Homework 6

Due **March 5 at 2pm in class**

Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. A neutron star has a fixed moment of inertia I, and rotates with a period P. It is slowing down at a rate given by $\dot{P} = dP/dt > 0$
   (a) What is the rate at which the rotational kinetic energy of the neutron star is being reduced?
   (b) What is the rate of energy loss for a neutron star with $I = 10^{45}$ g cm^2, $P = 0.033$ seconds, and $\dot{P} = 4 \times 10^{-13}$ (the values for the Crab pulsar)?

2. A pulsar is a rotating neutron star with a fixed moment of inertia I, and a rotation period P. Assume that the pulsar can be represented as a rotating magnetic dipole, with α being the constant angle between the magnetic dipole axis and the rotation axis of the neutron star. Let μ be the magnetic dipole moment of the neutron star, and assume this is constant in magnitude.
   (a) Use the dipole radiation formula (substituting the magnetic dipole moment for the electric dipole moment), and give a formula for the rate of energy loss of the pulsar.
   (b) Assume the source of this energy is the rotational kinetic energy of the neutron star. Using the result from problem (1), find the rate of slowing down of neutron star rotation $\dot{P}$.
   (c) If $I = 10^{45}$ g cm^2, $P = 0.033$ seconds, and $\dot{P} = 4 \times 10^{-13}$, and $\alpha = 90°$, estimate the value of μ implied. If the neutron star has a radius of 10 km, roughly what is the average surface magnetic field?

3. Assume a pulsar is born at time $t = 0$ with a rotation period P_o.
   (a) Using the formula given in Problem (2b) for $\dot{P}$ for a pulsar with a fixed I, μ, and α, show that $P\dot{P}$ = constant.
   (b) From the result of Problems (2b) and part (a) of this problem, find the value of the rotation period $P(t)$ at any later time t.
   (c) The "braking index" n of a pulsar is defined as $n \equiv 2 - \ddot{P}P/\dot{P}^2$. In the magnetic dipole model for the rate of slowing down of pulsars given above, what is the value of the braking index n?

4. Outline the scope of your project (i.e., describe all the steps you will take—can be vague, such as "install XXX software, download the data on object XXX, process data," etc.) and complete the first step.