ASTR 5590 - Chapter 11b

Timescales → reaching equilibrium

Recall ion E loss : $-\frac{dE}{dx} \approx \frac{Z^2 e^4 n_e}{2\pi\epsilon_0^2 m_e v^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$

generalize to any 2 particles (ion → 1, e^- → 2)

keep $$-\frac{dE}{dx} = \frac{Z_1^2 Z_2^2 e^4 n_2}{2\pi\epsilon_0^2 m_2 v_1^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

use Λ, generalize to thermal dist.

$$\ln\Lambda \approx 30 + \ln\left[\left(\frac{T}{10^6 K}\right)\left(\frac{n_e}{cm^{-3}}\right)^{-1/2}\right]$$

↳ v. slowly varying, const. effectively

b_{max} → Debye length, b_{min} → classical

HERE →

Now, can write $dE = \Delta KE$, $dx = v\Delta t$

so $$\frac{dE}{dx} = \frac{d\left(\frac{1}{2}m_1 v_1^2\right)}{v_1\, dt} = m_1 \frac{dv_1}{dt}$$

The timescale for a particle to lose its (or gain) energy through interactions is how long it takes to change the fraction of its velocity on the order of its velocity, $\tau_s' \approx \dfrac{v_1}{dv_1/dt}$ → "slowing down time"

Since $\dfrac{dv_1}{dt} = \dfrac{1}{m_1}\dfrac{dE}{dx}$

$$\boxed{\tau_s = \frac{2\pi\epsilon_0^2 m_1 m_2 v_1^3}{Z_1^2 Z_2^2 e^4 n_2 \ln\Lambda}}$$

valid for $m_1 \sim m_2$

If $m_1 \ll m_2$, $\frac{dv}{dt}$ just change in direction,

so $\tau_s \equiv \left[\dfrac{1}{v_1}\dfrac{dv_1(\parallel)}{dt}\right]^{-1}$

$$\simeq \frac{2\pi\epsilon_0^2 m_1 \mu v_1^3}{Z_1^2 Z_2^2 e^4 n_2 \ln\Lambda}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Max-Bolt: $\boxed{\langle v_2^3\rangle \approx \dfrac{3\sqrt{\pi}}{4}\left(\dfrac{2kT}{m_2}\right)^{3/2}}$ roughly

Consider plasma close to equil., $T_e \approx T_p$

$$\tau_s \propto m_1 m_2 v_1^3, \quad \tfrac{1}{2} m_1 \overline{v_1^2} = \tfrac{3}{2}kT$$

e^- moving faster, react 1ST b/c find other e^- more quickly, but they also find ions

Imagine $e^- + p^+$ out of equilibrium, how long does it take them to reach equil.? → collision time! ☆ How would find mean free path?

First, e^- interact both w/themselves + ... roughly similar timescales

p^+, on longer)

$$\boxed{\tau_s(e,e) \gtrsim \tau_s(e,p) \simeq \frac{2\pi \epsilon_0^2 m_e^2 v_e^3}{e^4 n_{e,p} \ln\Lambda}} \propto m_e^{1/2}$$

e^- & p^+ interaction mostly changes direction of e^- only: isotropizes dist.

If e^- out of MB dist., will achieve it again on this timescale

If p^+ out of equilibrium, takes longer for e^- to get back:

$$\tau_s(p,e) \approx \frac{3\pi^{3/2} \epsilon_0^2 m_p m_e}{e^4 n_e \ln\Lambda} \left(\frac{2kT_e}{m_e}\right)^{3/2}$$

$$\approx \left(\frac{m_p}{m_e}\right) \tau_s(e,p)$$

$$\approx 1836\, \tau_s(e,p)$$

Takes much longer due to reduced Δv given to p^+

protons, however, will equilibrate w/ themselves somewhat faster

$$\tau_s(p,p) \approx \left(\frac{m_p}{m_e}\right)^{1/2} \tau_s(e,p) \simeq 43\, \tau_s(e,p)$$

e^- b/c isotropic $t \sim \tau_s(e,e) \sim \tau_s(e,p)$

p^+ b/c isotropic $t \sim \tau_s(p,p) \sim 43\, \tau_s(e,p)$

Equipartition, when E shared by e^- & p^+,
achieved when $T_e \to T_p \Rightarrow \tau_s(p,e)$
$\approx 1836\, \tau_s(e,p)$