ASTR 4080 - Week 9

Early Universe radiation dominated

→ how does ε_{rad} vary w/ a?

→ how does $a(t)$ vary w/ t?

$$a \propto t^{1/2}$$

→ how does $T(a)$ behave?

$$T \propto a^{-1}$$

$\boxed{a_{rm} \sim 3\times10^{-4}, \quad t_{rm} \sim 50\,\text{kyr}}$

RADIATION

$$T \approx 10^{10}\,K \left(\frac{t}{1s}\right)^{-1/2}$$

$$\langle E \rangle = 2.7\,kT(t)$$

$$kT \approx 1\,\text{MeV}\,(\;)$$

$$= 3\,\text{MeV}\left(\frac{t}{1s}\right)^{-}$$

☆ LHC → ~ 7×10^6 MeV [7 TeV]

↳ physics well-understood

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nomenclature: ^{A}E → $p = {}^1H$

$A =$ mass #

deuterium $D = {}^2H$

1n + 1p

nucleons → $Z = \#_p$, $N = \#_n$

$A = Z + N$

Li ⇒ $Z=3$, Be ⇒ $Z=4$

$^4He \to Z=2, N=2$

$\boxed{\text{☆ Go to slide on Nuclear Binding E}}$

@ $t \ll 0.1\,s$, quarks + antiq. created + destroyed, in balance

T falls until strong force takes over

@ $t \sim 0.1\,s$, nucleus have formed $(p + n)$

- however, neutrons outside nuclei are unstable + decay

$$n \to p + e^- + \bar{\nu}_e$$

$$N(t) = N_0 e^{-t/\tau_n}, \quad \tau_N = 880\,s$$

$\Rightarrow$ neutrons need to be in a nucleus to survive

$\to$ radiation $kT \sim 10$ MeV $> m_e c^2$

so $e^- + e^+$ still created/destroyed

$$\gamma + \gamma \rightleftharpoons e^- + e^+$$

ν_e + e^-/e^+ interact as well

$$n + \nu_e \rightleftharpoons p + e^- \;/\; n + e^+ \rightleftharpoons p + \bar{\nu}_e$$

B/c of all the interactions, all particles in equilibrium ($kT \sim 3$ MeV)

Can use prev. expression for particle ratios

$\boxed{\text{Next Slide}}$ $(m_n - m_p)c^2 = 1.3$ MeV

for $kT \gg 1.3$ MeV, $\frac{n_n}{n_p} \sim 1$

but declines as T drops

At some T, the n + p decouple b/c they require a ν_e mediator

→ ν interact via the weak force

$$\sigma_w \sim 10^{-47} m^2 \left(\frac{kT}{1\text{ MeV}}\right)^2$$

$\rightarrow$ $T \propto a^{-1} \propto t^{-1/2}$, so $\sigma_w \propto t^{-1}$

interaction rate $\Gamma = n_\nu \sigma_w c \propto t^{-3/2} t^{-1} = t^{-5/2}$

$H^2 = H_0^2 \Omega_{r,0} a^{-4}$, $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$ (since $n_\nu \propto a^{-3} \propto t^{-3/2}$)

$H^2 = H_0^2 \Omega_{r,0} a^{-4}$, $a(t) = \left(\frac{t}{t_0}\right)^{1/2}$ [rad.-dom. flat]

$H \propto a^{-2} \propto t^{-1}$

$H \propto t^{-1}$, so Γ will intersect H at which point ν no longer interact when this occurs ($\Gamma \approx H$), n/p can't find any ν to interact with, so even though there's enough energy to convert b/t them, this process stops

↳ ratio "freezes out"

$kT_{freeze} \sim 0.8$ MeV (9×10^9 K) : $t_{freeze} \sim 1$

$$\frac{n_n}{n_p} \approx e^{-1.29/0.8} = 0.2$$

$\tau_n = 880s$, so ratio holds for a few minutes

☆ Explains why universe mostly H → not enough neutrons to combine

p+n → just σ_{np} ‖ p+p → repel, must overcome Coulomb barrier (eff. reduced σ_{pp})

$p + p \rightleftharpoons {}^2He$ (diproton or He-2)

↳ $\tau \sim 10^{-23}$ s back to $p + p$

↳ $\tau_{weak} \gtrsim 10^{-2}$ s : $\rightarrow D + e^+ + \nu_e$
(actual value unknown)

→ this can produce D in the sun, but not enough time (< 1 hr) in BBN

left w/ 2 n for every 10 p → max ^4He that can be made (relative to H) is 1 ^4He for every 8 p

$$Y_{max} = \frac{\rho(^4He)}{\rho_{bary}} = \frac{4}{12} = \frac{1}{3}$$

$$Y_p(\text{measured}) = 0.24 < Y_{max} \checkmark$$

<u>Why?</u> – some n might decay if they don't find a p quickly enough
– may not end up in He: stuck in D
– may get fused in ↑ element (Li, Be)

Deuterium ++

$\boxed{t = 2s}$ $\quad \frac{n_n}{n_p} = 0.2 \quad Y_{max} = \frac{1}{3}$

ν's have decoupled, so can't be used, but γs still interacting

$$p + n \rightleftharpoons D + \gamma$$

$$B_D = (m_n + m_p - m_D)c^2 = \underline{2.22\,MeV}$$

$E_\gamma > 2.22\,MeV$ can destroy the D

☆ same process as w/ atoms & recombination

$\boxed{?}$ What equation determines their relative #s? Saha Eq.!

Not identical, but same form:

$$\frac{n_D}{n_p n_n} = 6\left(\frac{m_n kT}{\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_D}{kT}\right)$$

$\uparrow T$: free n & p $\left(\frac{n_D}{n_p n_n} \to 0\right)$

$\downarrow T$: D preferred exponentially

When does <u>nucleosynthesis</u> happen?

↳ isn't a single instant in time

Define as when $\frac{1}{2}$ n are in D, or

$$\frac{n_D}{n_n} = 1$$

→ move n_p to other side, just need an expression for n_p

$$n_p \approx 0.8\, n_{bary} = 0.8\, \underbrace{\eta\, n_\gamma}_{\text{know well}}$$

$$n_\gamma = 0.2436 \left(\frac{kT}{\hbar c}\right)^3$$ → integral of BB equation

$$\Rightarrow \quad \underbrace{\frac{n_D}{n_n} \approx 6.5\,\eta \left(\frac{kT_{nuc}}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT}\right)}_{=1}$$

$$T_{nuc} \approx 7.6\times10^8\,\mathrm{K} \,/\, \frac{66\,\mathrm{keV}}{k} \,/\, B_D/34$$

↳ occurs @ $\underline{t = 200\,\mathrm{s}}$

200s is <u>not</u> $\ll \tau_n = 880s$

Let's ignore the gradual creation of D & assume all forms @ 200s

↳ $\#_n$ will have decreased

@ ~1s, have $\frac{n_n}{n_p} = \frac{1}{5}$

ratio changes as $n \to p + e^- + \nu_e$

$$f = \frac{n_n + \frac{\partial n}{\partial t}}{n_p + \frac{\partial p}{\partial t}}, \quad \frac{\partial n}{\partial t} = -(n_n - n_n e^{-t/\tau_n}), \quad \frac{\partial p}{\partial t} = -\frac{\partial n}{\partial t}$$

$$f = \frac{n_n e^{-t/\tau_n}}{n_p + n_n - n_n e^{-t/\tau_n}} = \frac{e^{-t/\tau_n}}{n_p/n_n + 1 - e^{-t/\tau_n}}$$

$t = 200$, $\tau_n = 880$, $\frac{n_p}{n_n} = 5$

↳ ≈ 0.15

Est. Y_{max} was too high, should be

$$Y_{max} = \frac{0.15}{0.2}\,\frac{1}{3} \approx \frac{1}{4}$$

$\boxed{\text{D to slides}}$