ASTR/PHYS 5590: High Energy Astrophysics

Week 3

HW01 due on Thursday
(HWs due weekly on Thursdays from now on)

Chapter 7: Charged Particles in Magnetic Fields

Chapter 11: Plasma Physics

Radiation of an accelerated charge

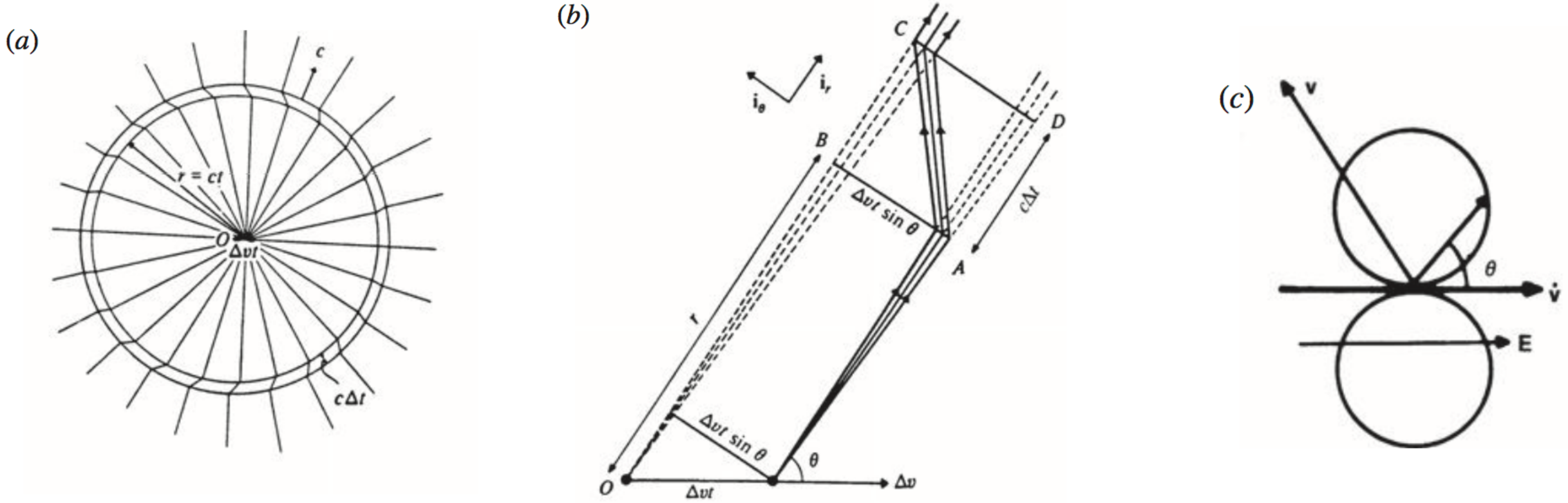


Fig. 6.1

Relativistic Case: electron radiation losses

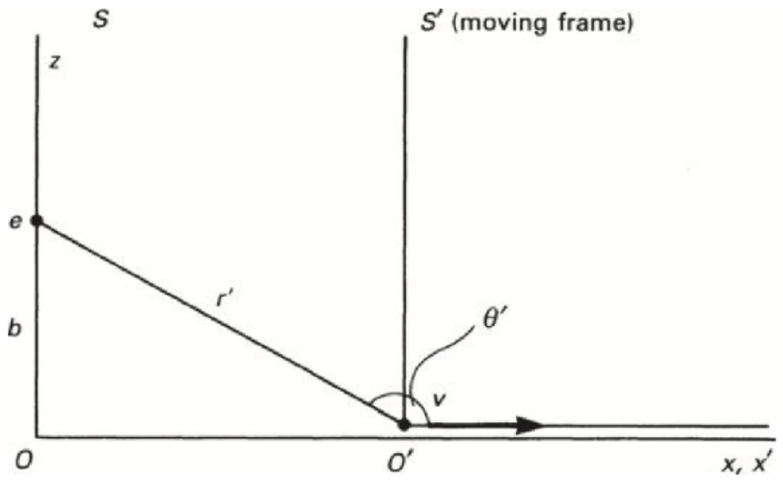


Fig. 5.3

$$\left.\begin{aligned} E_x &= E_{x'} & B_x &= B_{x'} \,, \\ E_y &= \gamma(E_{y'} + vB_{z'}) & B_y &= \gamma\left(B_{y'} - \frac{v}{c^2}E_{z'}\right) \,, \\ E_z &= \gamma(E_{z'} + vB_{y'}) & B_z &= \gamma\left(B_{z'} + \frac{v}{c^2}E_{y'}\right) \,. \end{aligned}\right\}$$

Since $B_{x'} = B_{y'} = B_{z'} = 0$ in S', we find

$$\left.\begin{aligned} E_x &= -\frac{\gamma zevt}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_x &= 0 \,, \\ E_y &= 0 & B_y &= -\frac{\gamma zevb}{4\pi\varepsilon_0 c^2[b^2 + (\gamma vt)^2]^{3/2}} \,, \\ E_z &= \frac{\gamma zeb}{4\pi\varepsilon_0[b^2 + (\gamma vt)^2]^{3/2}} & B_z &= 0 \,. \end{aligned}\right\} \qquad (5.18)$$

Notice that $B_y = -(v/c^2)E_z$.

$$\dot{\boldsymbol{v}}(t) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\boldsymbol{v}}(\omega) \exp(-\mathrm{i}\omega t)\, \mathrm{d}\omega \,, \tag{6.26}$$

$$\dot{\boldsymbol{v}}(\omega) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\boldsymbol{v}}(t) \exp(\mathrm{i}\omega t)\, \mathrm{d}t \,. \tag{6.27}$$

According to Parseval's theorem, $\dot{\boldsymbol{v}}(\omega)$ and $\dot{\boldsymbol{v}}(t)$ are related by the following integral:

$$\int_{-\infty}^{\infty} |\dot{\boldsymbol{v}}(\omega)|^2 \mathrm{d}\omega = \int_{-\infty}^{\infty} |\dot{\boldsymbol{v}}(t)|^2 \, \mathrm{d}t \,. \tag{6.28}$$

This is proved in all textbooks on Fourier analysis. We can therefore apply this relation to the energy radiated by a particle which has an acceleration history $\dot{\boldsymbol{v}}(t)$:

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}E}{\mathrm{d}t}\, \mathrm{d}t = \int_{-\infty}^{\infty} \frac{e^2}{6\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(t)|^2 \, \mathrm{d}t = \int_{-\infty}^{\infty} \frac{e^2}{6\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2 \, \mathrm{d}\omega \,. \tag{6.29}$$

Now, what we really want is $\int_0^\infty \cdots \mathrm{d}\omega$ rather than $\int_{-\infty}^\infty \cdots \mathrm{d}\omega$. Since the acceleration is a real function, there is another theorem in Fourier analysis which tells us that

$$\int_0^{\infty} |\dot{\boldsymbol{v}}(\omega)|^2 \, \mathrm{d}\omega = \int_{-\infty}^{0} |\dot{\boldsymbol{v}}(\omega)|^2 \, \mathrm{d}\omega \,,$$

and hence we find

$$\text{total emitted radiation} = \int_0^{\infty} I(\omega)\, \mathrm{d}\omega = \int_0^{\infty} \frac{e^2}{3\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2 \, \mathrm{d}\omega \,.$$

Therefore

$$I(\omega) = \frac{e^2}{3\pi\varepsilon_0 c^3} |\dot{\boldsymbol{v}}(\omega)|^2 \,. \tag{6.30}$$

Bremsstrahlung of a single electron, for a single impact parameter *b*

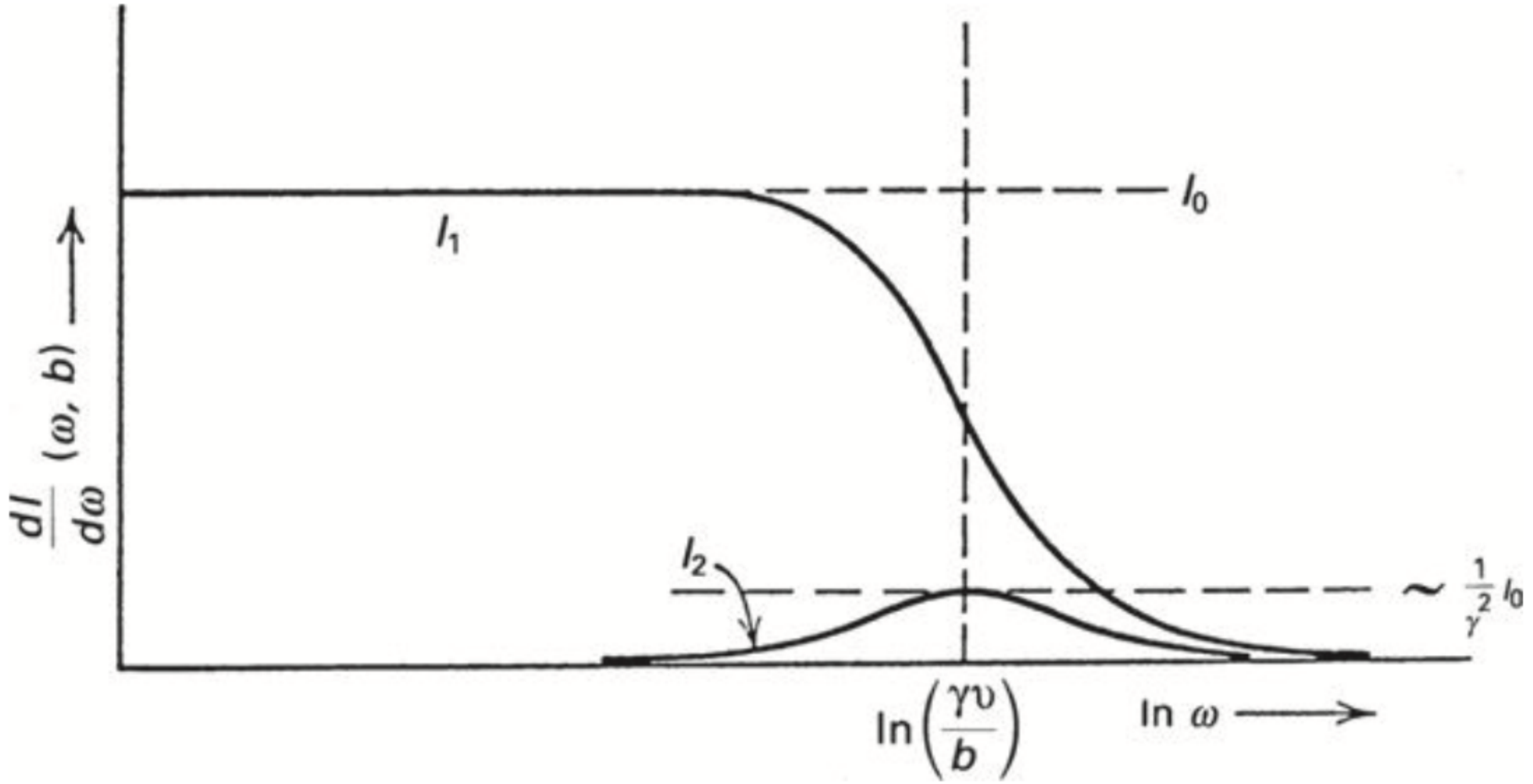
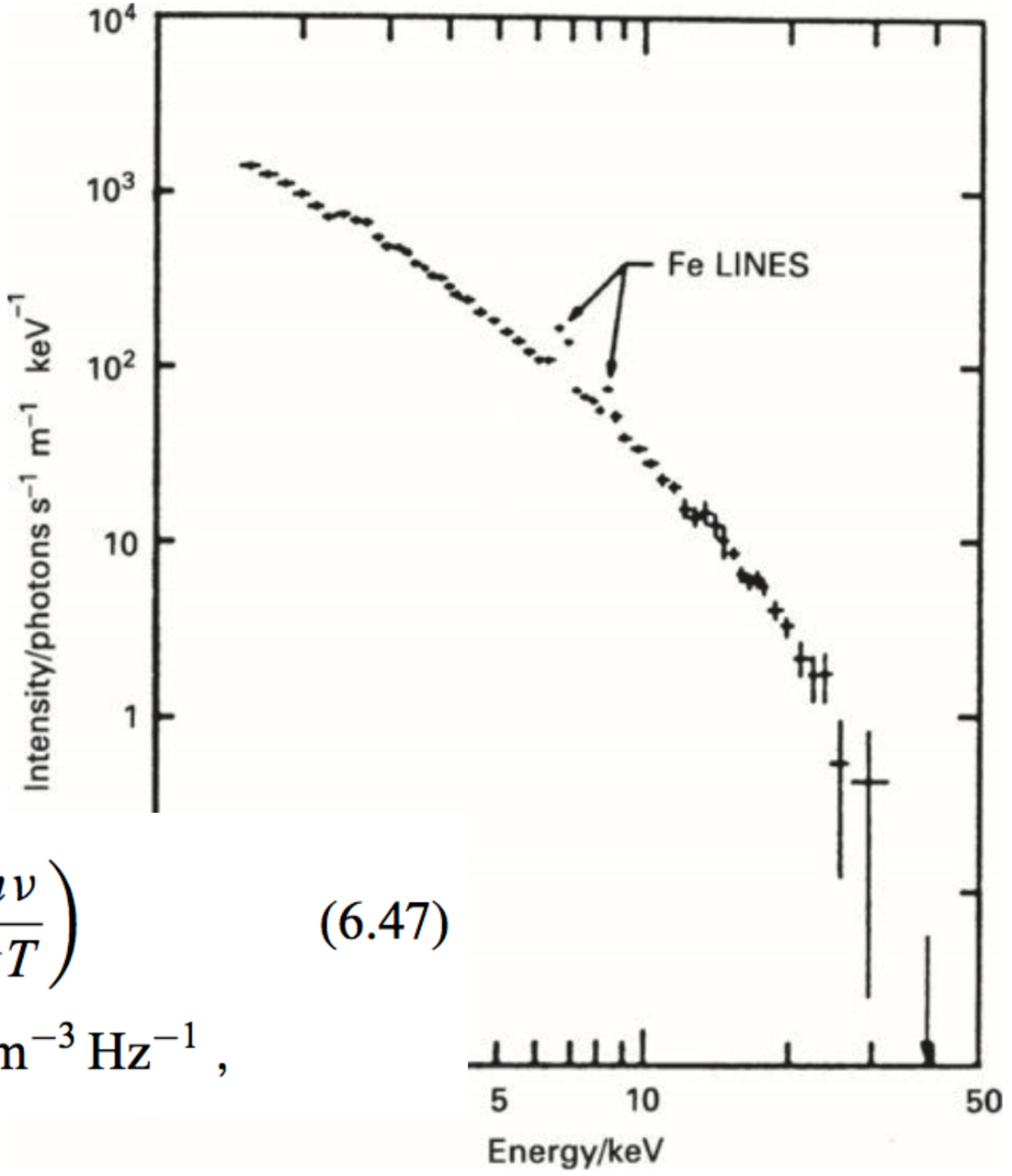


Fig. 6.2

Bremsstrahlung emissivity from a thermal distribution of electrons



$$\kappa_\nu = \frac{1}{3\pi^2}\left(\frac{\pi}{6}\right)^{1/2}\frac{Z^2e^6}{\varepsilon_0^3 c^3 m_e^2}\left(\frac{m_e}{kT}\right)^{1/2} g(\nu, T)\, N N_e \exp\left(-\frac{h\nu}{kT}\right) \qquad (6.47)$$

$$= 6.8\times 10^{-51} Z^2 T^{-1/2} N N_e\, g(\nu, T)\, \exp(-h\nu/kT)\ \mathrm{W\,m^{-3}\,Hz^{-1}}\ ,$$

Charged particle accelerated in a B field

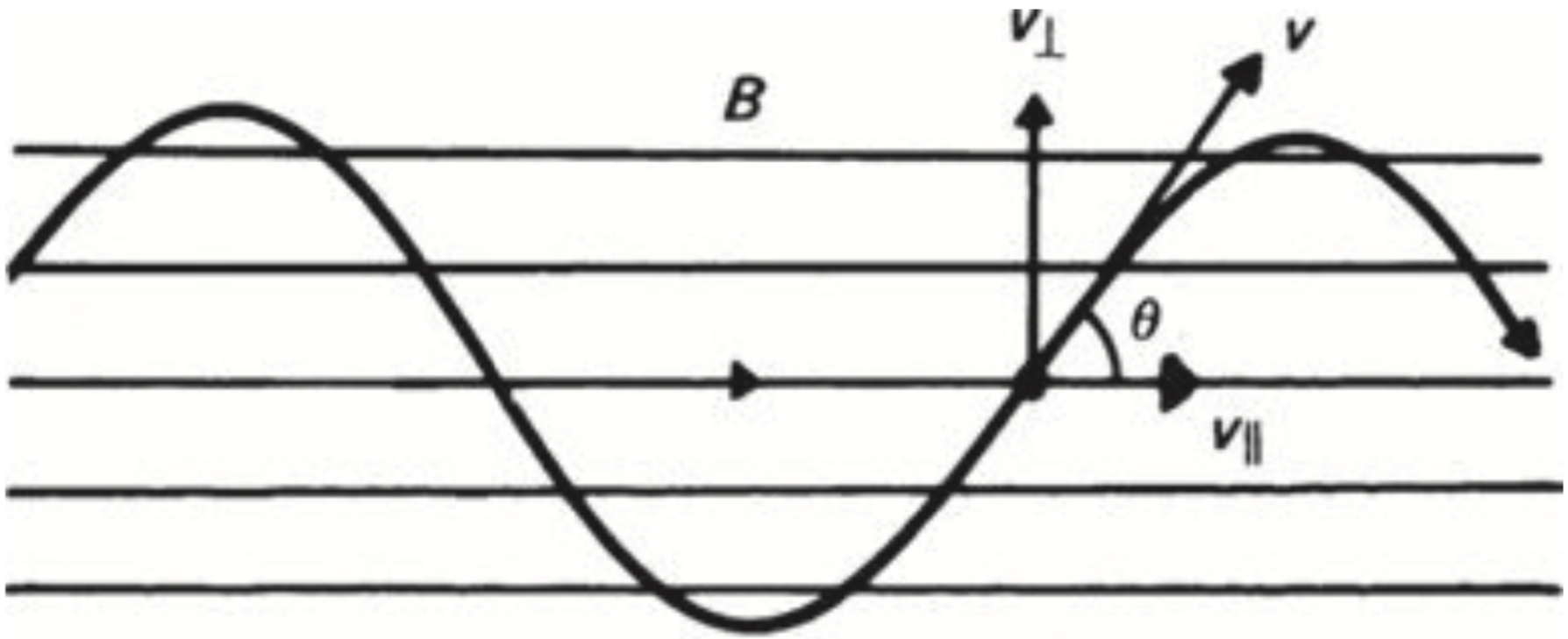


Fig. 7.1

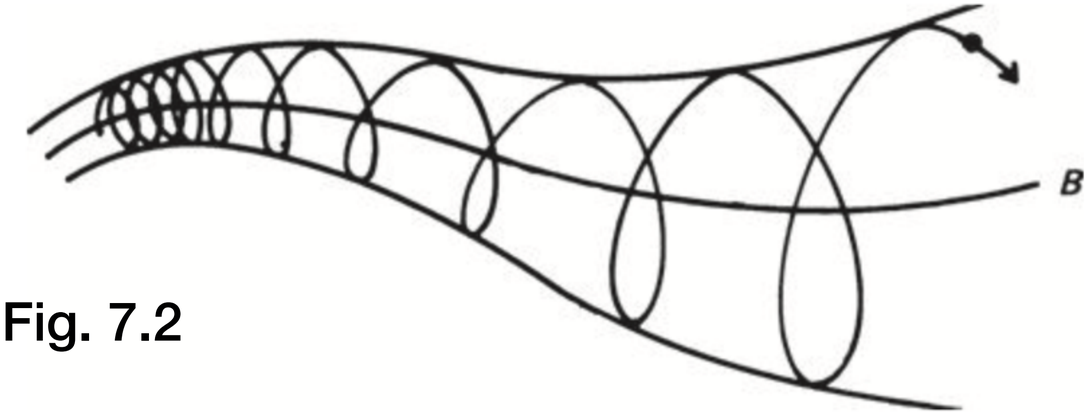
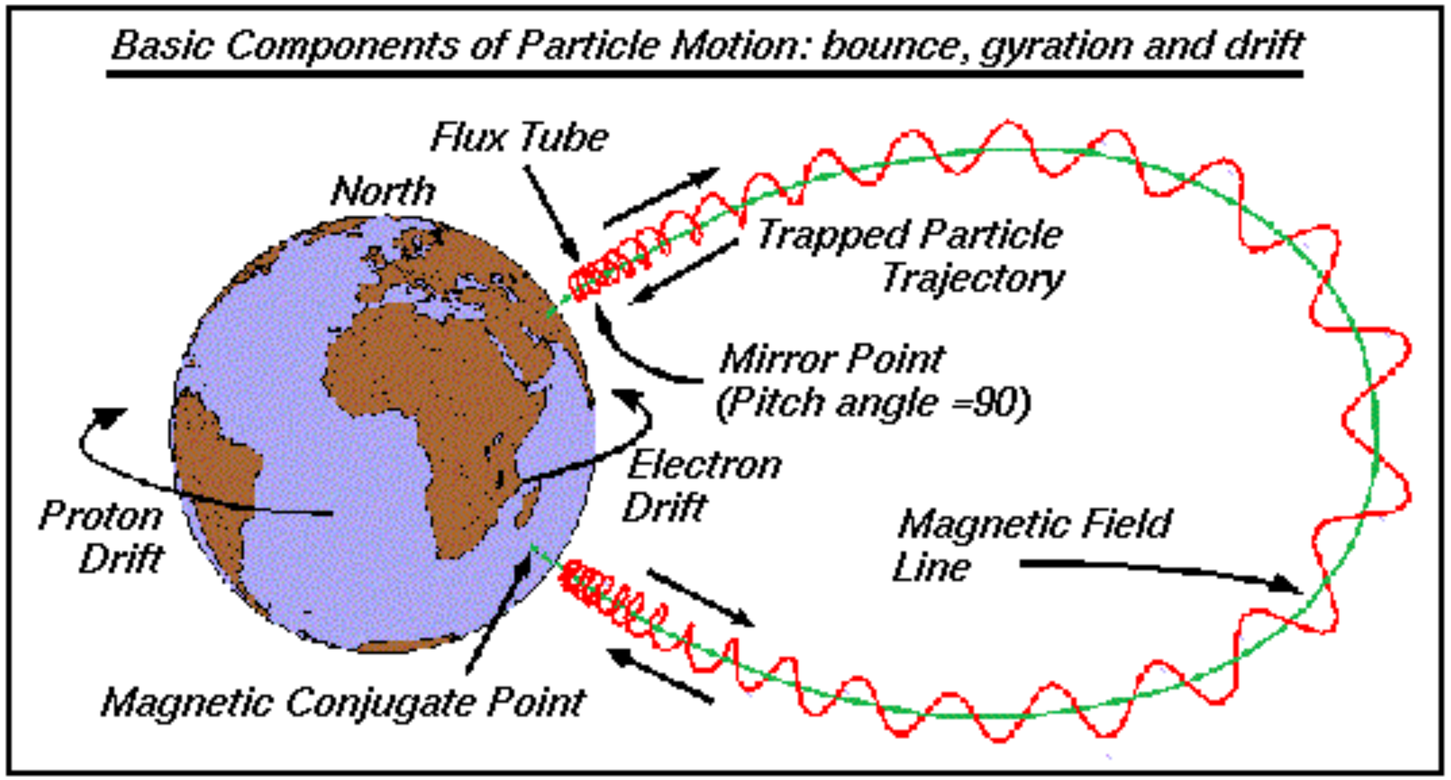
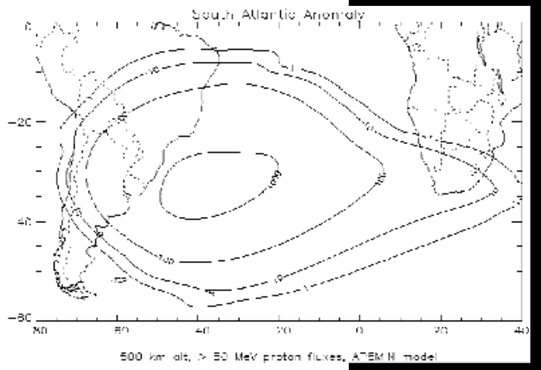
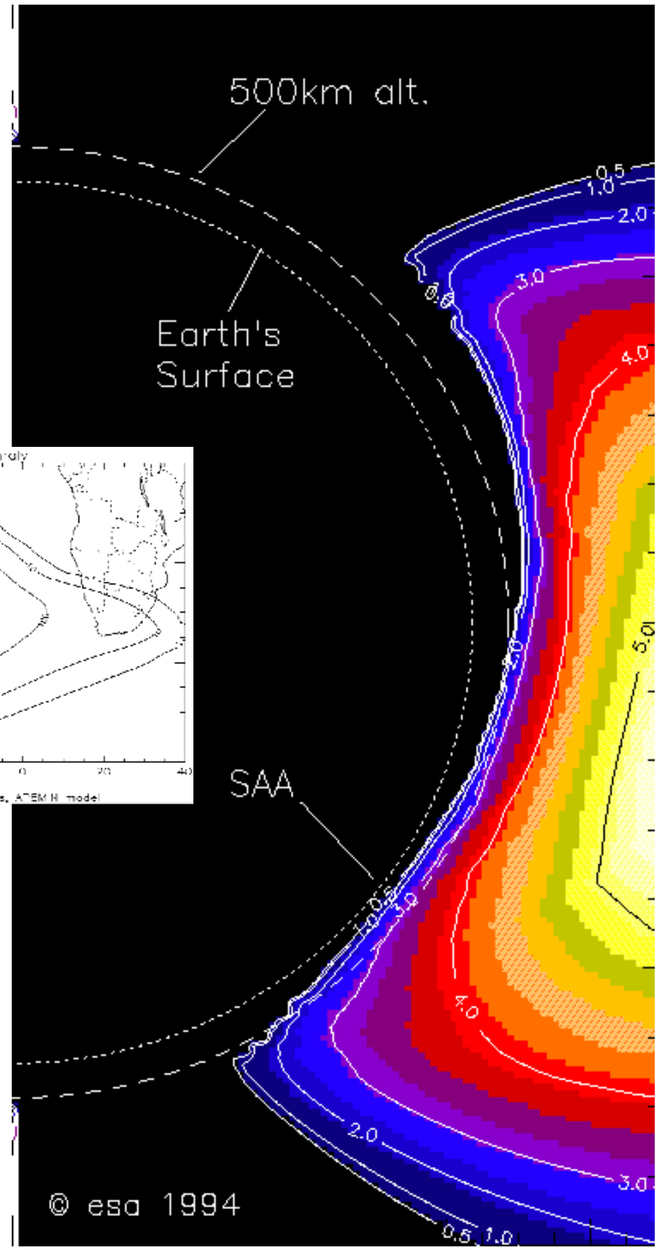


Fig. 7.2







https://image.gsfc.nasa.gov/poetry/tour/vanallen.html

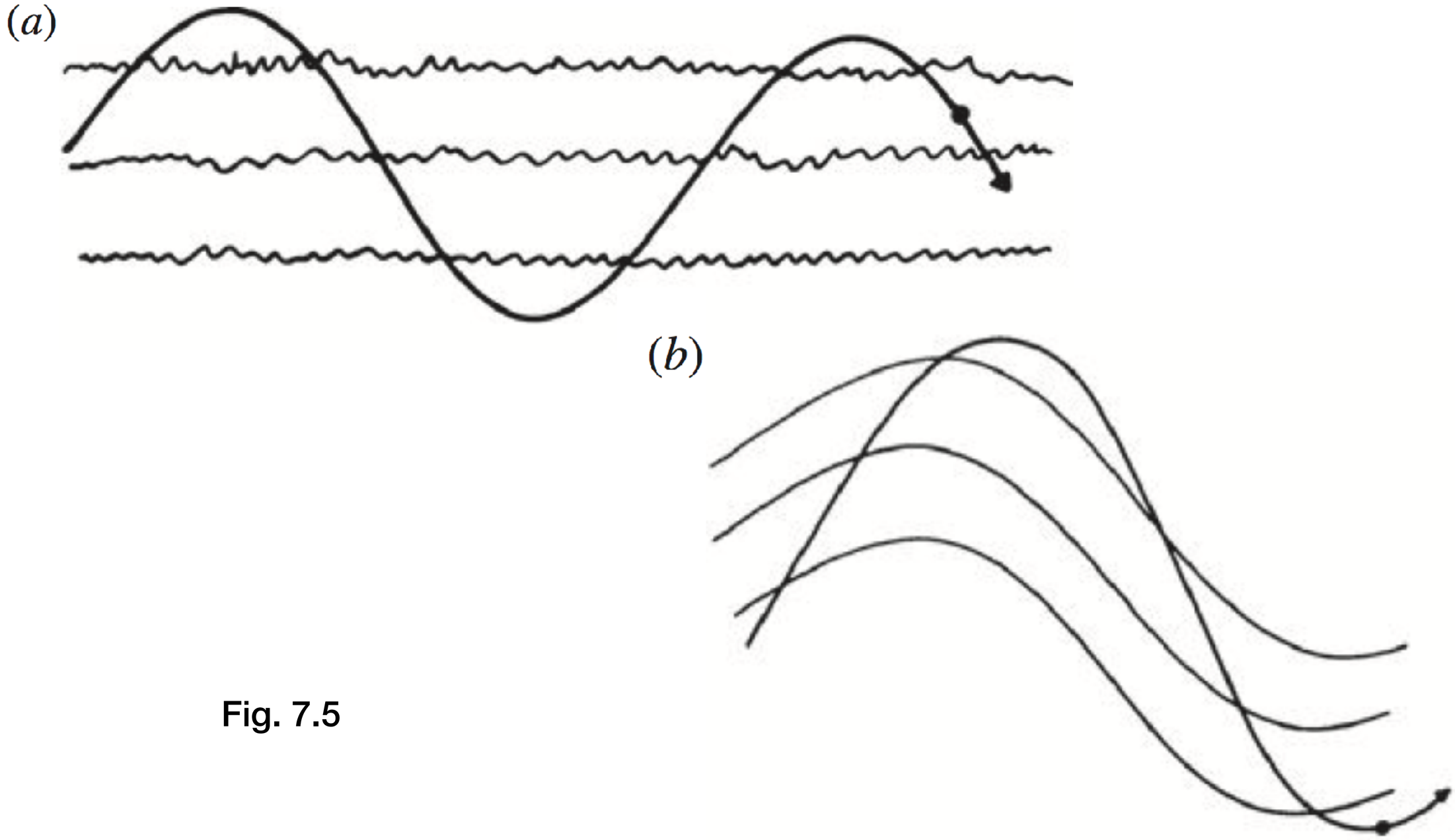


Fig. 7.5

“Swordy Plot”

particle flux of CRs at the Earth

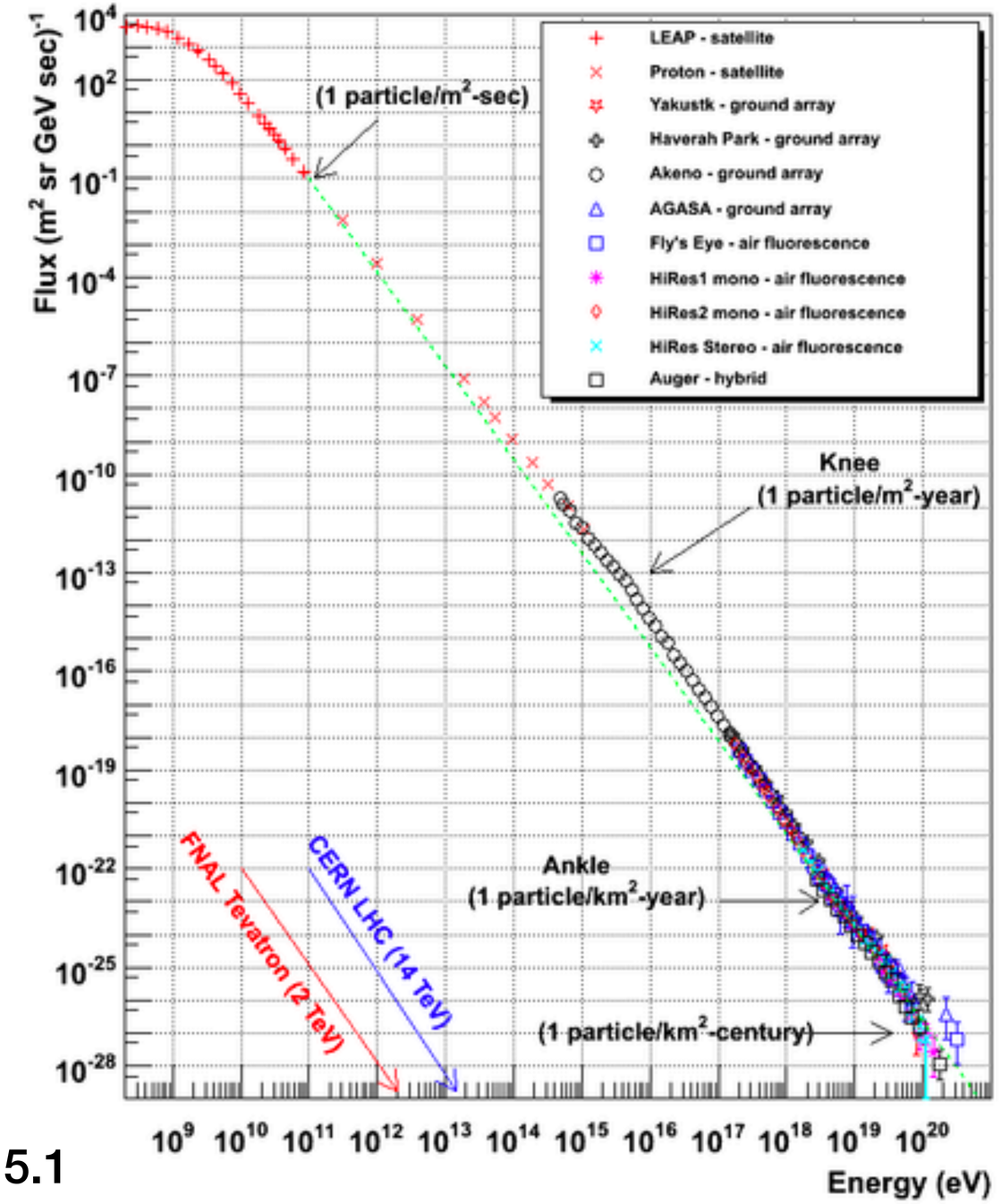


Fig. 15.1

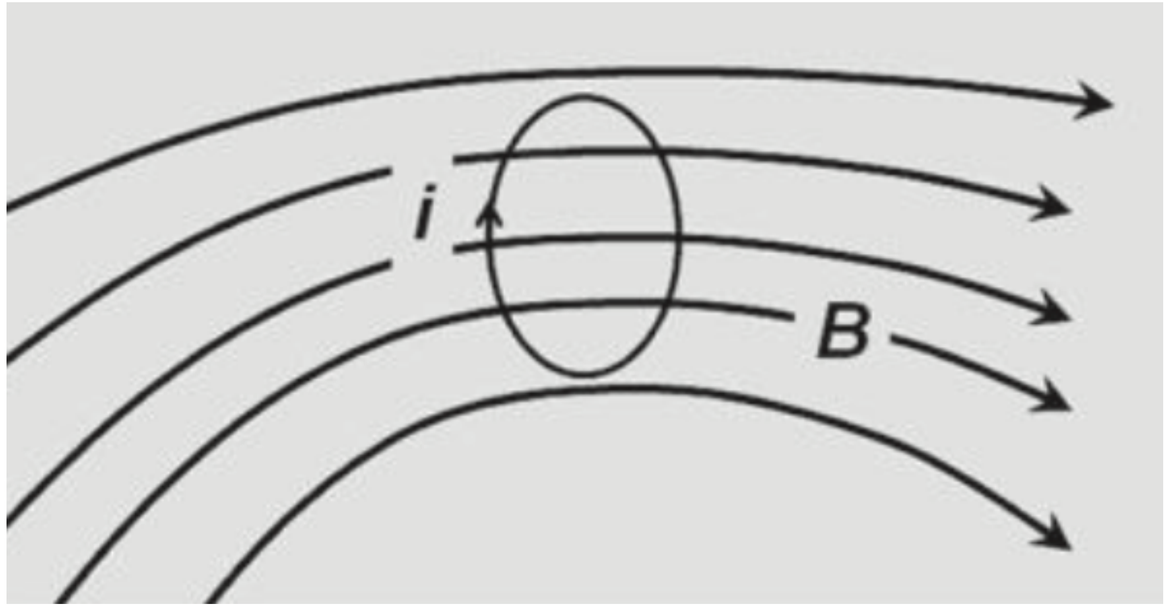


Fig. 11.2

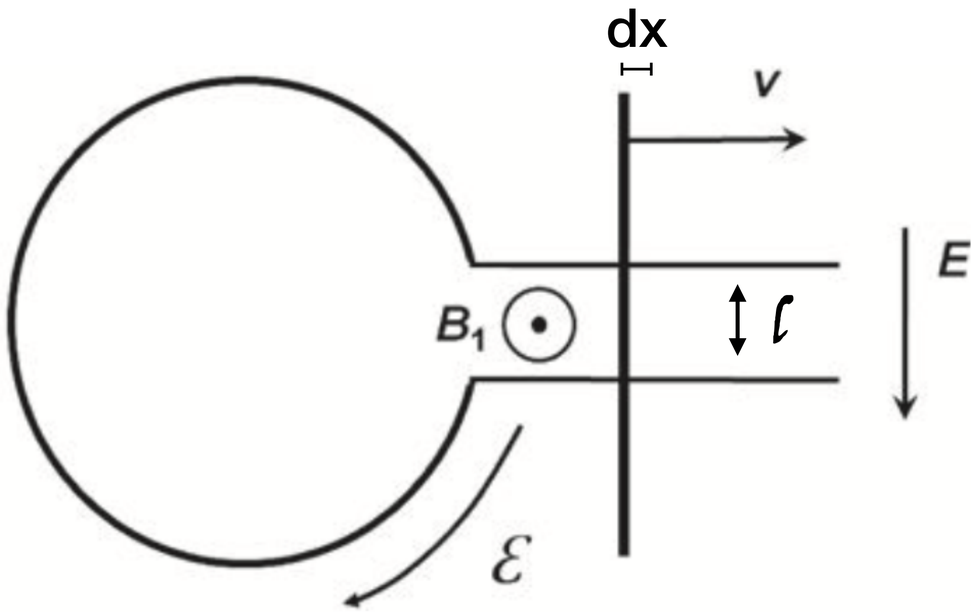


Fig. 11.3

Shock Waves - 2 frames

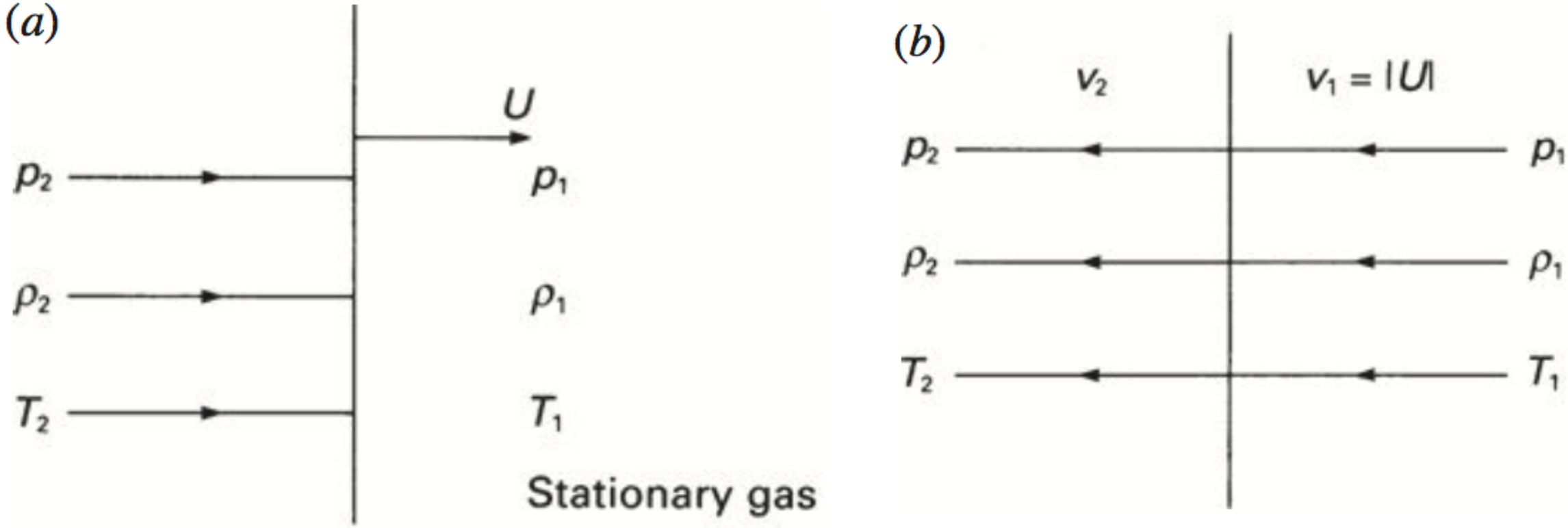


Fig. 11.6