ASTR 5590 - Accretion I

Accretion is essentially the process of a particle falling onto an object from very far away, liberating the KE it picked up from its initial gravitational PE

A proton falls from ∞

$$KE = PE$$

$$\frac{1}{2} m_p v_{ff}^2 = \frac{G M m_p}{r}$$

proton radiates KE away thru heat if it stays on the star

Imagine a stream of protons falling w/ some rate : $\dot{m}$

Energy per time, equivalent to lum.

of accretion:

$$L_{acc} = \frac{1}{2}\dot{m}v_{ff}^2 = \frac{GM\dot{m}}{R}$$

Put in terms of $r_g = \frac{2GM}{c^2}$

$$L_{acc} = \frac{1}{2}\dot{m}c^2\left(\frac{r_g}{R}\right)$$

which can be parameterized as

$$L_{acc} = \xi \dot{m} c^2$$

ξ (squiggle) $\rightarrow$ efficiency of conversion of KE in terms of rest mass E of particles (not actual conversion like in fusion)

for reference, H $\rightarrow$ He fusion, $\xi \sim 7\times10^{-3}$

Accretion onto WD, NS, give

$$\xi = \frac{r_g}{2R} = \frac{GM}{c^2R}$$

WD: $M \sim M_\odot$ $\quad \xi \sim 3\times10^{-4}$
$R \sim 5000\,\text{km}$

NS: $M \sim M_\odot$ $\quad \xi \sim 0.15$
$R \sim 10\,\text{km}$

↳ much more efficient E release than nuclear fusion!

For BHs, no surface where KE can be dissipated, but will be along the way thru interactions that remove angular momentum

$$E_{rot} = \frac{1}{2} I \Omega^2 \,; \quad L = I\Omega = \text{const.}$$

$$= \frac{1}{2} L^2 / I \,; \quad I = mr^2$$

$$\propto r^{-2}$$

$$E_{grav} = \frac{GMm}{r} \propto r^{-1}$$

so as $r \to 0$, $E_{rot} > E_{grav}$

even if E_{rot} starts out small

Ultimately, E lost until LSCO, results in $\xi \sim 0.06$ (non-rotating Schwarzschild)

~ 0.43 (max. rotating)
Kerr

In principle, could liberate *bleep* tons of E this way, but there's a limit: What? ☆

Radiation produced by infalling stuff exerts pressure on stuff falling behind it, which will prevent material from continuing to fall in, so $\dot{m}$ will be reduced

Balance grav. force & radiation force

$$f_{grav} = \frac{GM}{r^2}\left(m_p + \cancelto{0}{m_e}\right)$$

Radiation pressure most effective acting on e^- (Thomson scattering) & b/c plasma neutral (EM force much stronger than grav) on scales larger than Debye length

What is this?

e^- will pull p^+ with it

So, a single photon transfers a momentum of $p = h\nu/c$ to e^-

force is momentum per time:

$$f_{e^-} = R_{ph} \cdot p$$ hitting e^- in some time

R_{ph} = # photons per time

Have a flux of photons N_{ph} (# per area per time)

Then $N_{ph}\,\sigma_T$ = # ph. per time hitting e^-

$= R_{ph}$

If E liberated @ surface of object,

then $$F_{ph} = \frac{L}{4\pi R^2} = N_{ph}\, h\nu$$

So the total force

$$\sum f_{e^-} = N_{ph}\,\sigma_T\, p = N_{ph}\,\sigma_T\, h\nu/c$$
$$= \frac{L\,\sigma_T}{4\pi c R^2}$$

And the maximum possible L is when this equals f_{grav}

$$\frac{GMm_p}{R^2} = \frac{L\sigma_T}{4\pi c R^2}$$

$$L_{Edd} = \frac{4\pi c m_p GM}{\sigma_T}$$

$$= \frac{2\pi r_g m_p c^3}{\sigma_T}$$

$$= 1.3\times10^{31}\,W\left(\frac{M}{M_\odot}\right)$$

$$= 1.3\times10^{38}\,erg/s\left(\frac{M}{M_\odot}\right)$$

Assumes spherically symmetric accretion, but is a decent rule of thumb

Most sources stay below this value

Fig. 14.2

HERE

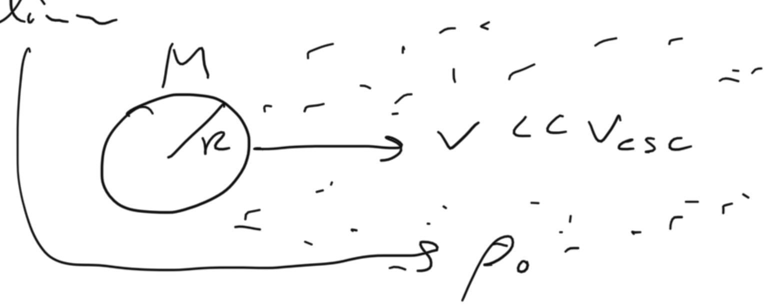
$$L_{Edd} = \frac{2\pi r_g m_p c^3}{\sigma_T} = 1.3\times10^{38}\,erg/s\left(\frac{M}{M_\odot}\right)$$

$$= \xi \dot{M}_{Edd} c^2$$

so $\dot{M}_{Edd} = \frac{2.2\times10^{-9}}{\xi}\left(\frac{M}{M_\odot}\right) M_\odot\,yr^{-1}$

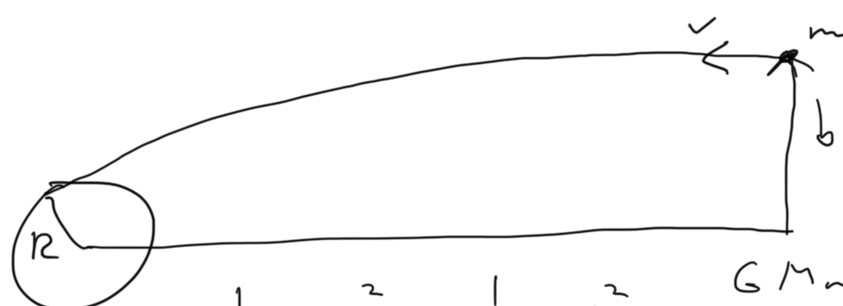
Bondi Accretion

Imagine a compact object (star) travelling thru a medium



Assume fluid w/ density ρ_0 is collisionless

E-conservation: $KE_i = KE_f + PE$



$$\frac{1}{2}mv^2 = \frac{1}{2}mv_f^2 - \frac{GMm}{R}$$

$L = mvb = mv_f R$

↳ defines cone w/in which star accumulates stuff from ρ_0

$A = \pi b^2$

so, mass accretion rate is just density of material times the volume of cylinder traversed per unit time

$$\dot{M} = \rho_0 v A = \pi b^2 v \rho_0$$

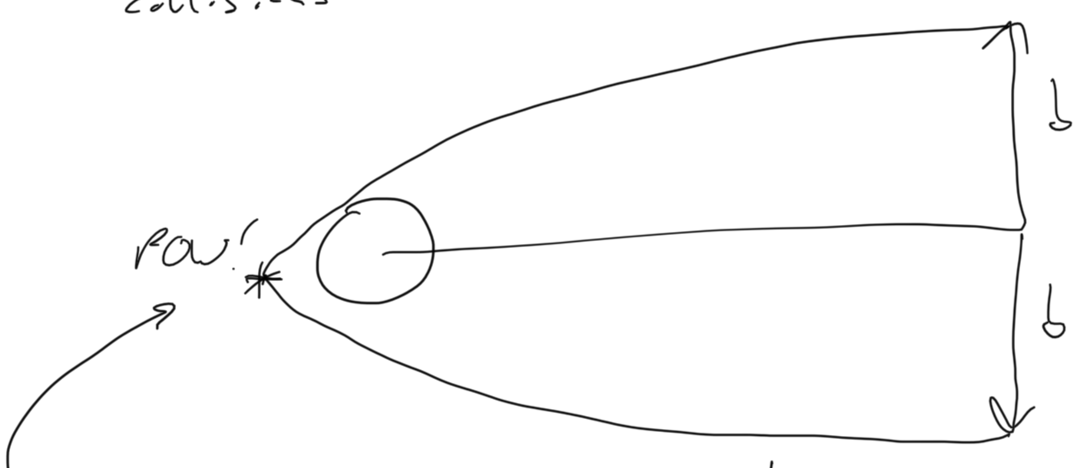
$$\frac{1}{2} m v^2 = \frac{1}{2} m v^2 \left(\frac{b}{R}\right)^2 - \frac{GMm}{R}$$

$$b^2 = R^2 + \frac{2GMR}{v^2} \simeq \frac{2GMR}{v^2}$$

for compact, massive objects

so $$\boxed{\dot{M} = \frac{2\pi GMR}{v} \rho_0}$$

In the collisional fluid case, particle collisions enhance accretion



most accretion occurs here

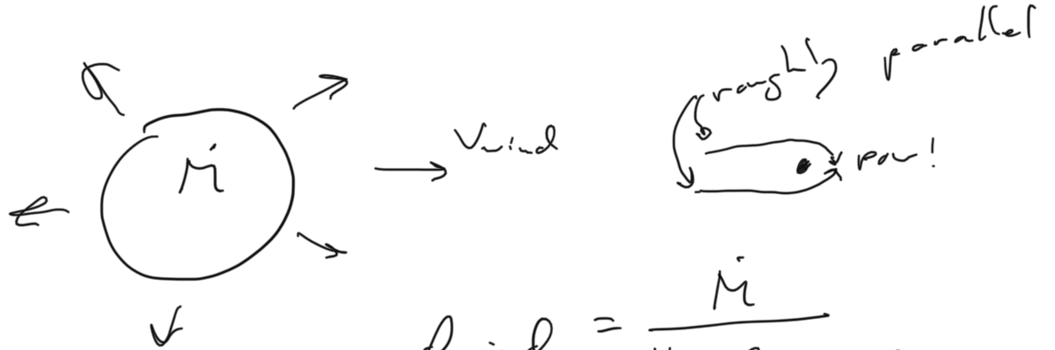
$$\dot{M} = \frac{4\pi G^2 M^2}{(\ldots)^{3/2}} \rho_0$$

<u>Bondi</u>

Accr

$$\boxed{\cdots \quad (v^2 + c_s^2) \quad \cdots}$$ Acc.

One way for a compact obj. in a binary to accrete from the wind of the companion



$$\rho_{wind} = \frac{\dot{M}}{4\pi r^2 v_{wind}}$$