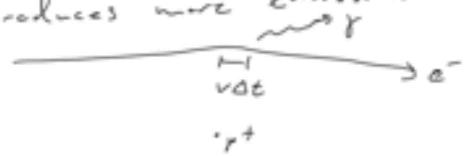
ASTR 5590 - Chapter 6

& ions & the radiation produced when they deflect, effectively

When e^- & ions interact, which produces more emission? why?



free-free or bremsstrahlung

Show Fig. 6.1

JJ Thomson derivation → velocity changed by Δv over time Δt

Before impulse, E_r → Coulomb + $E_\theta = 0$ in frame of moving e^-

During impulse $\quad \frac{E_\theta}{E_r} = \frac{\Delta v\, t \sin\theta}{c\,\Delta t}$

but $E_r = \frac{q}{4\pi\epsilon_0 r^2}$ w/ $r = ct$ so

$$E_\theta = \frac{q(\Delta v/\Delta t)\sin\theta}{4\pi\epsilon_0 c^2 r} \quad \text{(get rid of } t\text{)}$$

$$\frac{\Delta v}{\Delta t} \to |\vec{a}|$$

Need energy change w/ time, given by Poynting vector $S = |E \times H| = E^2/Z_0$

$Z_0 = (\mu_0/\epsilon_0)^{1/2}$ impedance of free space

→ E flow/area/time

area → $r^2 d\Omega$ ← solid angle

$$S\, r^2 d\Omega = -\left(\frac{dE}{dt}\right) d\Omega = \frac{q^2|a|^2 \sin^2\theta}{16\pi^2 \epsilon_0 c^3} d\Omega$$

w/ $d\Omega = 2\pi \sin\theta \, d\theta$, integrate $0 \to \pi$

$$\boxed{-\left(\frac{dE}{dt}\right) = \frac{q^2 |a|^2}{6\pi \epsilon_0 c^3} \quad \text{Larmor's formula}}$$

- Radiation is dipolar
  $E_\theta \propto \sin\theta$: most radiation $\perp$
- Radiation is polarized $(\vec{i}_\theta)$
- proper acceleration (in rest frame of particle)

Relativistic case

Note that $\frac{dE}{dt}$ is Lorentz invariant

$dE = \gamma dE'$ & $dt = \gamma dt'$

$$\left(\frac{dE}{dt}\right)_S = \left(\frac{dE'}{dt'}\right)_{S'} = \frac{q^2 |\vec{a}_0|^2}{6\pi \epsilon_0 c^3}$$

$$= \frac{q^2 \gamma^4}{6\pi\epsilon_0 c^3}\left[\vec{a}^2 + \gamma^2\left(\frac{\vec{v}\cdot\vec{a}}{c}\right)^2\right]$$

Comes from acceleration 4-vector:

$$\gamma\left[c\frac{\partial\gamma}{\partial t}, \frac{\partial(\gamma\vec{v})}{\partial t}\right]$$

$$= \left[\left(\frac{\vec{v}\cdot\vec{a}}{c^2}\right)\gamma^4 c, \gamma^2\vec{a} + \left(\frac{\vec{v}\cdot\vec{a}}{c^2}\right)\gamma^4\vec{v}\right]$$

(all quantities measured in S)

$\vec{a} = a_\parallel \vec{i}_\parallel + a_\perp \vec{i}_\perp$ & $|\vec{a}|^2 = |a_\parallel|^2 + |a_\perp|^2$

do math, get

$$\left(\frac{dE}{dt}\right)_S = \frac{q^2\gamma^4}{6\pi\epsilon_0 c^3}\left(|a_\perp|^2 + \gamma^2|a_\parallel|^2\right)$$

Need $a_\perp$ & $a_\parallel$, go back to Ch. 5

★ Show 5.18 eqn.

$a = \dot{\vec{v}}_x = -\frac{eE_x}{m}$

$$a_\parallel \quad \quad \quad m_e$$

$$a_\perp = \dot{v}_2 = -\frac{eE_2}{m_e}$$

Show Fourier trans. stuff

Transform to freq. space to get spectrum of emission → get Bessel functions

$$I(\omega) = \frac{Z^2 e^6}{24\pi^4 \epsilon_0^3 c^3 m_e^2 v^2} \frac{\omega^2}{\gamma^2 v^2}\left[\frac{1}{\gamma^2} K_0^2\left(\frac{\omega b}{\gamma v}\right) + K_1^2\left(\frac{\omega b}{\gamma v}\right)\right]$$

Comes from $I(\omega) = \frac{e^2}{3\pi\epsilon_0 c^3}\left[|a_\parallel(\omega)|^2 + |a_\perp(\omega)|^2\right]$

$$y = \frac{\omega b}{\gamma v}, \quad y \ll 1,\ K_0(y) = -\ln y,\ K_1(y) = \frac{1}{y}$$
$$y \gg 1,\ K_0(y) = K_1(y) = \left(\frac{\pi}{2y}\right)^{1/2} e^{-y}$$

High freq ($\omega\uparrow$), get exponential cutoff

Just like in Ch. 5, $\tau = \frac{2b}{\gamma v}$

& $\nu \approx \frac{1}{\tau} = \frac{\gamma v}{2b}$ so $\omega \approx \frac{\pi \gamma v}{b}$

or $\frac{\omega b}{\gamma v} \sim 1$

Above this freq., e^- can't react to impulse

At low freq. ($\omega\downarrow$), can pull out b^2

& brackets take form:

$$\left[1 + \frac{1}{\gamma^2}\left(\frac{\omega b}{\gamma v}\right)^2 \ln^2\left(\frac{\omega b}{\gamma v}\right)\right]$$

if this small, goes to 1

& emission is constant

★ Show Fig. 6.2

Again, this is radiation of 1 interaction!

integrate over b

If relativistic, need to include length contraction effect on the density e^- sees in its frame $N' = \gamma N$

$$I(\omega') = \int_{b'_{min}} 2\pi b' \gamma N v K \, db'$$

flat term

$$K = \frac{Z^2 e^6}{24\pi^4 \epsilon_0^3 c^3 m_e^2 b^2 v^2}$$

$$I(\omega') = \frac{Z^2 e^6 \gamma N}{12\pi^3 \epsilon_0^3 c^3 m_e^2} \frac{1}{v} \ln\left(\frac{b'_{max}}{b'_{min}}\right)$$

Non-rel.

$S = S'$ & $\gamma = 1$, just need

$$\ln\left(\frac{b_{max}}{b_{min}}\right) \equiv \ln \Lambda$$

What is b_{max} in this case?

exponential cutoff $\rightarrow \frac{\omega b}{v} = 1$

b_{min}, same options as before

$\rightarrow$ for us, $T\uparrow$ so need quantum b_{min}

$$\Lambda = \frac{2 m_e v^2}{\hbar \omega}$$

Can integrate over all freq. to get E loss of e^-:

$$-\left(\frac{dE}{dt}\right)_{brems} \simeq \int_0^{\omega_{max}} (\text{const}) Z^2 N \frac{1}{v} \ln \Lambda \, d\omega$$

where $\omega_{max} \approx m_e v^2 / 2\hbar$ so find

$\rightarrow (\text{const}) Z^2 N v$

& since $E \propto v^2$, $-\frac{dE}{dt} \propto E^{1/2}$

But still just 1 particle! Need to integrate over a population to get a useful result

Thermal Bremsstrahlung

Maxwellian dist

$\left(\frac{m_e}{\cdot} \right)^{3/2}$

$$N_e(v)dv = 4\pi N_e \left(\frac{m_e}{2\pi kT}\right) v^2 \exp\left[-\frac{m_e v^2}{2kT}\right] dv$$

Pretty sure eqn 6.45 has a typo, so skip to answer:

Show Eqn 6.47

gaunt factor $g(\nu, T)$ is of order 1 $(\bar{g} \sim 1.2)$

emissivity (emission per volume per freq)

$$\propto Z^2 T^{-1/2} N N_e \, g(\nu, T) \, e^{-h\nu/kT}$$

& $-\left(\frac{dE}{dt}\right) \propto Z^2 T^{1/2} \bar{g} N N_e$