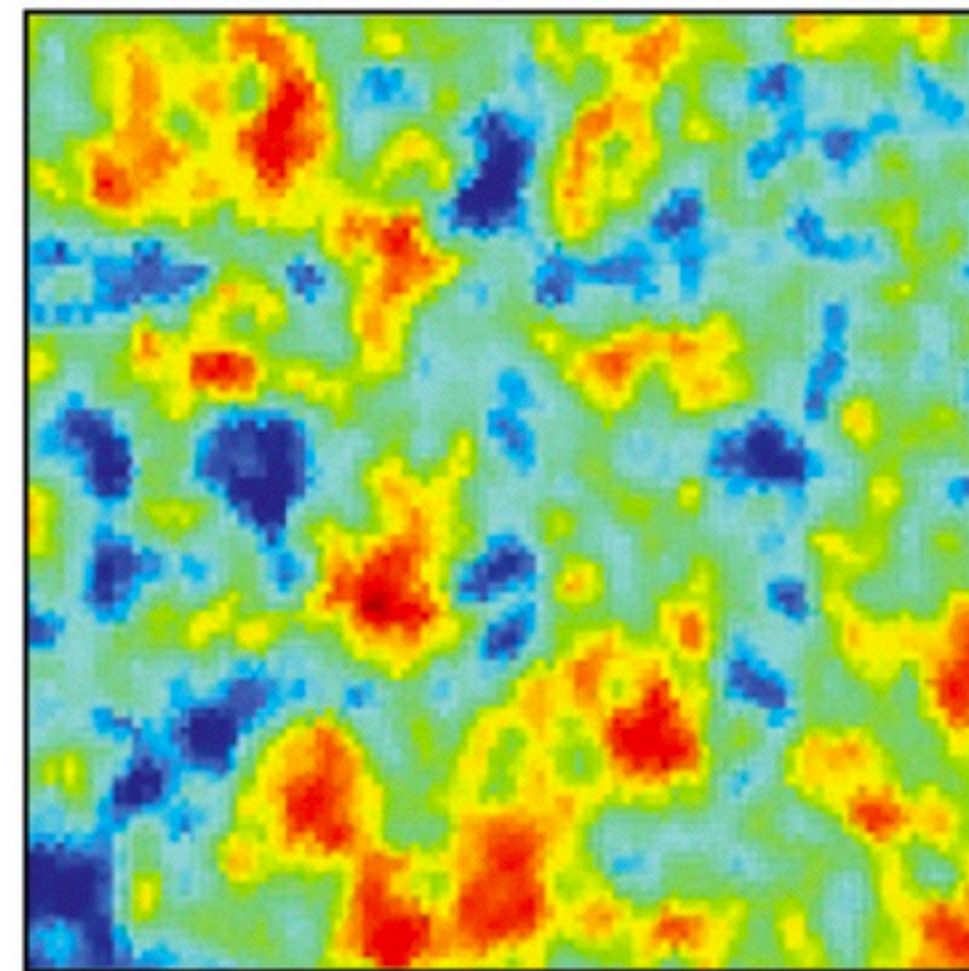
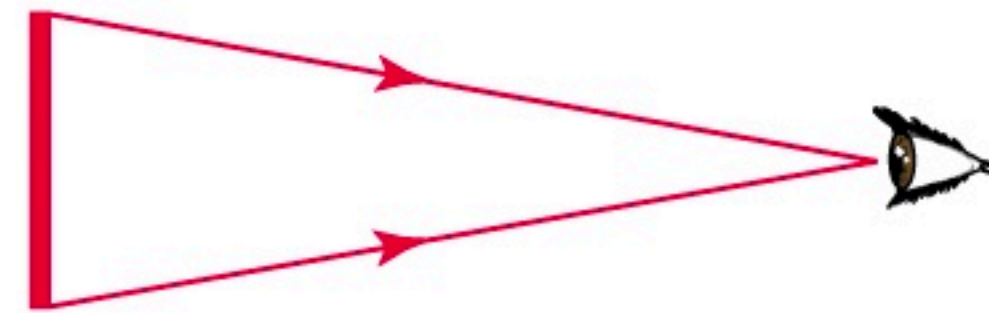
size scale of a DM potential well where baryon collapse reaches turnaround due to its pressure



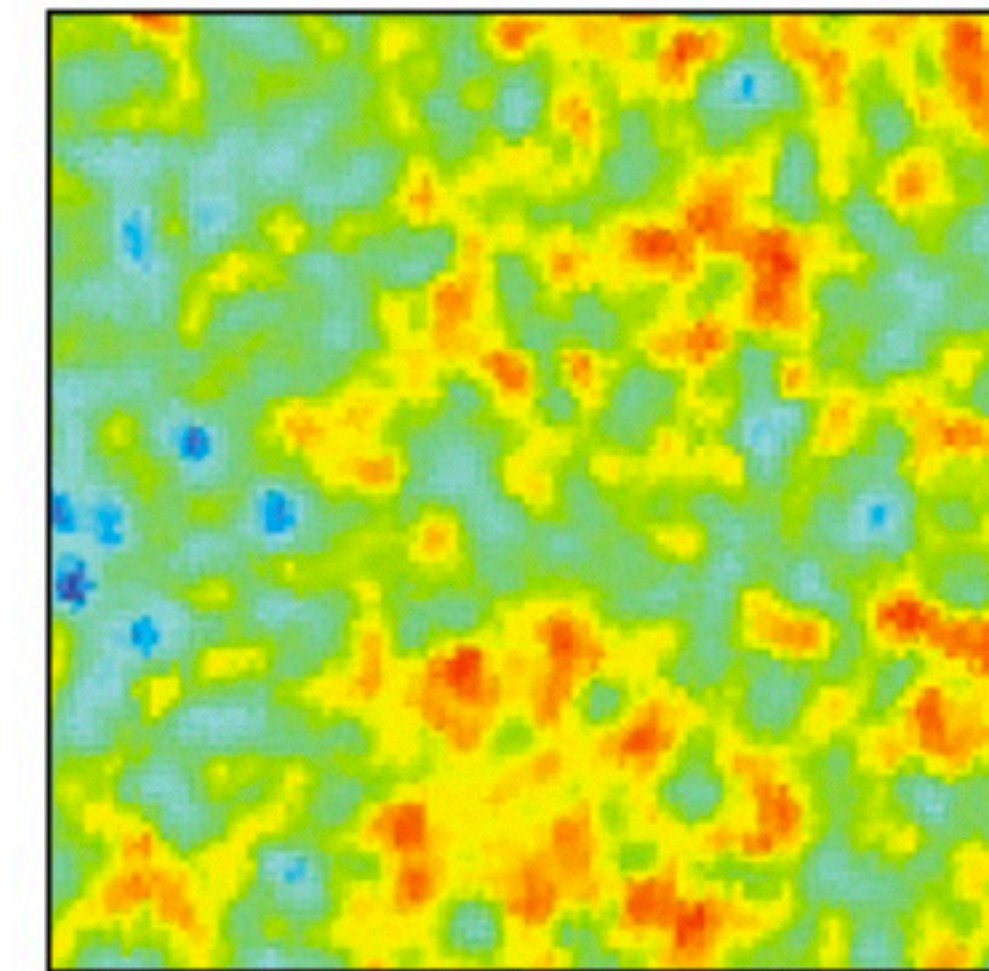
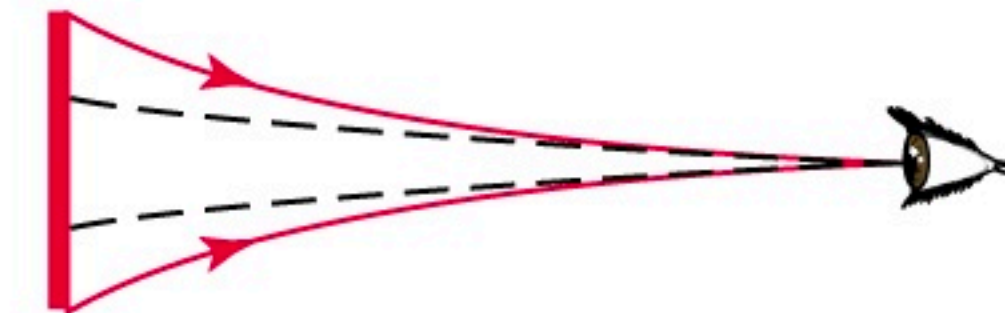
# CMB provides a giant triangle of known size!



a If universe is closed, "hot spots" appear larger than actual size



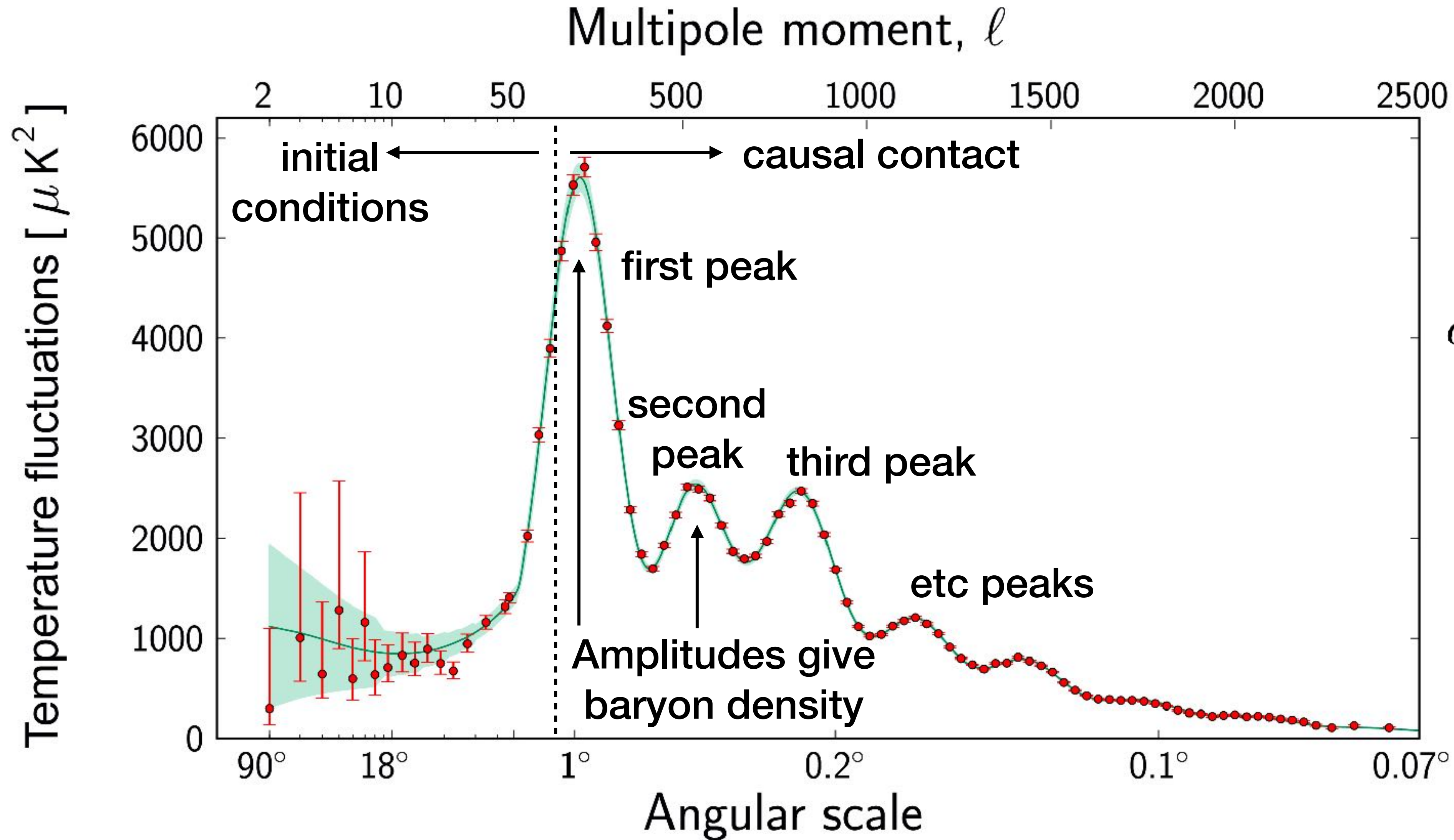
b If universe is flat, "hot spots" appear actual size



c If universe is open, "hot spots" appear smaller than actual size



# Acoustic peaks

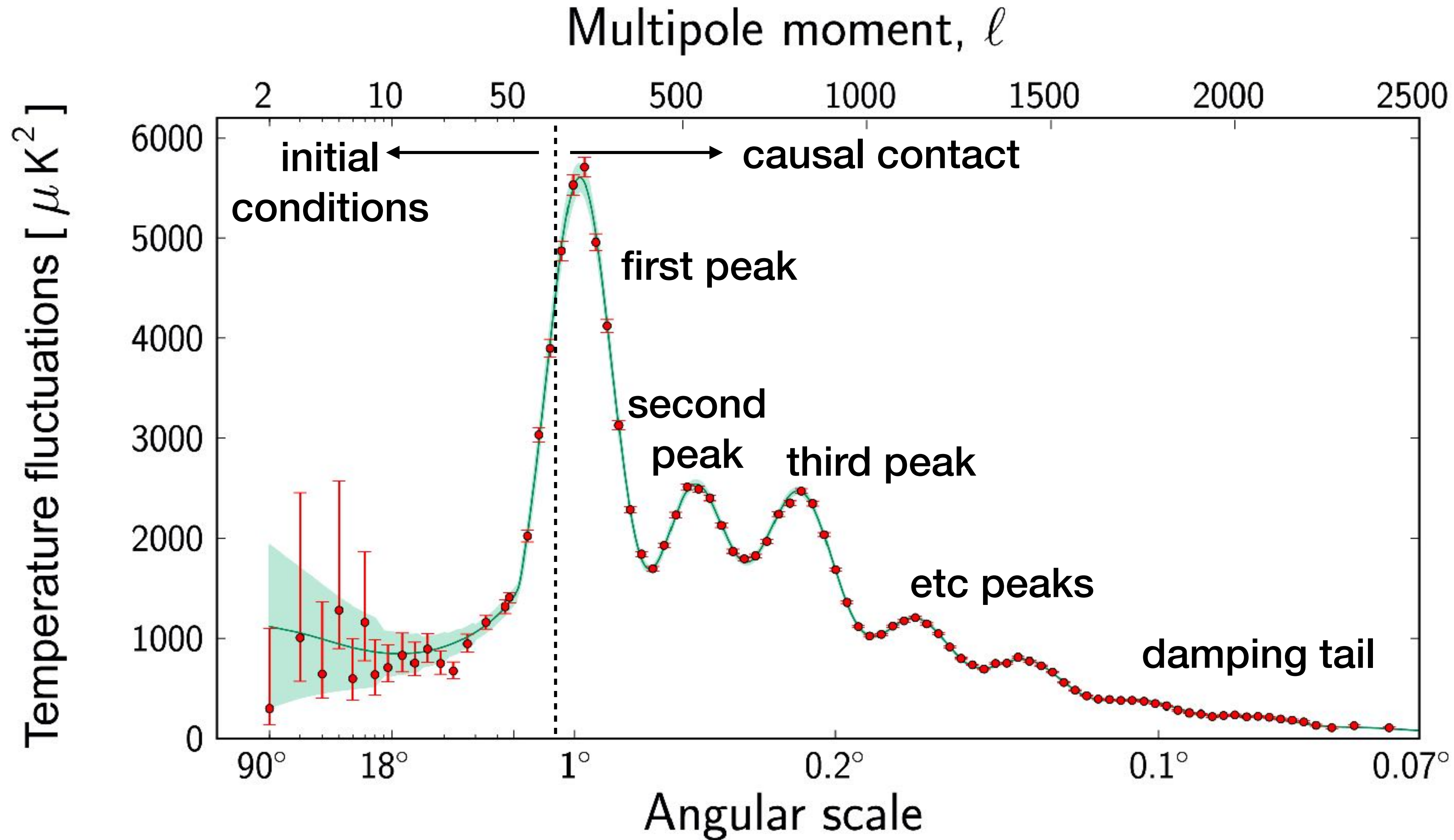


$$c_s = c\sqrt{w_{\text{pb}}}$$

$$\approx c\sqrt{\frac{1}{3}\left(1 - \frac{n_{\text{bary}}}{n_\gamma}\right)}$$



# Acoustic peaks



**First peak:**  
spatially flat

**Second peak:**  
existence of “dark baryons”

**Third peak:**  
amount of dark matter

**Damping tail:**  
photons can cross entire grav. fluct., wipes out signal

**[https://lambda.gsfc.nasa.gov/  
education/cmb\\_plotter/](https://lambda.gsfc.nasa.gov/education/cmb_plotter/)**



Fitting the power spectrum in detail yields narrow constraints

