ASTR 5590 - Particle Acceleration

Already discussed gaps in a Pulsar Magnetosphere where accel. can happen (likely producing beam @ poles and powering the PWN @ outer gap)

But, E fields quickly shorted by moving charge – need another way to produce CR e^- + ions

Need to explain how particles achieve such energies + their E dist.

$$E \gg mc^2 \,, \quad N(E)\,dE \propto E^{-2.5}\,dE$$

<u>Enrico Fermi</u>

1949 → imagined clouds in ISM moving randomly that a high E particle would encounter

↳ clouds would act like mirrors, much more massive than particles, so wouldn't be affected by collision

Head-on collisions, particle gains E,
& tail-on (both travelling same direction)
particle loses E

CoM frame is that of cloud, so
rel. transform into that frame particle's

$$E' = \gamma_V (E + Vp\cos\theta)$$

Lorentz factor
angle coll.

can skip

product $Vp = Vmu$ is + (head-on) $\uparrow E$
− (tail-on) $\downarrow E$

$$p_x' = p'\cos\theta = \gamma_V \left(p\cos\theta + \frac{VE}{c^2}\right)$$

After collision, $E_b' = E_a'$ & $p_x' \to -p_x'$

Transfer back to "rest" frame

$$E'' = \gamma_V (E' + Vp_x')$$

$$= \gamma (\gamma_V [E + Vp\cos\theta] + Vp_x')$$

but $p = \gamma m u$ & $E = \gamma m c^2$, so $p = E\frac{u}{c^2}$

$$E'' = \gamma_V^2 \left(E + Vp\cos\theta + V\left(p\cos\theta + \frac{VE}{c^2}\right.\right.$$

$E\frac{u}{c^2}$

$$= \gamma_V^2 E\left[1 + \frac{2Vu}{c^2}\cos\theta + \frac{V^2}{c^2}\right]$$

since $\gamma_V^2 = \left(1 - \frac{V^2}{c^2}\right)^{-1}$, multiply out to get terms $O(1)$, $O\left(\frac{V^2}{c^2}\right)$, $O\left(\frac{V^3}{c^3}\right)$

assuming clouds non-rel., can ignore ↑ ab...

$$E'' \approx \left(1 + \frac{V^2}{c^2}\right)\left(1 + \frac{2Vu}{c^2}\cos\theta + \frac{V^2}{c^2}\right)E$$

$$= E\left[1 + \frac{V^2}{c^2} + \frac{2Vu}{c^2}\cos\theta\left(1 + \cancelto{0}{\frac{V^2}{c^2}}\right) + \frac{V^2}{c^2}\left(1 + \cancelto{0}{\frac{V^2}{c^2}}\right.\right.$$

$$= E\left(1 + \frac{2Vu}{c^2}\cos\theta + \frac{2V^2}{c^2}\right)$$

Change in E is

$$\Delta E = E'' - E = 2E\frac{V}{c}\left(\frac{u}{c}\cos\theta + \frac{V}{c}\right.$$

Avg. over angles $0 \to \pi$ for a $u \to c$ particle (prob. of collision @ θ is $\gamma_V\left[1 + \frac{V}{c}\cos\theta\right]$ (rel. transformation))

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{8}{3}\left(\frac{V}{c}\right)^2$$

2^{nd} Order in $\frac{v}{c}$, so called

2^{nd} order Fermi acc.

If clouds separated by mfp $\sim L$, then
E gain per time is

$$\frac{dE}{dt} = \frac{4}{3}\left(\frac{v^2}{2c}\right)E = \alpha E$$

And the particle spectrum is

$$\frac{d\,N(E)}{dE} = -\left(1 + \frac{1}{\alpha\tau}\right)\frac{N(E)}{E}$$

time spent in accel. region

since $\frac{df}{dx} = \frac{f}{x}$ is of PL form

$$N(E) = \text{const.}\ E^{-\left(1+\frac{1}{\alpha\tau}\right)}$$

This picture explains PL form, but not that exponent is -2.5
→ clouds v too small, would take longer than a Hubble time to reach E_{obs}

– particle crosses accel. region, picks up

$\beta = \frac{E}{E_0}$ frac. E, & will have

some prob. P of leaving region,

so after k crossings, the # part.

drops by $N = N_0 P^k$ but they

have $E = E_0 \beta^k$ energy now

Killing k;
$$\frac{\ln\left(\frac{N}{N_0}\right) = k \ln P}{\ln\left(\frac{E}{E_0}\right) = k \ln \beta}$$

$$\frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{\ln P / \ln \beta}$$

(but, this is $N(\geq E)$, since some particles will continue to $k+1$, $k+2$, etc.)

Spectrum deriv. of this, or

$$N(E)dE \propto E^{-1+(\ln P/\ln\beta)} dE$$

$= -\frac{1}{\alpha \tau}$ from befo

Similar accel zone is a shock front

– consider strong shocks $M \gg 1$, so

$$\frac{\rho_2}{\rho_1} = 4, \quad v_2 = \frac{1}{4} v_1$$

↑E particles will scatter, be randomized in up or downstream regions, won't care about shock itself (small rel. to m.f.p.)

Want to figure out ΔE as particle passes from up → down, then back

☆ Show Fig 17.3

- frame (c) shows that when shock crossed, enters a flow of gas w/ $\frac{3}{4}U$ velocity directed at it
- when it crosses back, sees identical situation, so must gain same E!

Avg. over crossing angles gives

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2}{3} \frac{V}{c} \quad \left(V = \frac{3}{4} U \right)$$

so total fraction E gained is 2x this

From before, $\beta = \frac{E}{E_0}$, $\frac{E - E_0}{E_0} = \frac{4}{3} \frac{V}{c}$

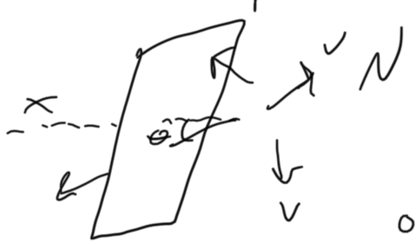
so $\beta = 1 + \frac{4}{3}\frac{V}{c}$

Since E gain $\propto \frac{V}{c}$ instead of $\left(\frac{V}{c}\right)^2$, this is referred to as 1^{st} order Fermi accel.

To get the index of E spectrum, need P.

particles per time crossing shock is $\frac{1}{4}Nc$ ($v \sim c$

(HW prob., $\frac{1}{4}$ result from kinetic th., calc. rate particles pass plane w/ angle)



flux thru surface: $N\langle v_x$

$$\langle v_x \rangle = \frac{\int_0^{\frac{\pi}{2}} v\cos\theta \cdot \sin\theta\, d\theta}{\int_0^{\pi} \sin\theta\, d\theta} = \frac{1}{4}$$

$0 \to \frac{\pi}{2}$ avg. same as $\frac{\pi}{2} \to \pi$, $\pi \to 2\pi$ don't cross

(incl. $-v_x$)

$x = \cos\theta$

$dx = -\sin\theta\, d\theta$

$$\frac{-v\int_0^1 x\, dx}{-\int_1^{-1} dx} = \frac{v\,\frac{1}{2}(1-0)}{1-(-1)}$$

Particles pass both ways in even #s, except those farthest from shock in downstream direction are being advected

Rate particles removed $NV = \frac{1}{4}N$

away. Rate particles

Figure again

Fraction particles lost per time is

$$\frac{\frac{1}{4} N V}{\frac{1}{4} N c} = \frac{V}{c}$$

Since $V \ll c$, small # lost, & $P = 1 - \frac{V}{c}$

over 1 cycle

Thus $\ln P = \ln\left(1 - \frac{V}{c}\right) \approx \frac{V}{c}$

(Taylor series $\ln x \approx \sum (-1)^n (x-1)^n$
$0 < x \leq 2$, or $\ln x \approx 1 - x$)

and $\ln \beta = \ln\left(1 + \frac{4}{3}\frac{V}{c}\right) = \ln\left(1 + \frac{4}{3c}\left(\frac{3}{4}V\right.\right.$

$$\approx -\frac{V}{c}$$

or $\frac{\ln P}{\ln \beta} = -1$ & $N(E)\,dE \propto E^{-1+}$

we find $\underline{N(E)\,dE \propto E^{-2}\,dE}$

close to $E^{-2.5}$!

In practice, situation more complex, particles can remove E from shock, end up w/steeper particle dist.

Galaxy clusters

- Weak shocks manage to re-accel. mildly relativistic particles (diffuse shock accel → 1st order Fermi can't do it, or explain spectral index
- Turbulence generated by merger can create magnetic mirrors, operate like 2nd order Fermi