Fun Group Problems

Form 4 groups, 2 of undergrads and 2 of grad students. Each group should tackle 1 problem, one of the U or G problems for the undergrad and grad groups respectively.

Please work together: first discuss approaches to the problem and what concepts apply to the situation. Draw pictures or diagrams to illustrate the problem, break it down into parts, explicitly list your assumptions, and use that information to derive mathematical expressions that can be used to calculate estimates.

1. (U) Explosions in the Galaxy, caused by supernovae or compact object outbursts, can create "light echoes" or "dust scattering halos," where surrounding clouds light up days, months, or even years after the light from the explosion itself is observed. What is physically happening (in a geometric sense), and what would observations of this phenomena look like? What can be learned from such observations, either about the explosion or the reflecting environment? Derive an expression for physical distances in terms of observed quantities (angular separation, time delays, etc.). Can the distance to the explosion, if unknown, be derived from such data? Are certain assumptions required to do so?

2. (U) Imagine a dark star with a solid surface with mass M and radius R that is accreting matter falling on it from all directions at some rate $\dot{m}$. Derive 2 expressions for the maximum luminosity of the star as a function of $\dot{m}$ and, in the first case, the velocity of impact of the particles, and, in the second case, M and R. What assumptions have you made to derive these expressions? If the luminosity is high enough, it can prevent material from accreting onto the star; explain why this is the maximum luminosity the star can obtain. By balancing inflow and outflow forces, this luminosity can be derived: attempt to do so!

3. (G) White dwarfs and neutron stars are supported by degeneracy pressure, which is essentially the maximum amount electrons or neutrons can be compressed in a given volume. Derive an expression for the radius of a star supported by degeneracy pressure and estimate the radius of a white dwarf or neutron star under simple assumptions. Why might this result differ from actual sizes of these stars?

4. (G) About 160 kya, a supernova occurred in the Large Magellanic Cloud we call 1987A. In a deep mine neutrino facility in Japan, where flashes could be detected from interactions of neutrinos with 3,000 tons of water. Out of 10^{58} neutrinos estimated to be created in the supernova, 12 were detected. What was the detection efficiency of the facility? What assumptions do you have to make this calculation? The neutrinos arrived within a 15 s window and had a range of energies: 7.5-35 MeV. How can this information be used to place a limit on the mass of neutrinos? What is the limit?