Homework 1

Due **January 21 at 2pm in class**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. A heavy ion of charge Z moves at a velocity v through a cold, neutral gas. The ion moves rapidly but is nonrelativistic. Let $\partial^2 N(E)/\partial E \partial x$ be the number of ionized electrons produced per unit length x of the path of the ion and per unit electron energy E. (The electron energy includes the ionization energy I necessary to free the electrons.)

   (a) Show that the energy spectrum of ionized electrons is

   $$\frac{\partial^2 N(E)}{\partial E \partial x} = \frac{2\pi Z^2 e^4 n_e}{m_e v^2 E^2} \, .$$

   (b) Integrate this result to give the previous result for the total ionization loss of the heavy ion, dE/dx.

2. A 1 GeV cosmic ray proton (kinetic energy = 1 GeV) moves on a straight path through the interstellar medium (ISM) of our galaxy. Assume that the ISM is mainly neutral hydrogen gas and has an average density of 1 atom per cm^3. Estimate the distance the proton travels (in pc) before it loses most of its energy to ionization losses in the ISM.

3. Use Figure 15.1 (`www.physics.utah.edu/~whanlon/spectrum.html`) in the text to determine the flux of cosmic ray protons at the Earth. Assume this same flux is typical throughout the disk of our galaxy.

   (a) Estimate the kinetic energy density in cosmic rays in the disk of our galaxy, in $\mathrm{eV/cm}^3$.

   (b) Estimate the rate of ionization of per hydrogen atom in ISM of our galaxy by cosmic rays. Give the rate as ionizations/second/(interstellar hydrogen atom).

4. Ultra-high energy cosmic rays (UHECRs) have energies which exceed 10^{20} eV. It is remarkable that these elementary particles have a macroscopic amount of energy.

   (a) For comparison, calculate the kinetic energy (in eV) of a baseball (mass ≈ 140 g) thrown at 80 km/hour (roughly 50 mph)?

   (b) Why aren't people or other object knocked down or damaged by UHECRs?

   (c) UHECRs can lose energy by interacting with photons. In the rest frame of the UHECR, even photons from the Cosmic Microwave Background (CMB) will appear as very energetic photons, and interactions between these photons and UHECRs can produce pions and other particles. Consider a typical CMB photon with an energy of 0.001 eV. If the photon is moving directly toward a UHECR proton with an energy of 10^{20} eV in the opposite direction of the CR's velocity, estimate the energy of the photon as seen in the UHECR's rest frame (in MeV)?

5. It is known that all of the space between the galaxies in clusters of galaxies is full of very hot gas. Typically, there is about 10^{14} $M_\odot$ of this gas, at a temperature of $T \approx 7 \times 10^7$ K and an electron density of $n_e \approx 10^{-3}$ cm^{-3}. The primary emission from such gas is thermal bremsstrahlung. Assume the gas is mainly ionized hydrogen.

   (a) In what part of the spectrum (radio, infrared, optical, ultraviolet, X-ray, gamma–ray) is most of the energy from this gas radiated?

   (b) Estimate the luminosity of the thermal bremsstrahlung emission from this gas.