Week 2

Lecture material, handouts, and HW posted to the website

Chapter 5: Ionization losses by energetic ions plowing through a neutral medium

Chapter 6: Radiation losses by energetic electrons getting deflected by ions in a plasma

“Swordy Plot”

particle flux of CRs at the Earth

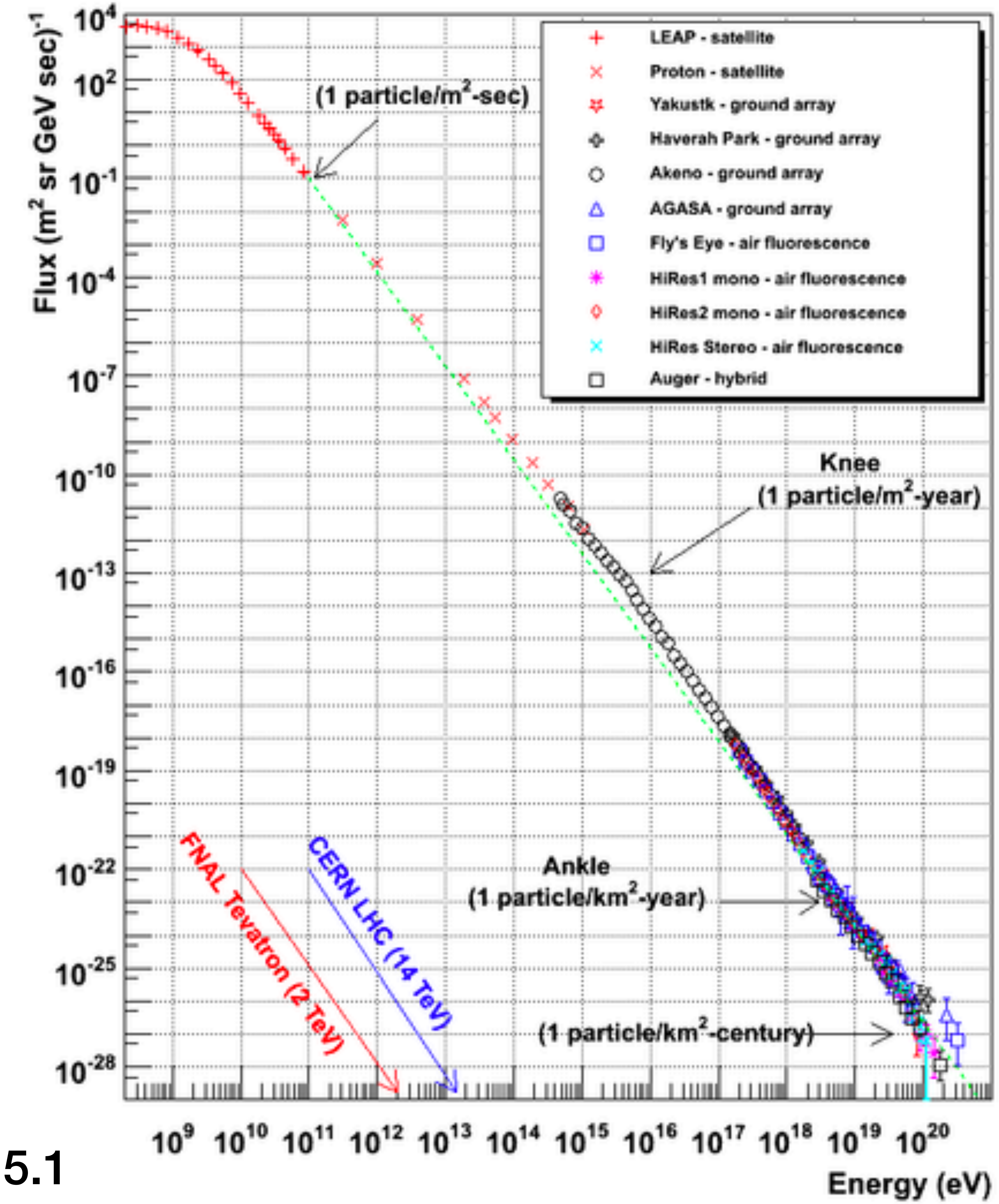


Fig. 15.1

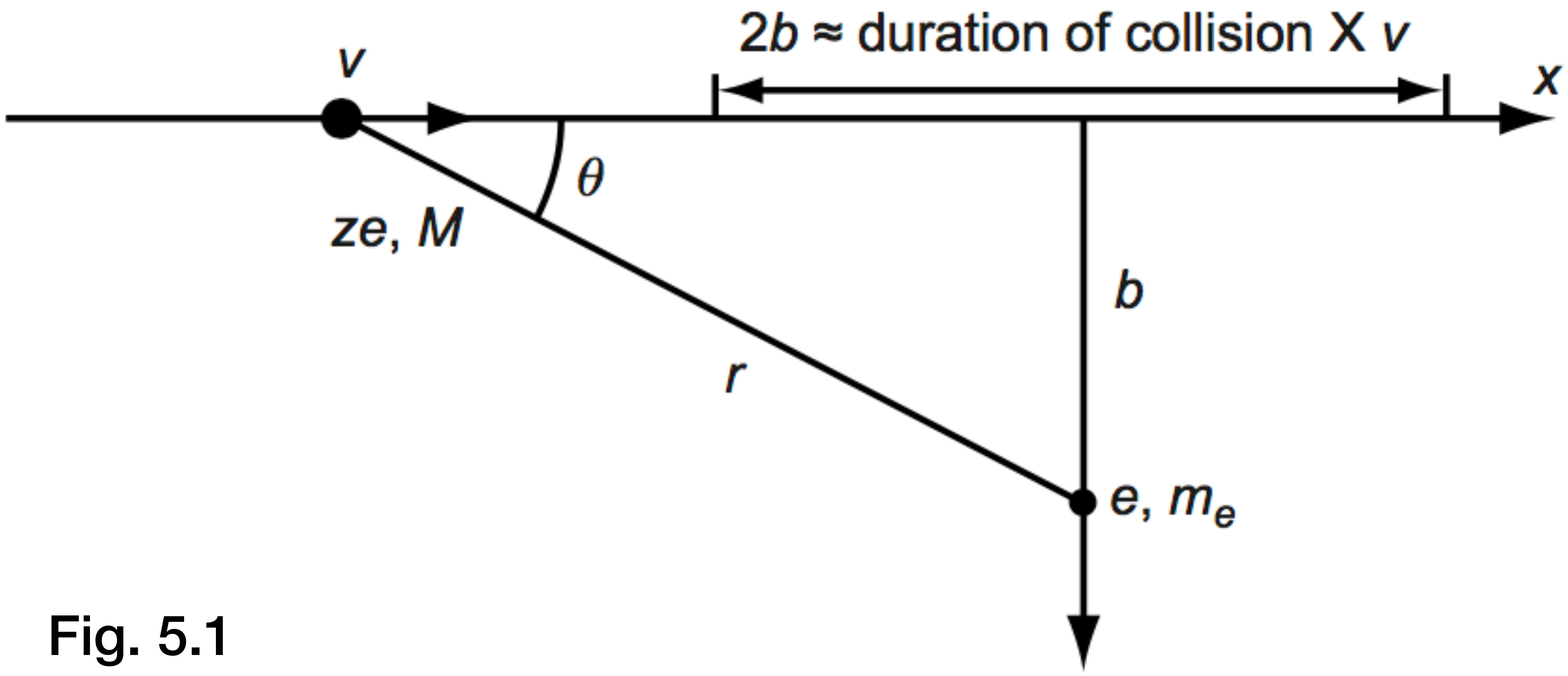


Fig. 5.1

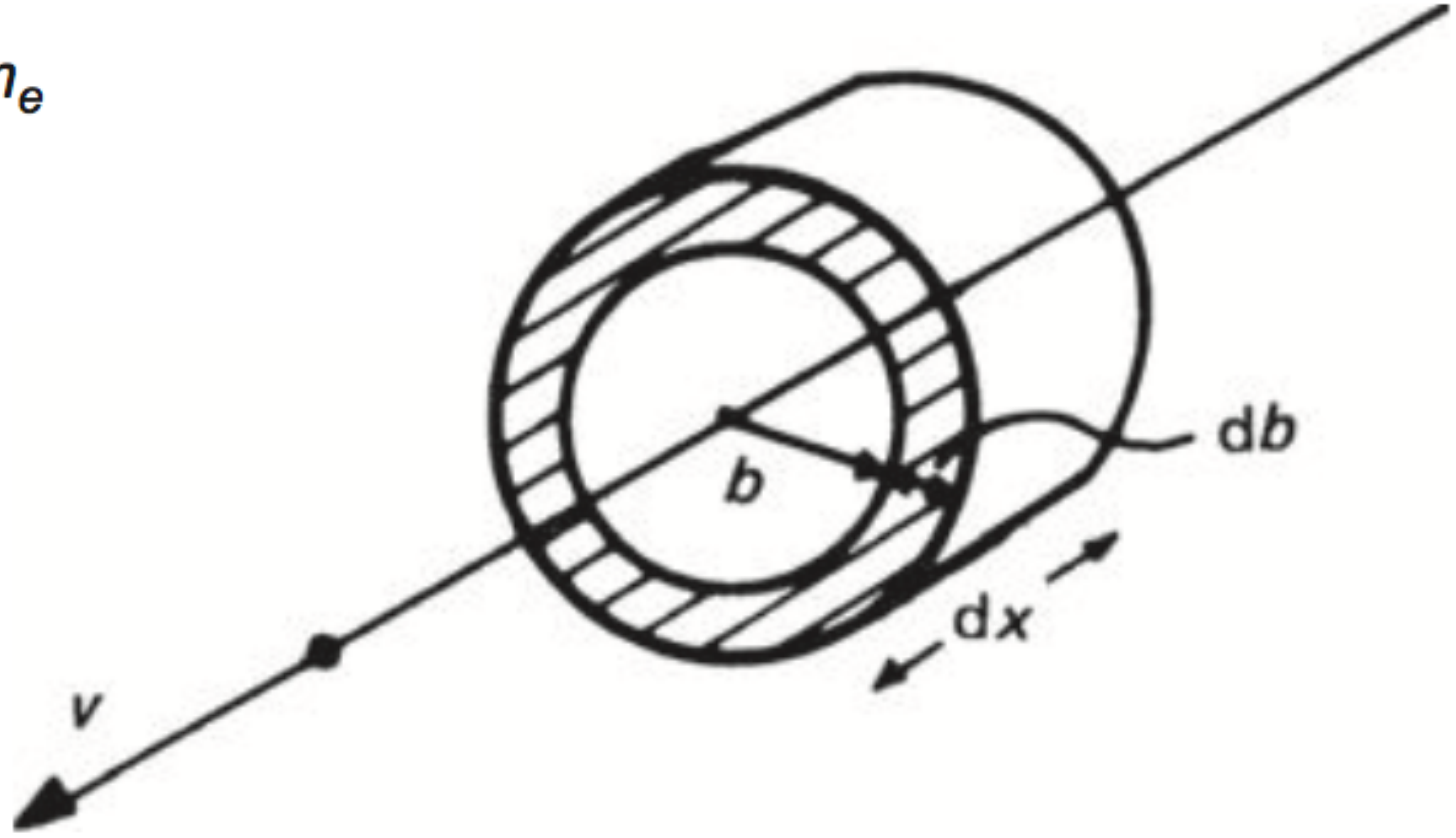


Fig. 5.2

Relativistic Case: Ionization losses

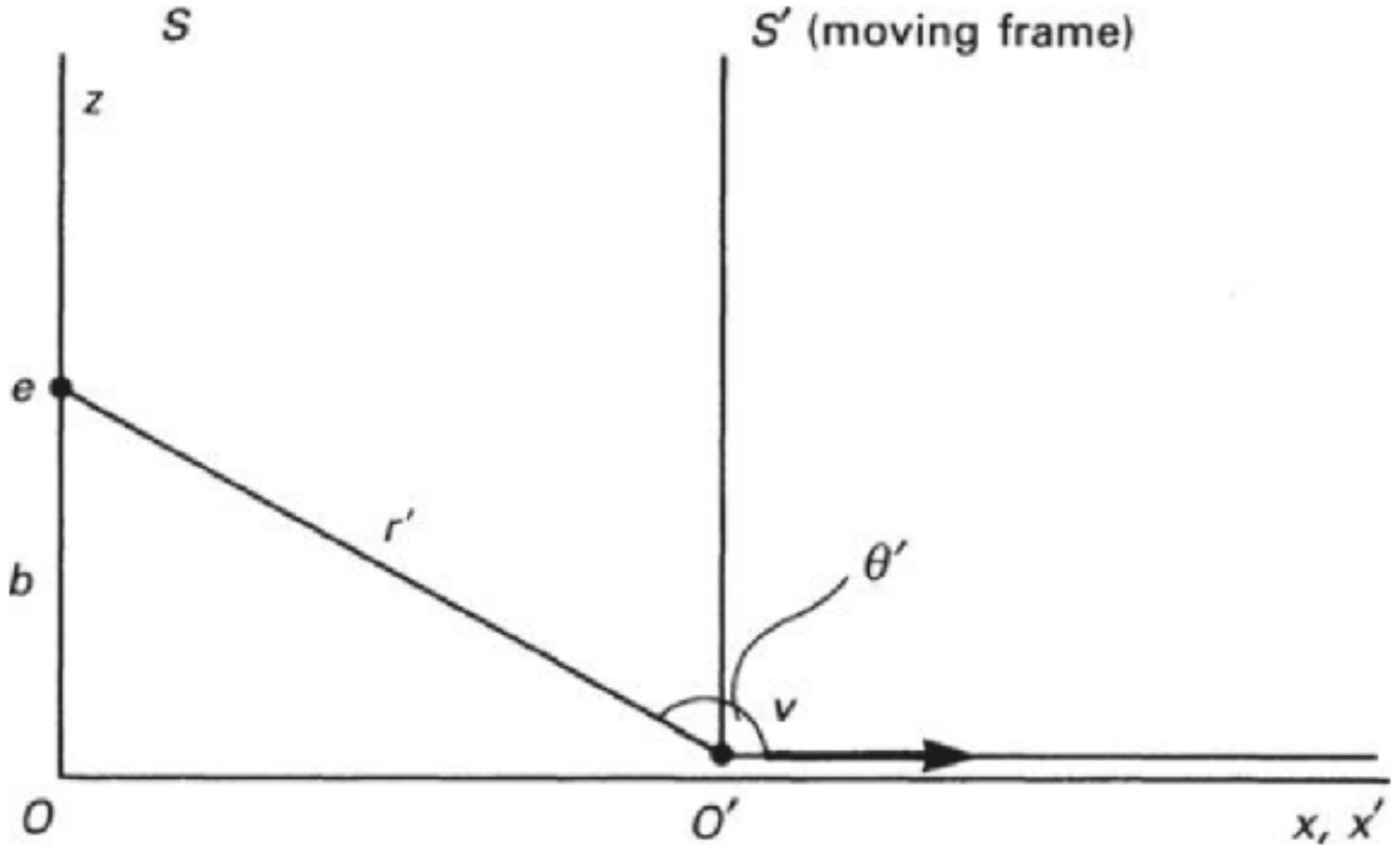


Fig. 5.3

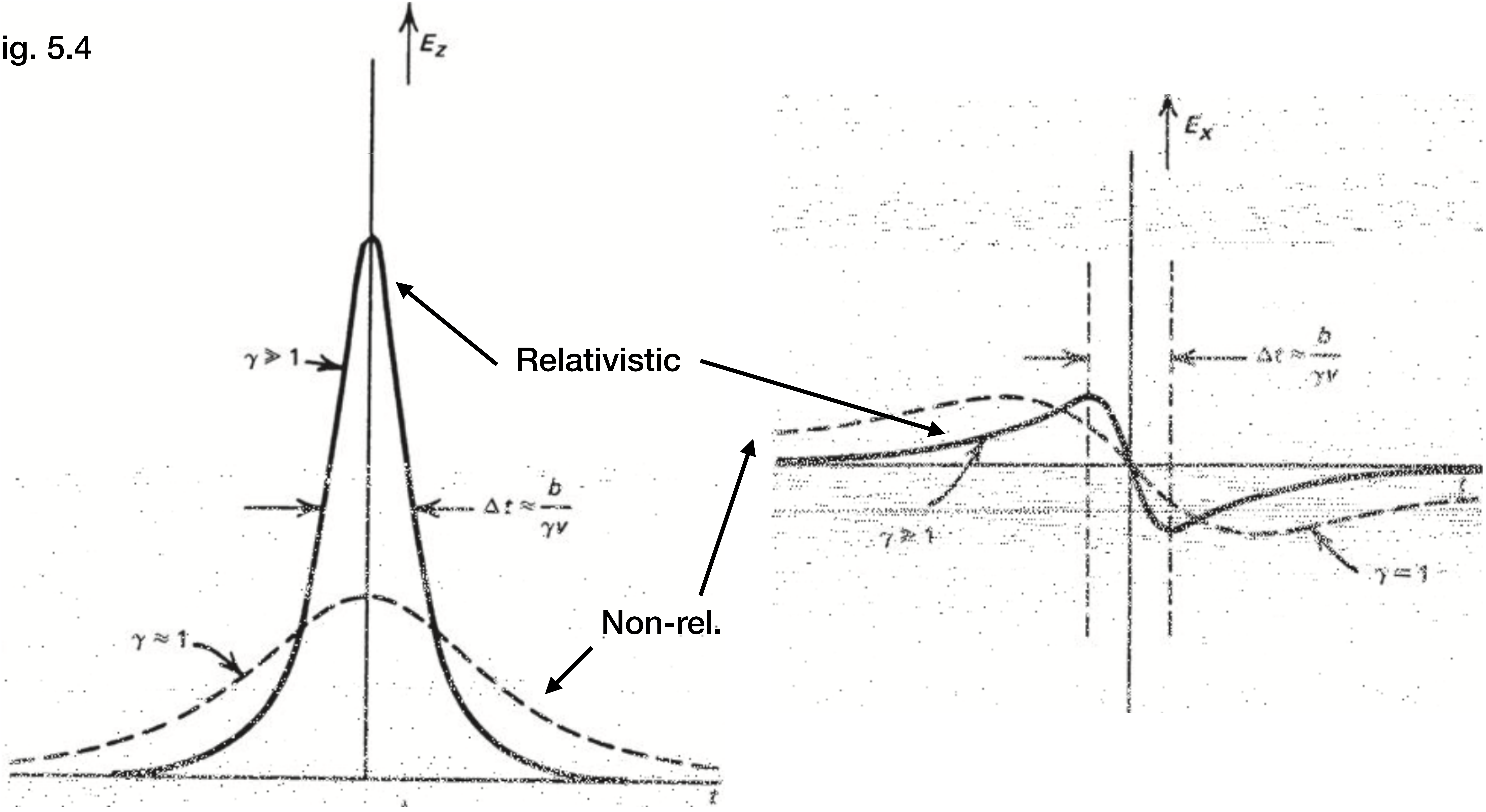
$$\left.\begin{array}{ll} E_x = E_{x'} & B_x = B_{x'} \, , \\ E_y = \gamma(E_{y'} + vB_{z'}) & B_y = \gamma\left(B_{y'} - \dfrac{v}{c^2}E_{z'}\right) \, , \\ E_z = \gamma(E_{z'} + vB_{y'}) & B_z = \gamma\left(B_{z'} + \dfrac{v}{c^2}E_{y'}\right) \, . \end{array}\right\}$$

Since $B_{x'} = B_{y'} = B_{z'} = 0$ in S', we find

$$\left.\begin{array}{ll} E_x = -\dfrac{\gamma zevt}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_x = 0 \, , \\ E_y = 0 & B_y = -\dfrac{\gamma zevb}{4\pi\varepsilon_0 c^2[b^2 + (\gamma vt)^2]^{3/2}} \, , \\ E_z = \dfrac{\gamma zeb}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_z = 0 \, . \end{array}\right\} \quad (5.18)$$

Notice that $B_y = -(v/c^2)E_z$.

Fig. 5.4



E_z
$\gamma \gg 1$
$\Delta t \approx \frac{b}{\gamma v}$
$\gamma \approx 1$
Relativistic
Non-rel.
E_x
$\Delta t \approx \frac{b}{\gamma v}$

Bethe-Bloch Formula

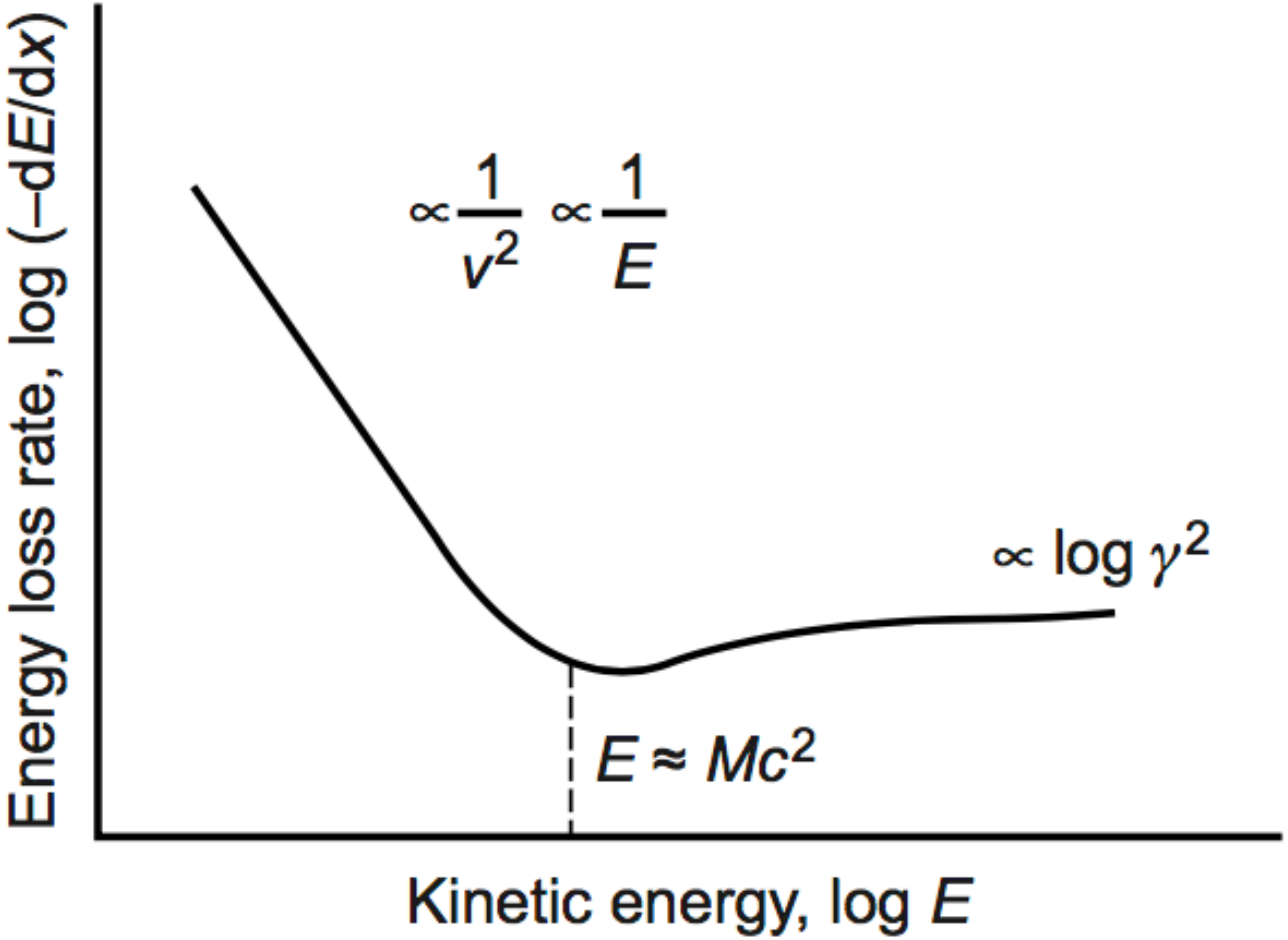


Fig. 5.5

Radiation of an accelerated charge

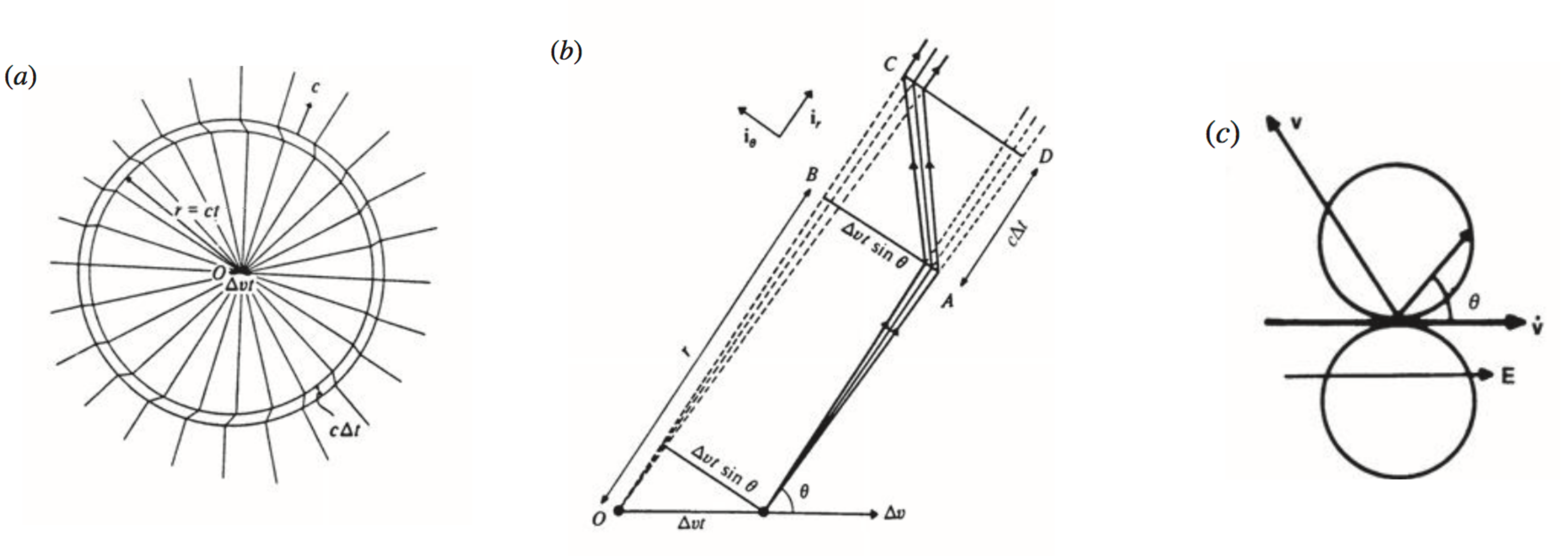


Fig. 6.1

Relativistic Case: electron radiation losses

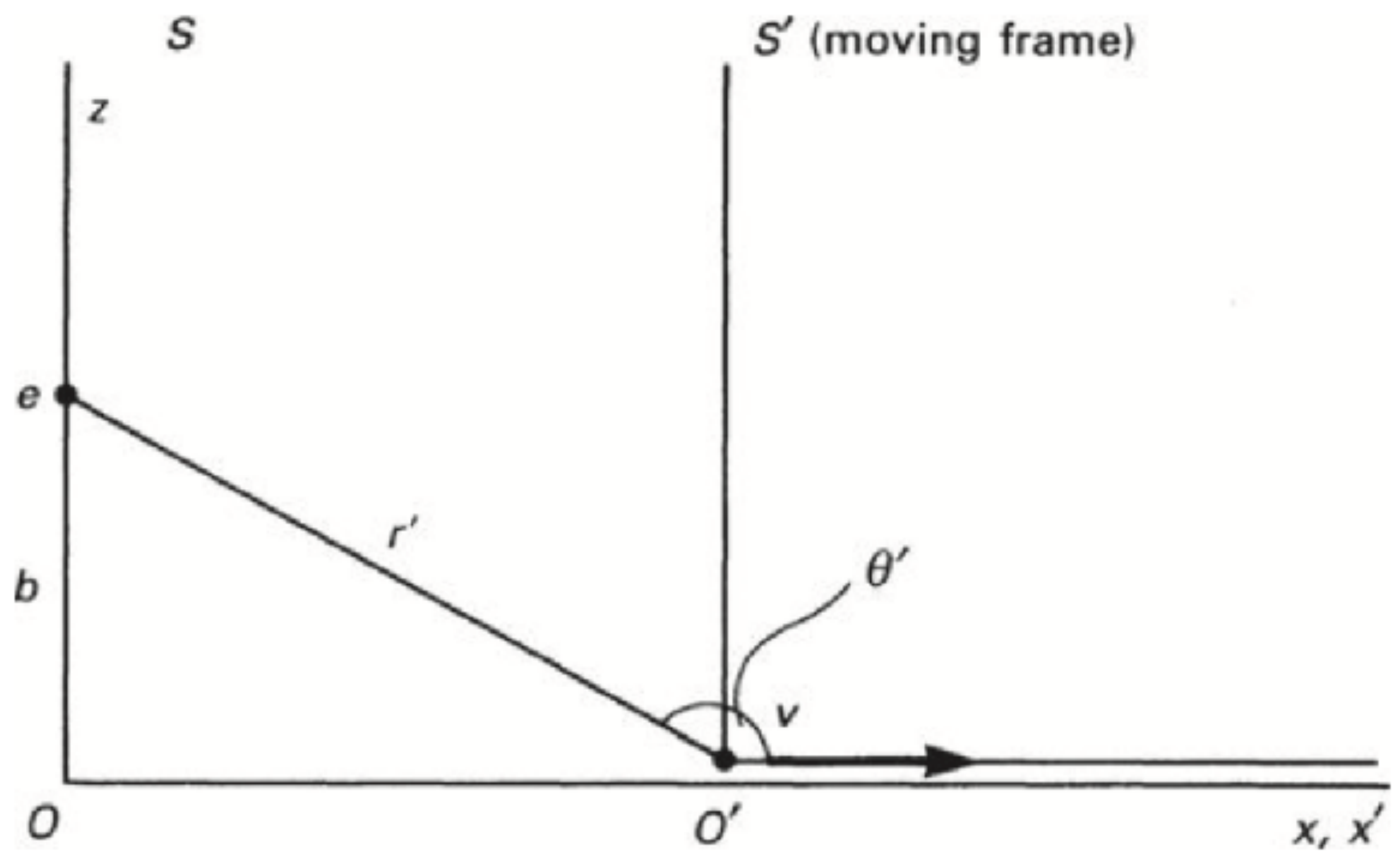


Fig. 5.3

$$
\left.\begin{aligned}
E_x &= E_{x'} & B_x &= B_{x'} \,, \\
E_y &= \gamma(E_{y'} + vB_{z'}) & B_y &= \gamma\left(B_{y'} - \frac{v}{c^2}E_{z'}\right) \,, \\
E_z &= \gamma(E_{z'} + vB_{y'}) & B_z &= \gamma\left(B_{z'} + \frac{v}{c^2}E_{y'}\right) \,.
\end{aligned}\right\}
$$

Since $B_{x'} = B_{y'} = B_{z'} = 0$ in S', we find

$$
\left.\begin{aligned}
E_x &= -\frac{\gamma zevt}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_x &= 0 \,, \\
E_y &= 0 & B_y &= -\frac{\gamma zevb}{4\pi\varepsilon_0 c^2[b^2 + (\gamma vt)^2]^{3/2}} \,, \\
E_z &= \frac{\gamma zeb}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_z &= 0 \,.
\end{aligned}\right\} \qquad (5.18)
$$

Notice that $B_y = -(v/c^2)E_z$.

$$\dot{\boldsymbol{v}}(t) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\boldsymbol{v}}(\omega) \exp(-\mathrm{i}\omega t)\, \mathrm{d}\omega \,, \tag{6.26}$$

$$\dot{\boldsymbol{v}}(\omega) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\boldsymbol{v}}(t) \exp(\mathrm{i}\omega t)\, \mathrm{d}t \,. \tag{6.27}$$

According to Parseval's theorem, $\dot{\boldsymbol{v}}(\omega)$ and $\dot{\boldsymbol{v}}(t)$ are related by the following integral:

$$\int_{-\infty}^{\infty} |\dot{\boldsymbol{v}}(\omega)|^2 \mathrm{d}\omega = \int_{-\infty}^{\infty} |\dot{\boldsymbol{v}}(t)|^2\, \mathrm{d}t \,. \tag{6.28}$$

This is proved in all textbooks on Fourier analysis. We can therefore apply this relation to the energy radiated by a particle which has an acceleration history $\dot{\boldsymbol{v}}(t)$:

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}E}{\mathrm{d}t}\, \mathrm{d}t = \int_{-\infty}^{\infty} \frac{e^2}{6\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(t)|^2\, \mathrm{d}t = \int_{-\infty}^{\infty} \frac{e^2}{6\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2\, \mathrm{d}\omega \,. \tag{6.29}$$

Now, what we really want is $\int_0^\infty \cdots \mathrm{d}\omega$ rather than $\int_{-\infty}^{\infty} \cdots \mathrm{d}\omega$. Since the acceleration is a real function, there is another theorem in Fourier analysis which tells us that

$$\int_0^{\infty} |\dot{\boldsymbol{v}}(\omega)|^2\, \mathrm{d}\omega = \int_{-\infty}^{0} |\dot{\boldsymbol{v}}(\omega)|^2\, \mathrm{d}\omega \,,$$

and hence we find

$$\text{total emitted radiation} = \int_0^{\infty} I(\omega)\, \mathrm{d}\omega = \int_0^{\infty} \frac{e^2}{3\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2\, \mathrm{d}\omega \,.$$

Therefore

$$I(\omega) = \frac{e^2}{3\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2 \,. \tag{6.30}$$

Bremsstrahlung of a single electron, for a single impact parameter *b*

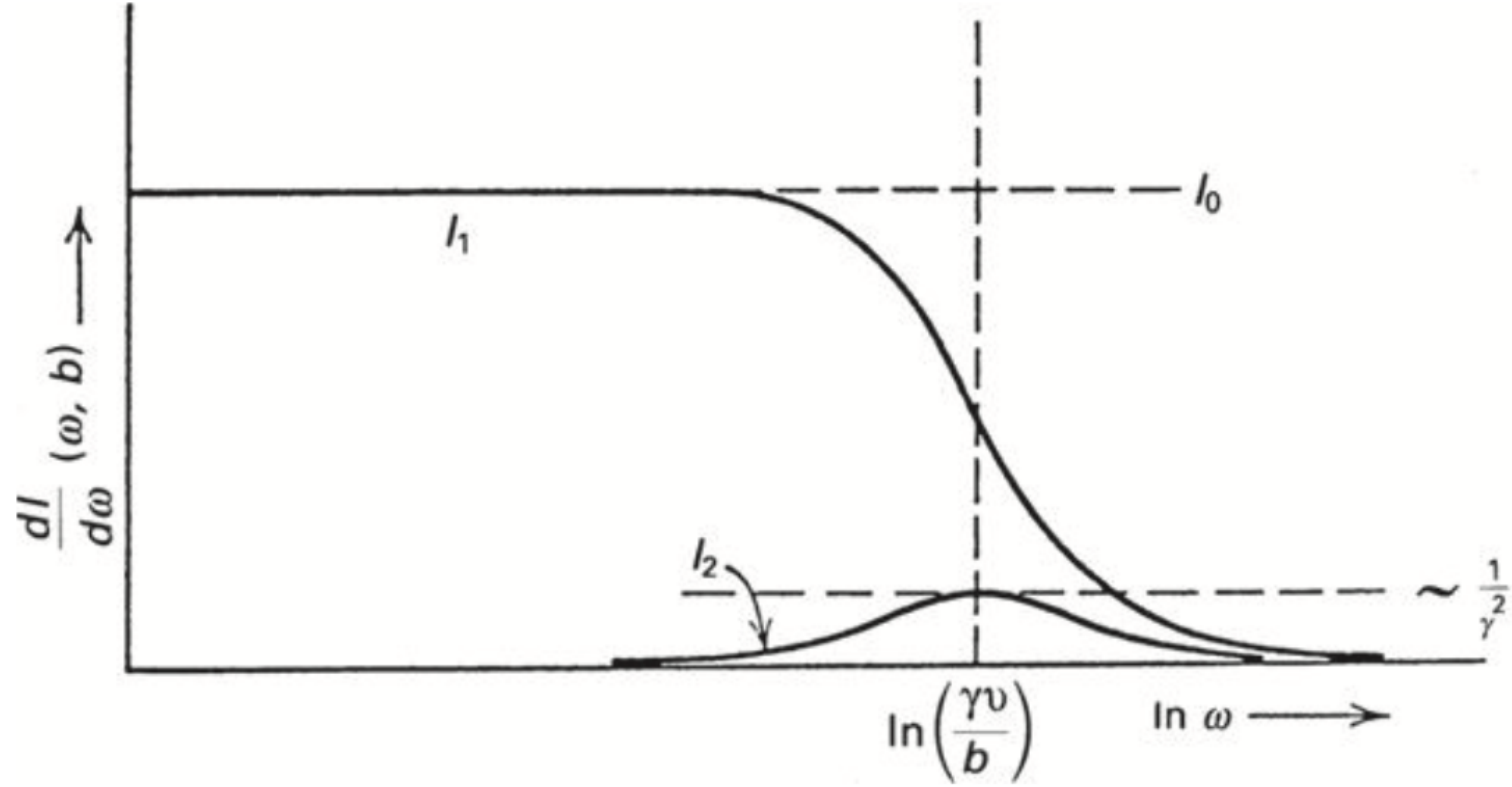
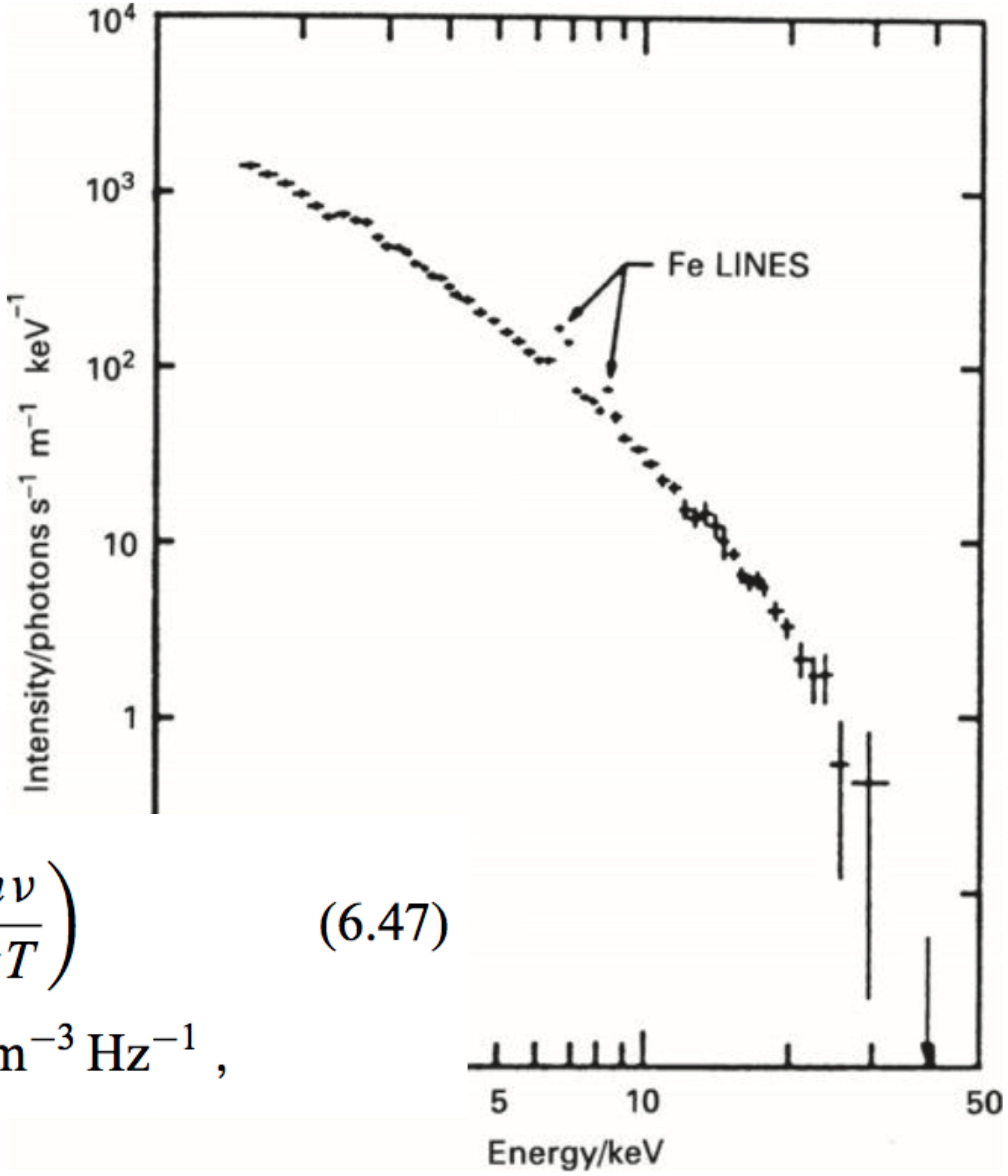


Fig. 6.2

Bremsstrahlung emissivity from a thermal distribution of electrons



$$\kappa_\nu = \frac{1}{3\pi^2}\left(\frac{\pi}{6}\right)^{1/2}\frac{Z^2e^6}{\varepsilon_0^3c^3m_e^2}\left(\frac{m_e}{kT}\right)^{1/2}g(\nu, T)\,N N_e \exp\left(-\frac{h\nu}{kT}\right) \qquad (6.47)$$

$$= 6.8\times 10^{-51} Z^2 T^{-1/2} N N_e\, g(\nu, T)\exp(-h\nu/kT)\ \ \mathrm{W\,m^{-3}\,Hz^{-1}},$$