ASTR 5590 - Accretion II

We'll skip a lot of the details, so if you're interested in the detailed structure & physics of discs, see 14.3 & 14.4

Thin Discs

$$M_{disc} << M_s \quad , \quad \frac{\partial p}{\partial z} \approx \frac{p}{H}$$

pressure

thickness

Hydrostatic Pressure Support

$$\frac{\partial p}{\partial z} = - \frac{G M_s \rho \sin\theta}{r^2} \quad \sim z/r \quad \sim H/r$$

Gas moves on Keplerian orbits @ any radii losing small E & moving into a ↓r circular orbit

$$\frac{v_\phi^2}{r} = \frac{G M_s}{r^2}$$

centrifugal force = grav. force

$$\frac{p}{H} = \frac{v_\phi^2 \rho H}{r^2} \quad , \quad \text{but} \quad c_s = \sqrt{\frac{\partial p}{\partial \rho}} \sim \sqrt{\frac{p}{}}$$

$$\frac{P}{\rho} = c_s^2 = v_\phi^2 \frac{H^2}{r^2}$$

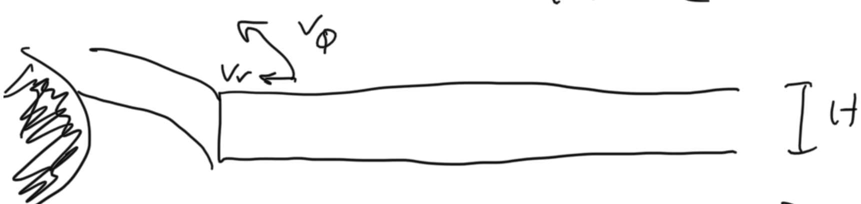
$V\ \rho$

★ Speed rel. to the sound speed is what?

$$\frac{H}{r} = \frac{1}{M} \leftarrow \text{Mach \#}$$

Thus, since by assumption $r \gg H$ implies $M \gg 1$ or, in order for disk to stay thin, pressure can't be high enough to "puff up" the disk

Amount of matter moving thru the disk $\dot{m}$ = amount of matter crossing area @ radius r per time



$$\dot{m} = \rho v_r \times 2\pi r H$$

matter flux (per area per time)

→ Area crossed @ r

$\dot{m}$ can be anything, let's say we're accreting @ Edd. lum. so

$$L = \frac{1}{2} \dot{m} c^2 \left(\frac{r_g}{r}\right) = 1.3 \times 10^{[illegible]} \text{ erg/s}$$

Consider $r = 10 r_g$, then $\dot{m} \sim 10^{15}$ kg/s

Still need v_r, H, ρ

from before $H \sim \frac{r}{M} = r \frac{c_s}{v_\varphi}$

@ $R = 10 r_g$, disk temperature around $\sim 1 M_\odot$ CO is $10^7 K$, which sets c_s, & v_φ given by Kepler's law, yield $M \sim 200$ (so have H)

No obvious way to estimate v_r

can parameterize $v_r = \beta c_s$

↳ interactions/shear will cause particles to move inward so $\beta \ll 1$

Apparent $\beta \sim 0.01$ works well in detailed disk models, so we'll assume that

Everything but ρ, but now we can calculate $\rho \sim 0.1 \beta^{-1}$ kg m^{-3}

~ 10 kg m^{-3}

Pretty dense!

From the mean free path, find that molecular (small scale) viscosity can't operate (needed to move mass inward, thru viscosity → collisions)

But, the conditions do allow for turbulent viscosity

$$\nu_{turb} \sim \lambda_{turb} v_{turb}$$

scale of eddies

rot. vel. o[f] eddi[es]

Shakura & Sunyaev introduced a parameter to scale ν_{turb} in order to derive the structure of such disks:

$$\nu = \alpha c_s H$$

called α-discs

Since these discs have B-fields, are susceptible to MHD instabilities such as the MRI that helps transport material <u>in</u> & angular momentum <u>out</u>

Long story short, you get dissipation of energy, emitted as radiation, allowing $\dot{m}$ <u>in</u> & L <u>out</u>

$$L(r)\Delta r = -\left(\frac{dE}{dt}\right) = \frac{3G\dot{m}M_s}{2r^2}\Delta r$$

includes a surface, which a BH doesn't have, & material is "eaten" w/o radiative loss once it crosses the ISCO @ radius r_I ($=3r_S$ for a=

$$L(r)\Delta r = \frac{3G\dot{m}M_{BH}}{2r^2}\left[1-\beta\left(\frac{r_I}{r}\right)^{1/2}\right]\Delta r$$

$\beta \rightarrow$ fraction of maximal ang. mom. mater[ial] has when it arrives @ $r = r_I$

$$\beta \leq 1$$

Total luminosity emitted by disc is

$$L = \int_{r_*,\, r_I}^{\infty}\left(-\frac{dE}{dt}\right)2\pi r\,dr$$

$$L = \frac{GM\dot{M}_*}{2r_*} \; ; \; L = \left(\frac{3}{2} - \beta\right) \frac{G\dot{M}M}{r_I}$$

Star ... BH

Consider a $1 M_\odot$ NS or BH

$R_{NS} \sim 10\,\mathrm{km}$, $R_{BH} \sim 3\,\mathrm{km}$

$r_I(a=0) = 9\,\mathrm{km}$

$r_* \sim r_I$

if $\beta \sim 1$, then

$$L_{NS} = \frac{G\dot{m}M_\odot}{2R_{NS}} \sim \frac{G\dot{m}M_\odot}{2r_I} = L_{BH}$$

- discs are basically the same, hard to tell difference!

Temperature structure

- disc optically thick, emits as BB
- emits from an area on top/bottom at each radius: $2\pi r \Delta r \times 2$

$$L\Delta r = A\sigma T^4 = 4\pi r \Delta r \sigma T^4$$

$$= \frac{3G\dot{m}M_*}{2r^2} \Delta r$$

$$T = \left[\frac{3 G \dot{m} M_*}{8 \pi \sigma r^3} \right]^{1/4}$$

$T \propto r^{-3/4}$ ($T \uparrow$ as $r \downarrow$, so $c_s \uparrow$, may invalidate thin disk assumpt

The spectrum of the disk is then

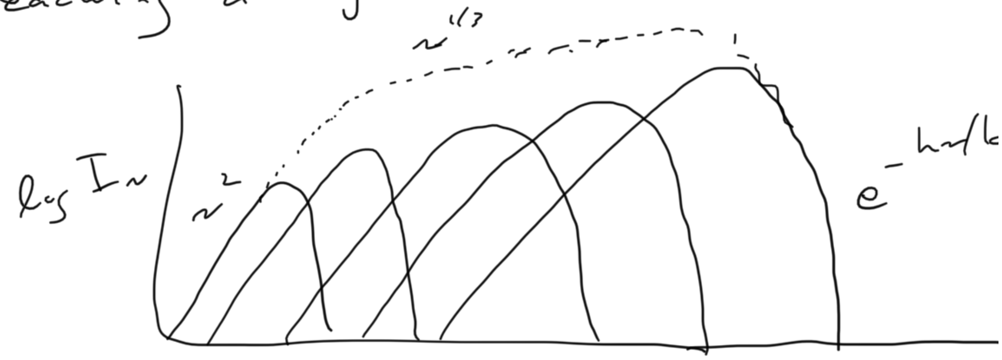
$$I(\nu) \propto \int_{r_I}^{r_{max}} 2\pi r \, B(T, \nu) \, dr$$

$$B(T, \nu) \propto \nu^3 \left(e^{h\nu/kT} - 1 \right)^{-1}$$

- can convert to

$$I(\nu) \propto \nu^{1/3} \underbrace{\int_{x(r_I)}^{x(r_{max})} x^{5/3} \left(e^{x} - 1 \right)^{-1} dx}_{\text{just some \#}}$$

- Just a sum of BB w/ peaks reaching a higher normalization as $\nu^{1/3}$



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