Homework 8

Due **April 16 at 2pm via Canvas upload**
Please show all work, writing solutions/explanations clearly, or no credit will be given. You are encouraged to work together, but everyone must turn in independent solutions: do not copy from others or from any other sources.

1. Consider a high mass X-ray binary system containing an O star with a mass of M_1 and a compact object (neutron star or black hole) of mass M_2. Assume that the orbit is circular, and that the total orbital velocity of the two stars is $v_{\rm orb}$. Assume that the O star is producing a steady stellar wind with a mass loss rate of $\dot{M}_w$. Assume the wind is spherically symmetric, and that it has reached its terminal speed of v_w by the time it reaches the compact object. Assume that the wind is highly supersonic at this point. Assume that the compact object accretes gas from the wind with a rate given by the Bondi accretion formula.

   (a) Show that the accretion rate by the compact object can be written as

   $$\dot{M}_2 = \left(\frac{M_2}{M_1 + M_2}\right)^2 \left(\frac{v_{\rm orb}}{v_w}\right)^4 \dot{M}_w \, .$$

   (b) The high mass X-ray binary Cyg X-1 contains an O9.7 Iab supergiant star with a mass of about 33 $M_\odot$ and a black hole with a mass of about 16 $M_\odot$. The orbital period is 5.6 days. Assume the O supergiant has a wind with $\dot{M}_w \approx 10^{-6}\, M_\odot$/yr and a velocity of about 1000 km/s. Estimate the accretion rate by the black hole $\dot{M}_2$ in $M_\odot$/yr.

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2. A black hole with a mass of 10 $M_\odot$ is accreting matter through an accretion disk. The inner radius of the accretion disk is the radius of the last stable orbit (three Schwarzschild radii). The black hole accretes the angular momentum of the material in this last orbit (that is, $\beta = 1$). The accretion rate is the Eddington rate for the black hole. **(a) What is the accretion rate (in $M_\odot$/yr)?** Assume the viscosity is given by the "α–disk" theory with $\alpha = 1$. Assume that gas pressure is greater than radiation pressure. Estimate

   (b) the disk temperature T (in K),

   (c) the disk Keplerian velocity v_ϕ (in km/s), and

   (d) the disk thickness h (in cm),

   all at a radius of $r = 10^8$ cm from the black hole.

3. The conventional theory for quasars is that they are powered by accretion onto massive black holes at the centers of galaxies. Assume a quasar contains a black hole with a mass of $10^9\ M_\odot$ which is accreting matter through an accretion disk. The inner radius of the accretion disk is the radius of the last stable orbit (three Schwarzschild radii). The black hole accretes the angular momentum of the material in this last orbit (that is, $\beta = 1$). The accretion rate is the Eddington rate for the black hole.

   (a) What is the accretion rate (in $M_\odot$/yr)?

   (b) What is the luminosity produced by this accretion (in ergs/s)?

   (c) What is the temperature (in K) in the accretion disk at a radius which is twice the inner radius of the disk?

   (d) In what part of the spectrum (radio, infrared, optical, UV, X-ray, or γ–ray) would most of the disk luminosity appear?