ASTR 5590 - Black Holes

☆ Refer to review sheets on BHs

Classically defined as having an escape velocity > speed of light

$$E_{esc} = \frac{1}{2} m v_{esc}^2 - \frac{GMm}{r} = 0$$

$$v_{esc} = \sqrt{\frac{2GM}{r}} = c \quad \text{(for a BH)}$$

$$\text{so} \quad r_S = \frac{2GM}{c^2}$$

→ light can't escape if inside this radius, so object smaller than this size must be <u>black</u>

Just happens to be radius of such an object in GR, derived from the metric: $R_S = \dfrac{2GM}{c^2} = 3\,\mathrm{km}\left(\dfrac{M}{1M}\right.$

Schwarzschild Radius for a point mass

In GR, R_s applies to radius at which spacetime curvature prevents particles from moving radially away

$$ds^2 = \alpha\, dt^2 - \frac{1}{c^2}\left[\alpha^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$$\alpha = 1 - \frac{2GM}{rc^2}$$

$ds \rightarrow$ distance in space-time b/t 2 events (invariant regardless of definition of coords)

Special Relativity, use the Minkowski metric

$$ds^2 = dt^2 - \frac{1}{c^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Far from the point mass, $r \rightarrow \infty$, $\alpha \rightarrow 1$ & metrics are the same

Nearer the mass, the metrics give

different answers

- if someone @ r & not moving
  ($dr = d\theta = d\varphi = 0$)

  the distance b/t 2 events (i.e., time)

  is $ds = \alpha^{1/2} dt = dt'$

  observer @ r ↗ ↖ observer @ $r \to \infty$

Gravitational field affects time experienced (rel. to someone far aw

↳ gravitational time dilation

$$\Delta t' = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} \Delta t$$

emitted @ r   received @ ∞

$$\nu_\infty = \nu_{em}\left(1 - \frac{2GM}{rc^2}\right)^{1/2}$$ grav. redshift

SKIP?

- now consider inclusion of motion in radial direction ($d\theta = d\varphi = 0$)
  (free fall towards M)

Proper time is [$dt \to r @ \infty$, dt' is @ r]

$$ds = d\tau = dt\left[\alpha - \frac{1}{c^2\alpha}\left(\frac{dr}{dt}\right)^2\right]^{1/2}$$

$$= dt' \frac{1}{\alpha^{1/2}} \left[\alpha - \frac{1}{c^2 \alpha} \frac{dr^2}{\frac{1}{\alpha} dt'^2} \right]$$

$$= dt' \left[1 - \frac{dr^2}{c^2 \alpha} \frac{1}{dt'^2} \right]^{1/2}$$

An increment of "geodesic distance" is actually $dx = \alpha^{-1/2} dr$

$$d\tau = dt' \left[1 - \frac{1}{c^2} \frac{dx^2}{dt'^2} \right]^{1/2} = dt_\infty$$

v^2 : speed person @ r is moving thru S (∞ frame)

Lorentz factor γ

↳ time dilation

If $r \leq r_g = \frac{2GM}{c^2}$, $\nu_\infty \to 0$

☆ no information can be transfered
— also no light

GR also tells us that KE of a particle contributes to the grav force felt, so a particle w/ ang. mom. close to a BH will eventually reach distance where this "extra gravity a ...

becomes larger than the centrifugal force, so it must fall in if

$$\ell = m v_\phi r \le 2 r_g c \cdot m$$

Last stable circular orbit @ $r = 3 r_g$

There is <u>NO</u> singularity @ r_g : just looks that way since choice of coords in metric

There <u>IS</u> a physical singularity @ $r = 0$

↳ likely a conseq. of GR not being quantized

HERE

Real BHs are almost certainly rotating described by the Kerr metric

Parameterize ang. momentum J by

$$a = \frac{J}{Mc} \quad (\text{units of distance})$$

event horizon becomes

$$r_+ = \frac{GM}{c^2} + \left[\left(\frac{GM}{c^2}\right)^2 - a^2\right]^{1/2}$$

Maximally spinning BH has $a = \frac{GM}{c^2}$

[illegible]

[Observationally, "a" is in units of $\frac{GM}{c^2}$
so its maximum value is 1]

half the size of a non-spinning BH

"Frame dragging" → particles are forced
to move in direction of spin

B/c event horizon smaller, particles can
orbit closer, thus more of their
rest mass E can be radiated during inf.

→ non-rotating: ~6% of mc^2
(binding E of particle @ $r = 3r_s$)

→ for particles orbiting in same direction
as rotation, can get ~40% of mc^2
for $a = 1$ spinning BH

Inside the "static" radius, where particles
cannot have $d\theta = 0$, is called <u>ergosphere</u>

– if a particle ends up on a
"negative energy" orbit inside
Then splits into 2 w/ one falling in
& the other escaping, the latter

[illegible]

particle can leave w/ more E
than the original particle fell in with

↳ garbage dump power for an advanced alien civ. → gain up to 30% of mc^2

↳ called the Penrose process

Hawking radiation: $T = \frac{\hbar c^3}{8\pi G M k} \approx \frac{10^{-7}}{(M/M_\odot)}\,K$

→ merging QM near event horizon