Homework 3

Due **January 30 at 2pm in class**
Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Use the same dipole model for the magnetosphere of a planet or star as in Problem 2.4. Consider a charged particle which starts on the magnetic equatorial plane of the body, with a pitch angle α and a radius from the center r_o. Assume that the radius from the center of the body always satisfies $r \gg r_c$, where r_c is the cyclotron radius. Assume that the particle follows field lines in its motion. Let R be the radius of the surface of the spherical planet or star.

    (a) Given that the particle hits the surface of body, at what latitude ($l = \pi/2 - \theta$) does it hit?

    (b) If $r_o = 2R$, what is the maximum value of the pitch angle such that the particle hits the body? At what latitude l does it hit?

2. The strength of the average dipole magnetic field at the surface of the Sun is about $B_\odot \approx 2$ Gauss. Assume that this is the rms field (square root of the average of B^2). Assume this field remains frozen–in to the plasma in the Sun during its evolution. (In this problem, ignore the fact that the field is much stronger in active regions, and the possibility that fields beneath the solar surface may be much stronger.)

    (a) Estimate the dipole moment μ (gaussian units) of the field today.

    (b) Estimate the values of B and μ if the Sun collapsed to become a white dwarf star (same mass, radius of 10^9 cm).

    (c) Estimate the values of B and μ if the Sun collapsed to become a neutron star (same mass, radius of 10^6 cm).

3. Consider a strong (Mach number $\mathcal{M} \gg 1$), planar, perpendicular shock propagating into an unmagnetized, $\gamma = 5/3$ gas.

   (a) Derive the relation between the post-shock temperature T_s and shock velocity v_s for this strong shock.

   (b) What is the post-shock temperature for strong shock with a velocity of $v_s = 1000$ km/s moving into an ionized hydrogen plasma with $n_e = n_p = 1\ \text{cm}^{-3}$ on the assumption that the protons and electrons are heated to equal temperatures at the shock?

   (c) What is the temperature in part (b) if the protons are heated but the electrons are not?

4. Imagine that you are making a series of i measurements, where each data point D_i consists of the number of events recorded. For example, you want to measure the count rate of a radioactive source, so each D_i is the number of counts detected in some interval of time. Let's assume half of the data points lie above the true value, M_t, by an amount σ, and the other half of the measurements lie below M_t by the same amount so that the average of all measurements is equal to M_t. Because measurements are in terms of counts $C = D_i$, the uncertainty in D_i can be estimated as $\sqrt{C}$.

   (a) Show that $\chi^2 = \frac{NM_t}{M_t^2-\sigma^2}\left[(M_t - M)^2 + \sigma^2(\frac{2M}{M_t} - 1)\right]$, where N is the number of measurements and M is the model being evaluated in order to find M_t.

   (b) Using χ^2, derive an expression for the best-fit value of M in terms of M_t and σ. Explain qualitatively what causes $M \neq M_t$.

   (c) For models that do a good job of describing data, the typical difference between a given D_i and M is the uncertainty on D_i. Describe how you could mitigate the bias on M by changing the bin size of D_i. Would using the alternate definition of $\chi^2 = \sum_i (D_i - M_i)^2/M_i$ alleviate the bias or change its sense? Explain.